MARVIN ZELEN (Editor). Statistical Theory of Reliability. The University of Wisconsin Press, Madison, Wisconsin, 1963. \$5.00 xvii + 166 pages.

## Review by Albert Madansky

## The RAND Corporation

This book is a collection of six expository papers on various major areas of work in the theory of reliability, presented at a seminar conducted by the Mathematics Research Center, United States Army, at the University of Wisconsin, May 8–10, 1962. In his Preface, Zelen says that "... few of the major results [summarized in this volume] have found their way into recent books on probability and statistics." Indeed, it has been the tendency of late for workers in reliability theory to publish their work only in reports for sponsoring agencies, and so few of the major results have even found their way into recent journals. It is for this reason that this book will be welcomed by those concerned with reliability theory.

Though the book is entitled "Statistical Theory of Reliability," only one of the six papers, "Confidence Limits for the Reliability of Complex Systems," by Joan Raup Rosenblatt, deals with a problem of statistical inference in reliability theory. The general problem dealt with by her is that of obtaining both point and interval estimates of the parameter  $R = \Pr\{f(X_1, \dots, X_k) \ge c\}$  when one has a random sample of each of the independent  $X_i$ 's. Letting  $(x_{i1}, \dots, x_{in_i})$  denote the observations on  $X_i$ ,  $i = 1, \dots, k$ , Rosenblatt gives the asymptotic distribution of the unbiased estimate  $\hat{R}$  of R given by

$$\hat{R} = \sum_{i_1=1}^{n_1} \cdots \sum_{i_k=1}^{n_k} \Phi(x_{1i_1}, \cdots, x_{ki_k}) / \prod_{i=1}^k n_i,$$

where

$$\Phi(x_1, \dots, x_k) = 1 \quad \text{if } f(x_1, \dots, x_k) \ge c,$$
  
= 0 otherwise,

and suggests its use in obtaining approximate confidence limits for R. In addition, an extensive bibliography of work on this problem for various functions f and distributions of the  $X_i$  is given.

The first paper in the volume, "A Survey of Some Mathematical Models in the Theory of Reliability," by George H. Weiss, is a fitting introduction to the problems surveyed more extensively in the remaining papers by Barlow, Hunter, Proschan, and Wolman. In this paper, various models of the reliability of a system based on the topological connections of the statistically independent components of the system are first described, one of which is considered in more detail in Proschan's paper. Next follows a description of various time dependent functions which have been used to express the reliability of a component, but with little discussion of their justifications. (In the paper, "Problems in System

Reliability Analysis," Wolman presents a reliability growth model, and so provides another example of a time dependent reliability function, along with a theoretical justification of it.) The final sections of Weiss's paper review some results of renewal theory needed for a study of maintenance policies, and provide at least a bibliographic survey of the work on optimal maintenance policies.

The major part of the book (the papers by Barlow, Hunter, and Proschan, as well as the final section of Weiss's paper) is devoted neither to the probabilistic or topological description of the reliability of a system from component reliability, nor to the problems of statistical inference about reliability, the traditional problems of reliability theory. Proschan's paper, "Redundancy for Reliability Improvement," addresses the question of how to alter a system by use of redundancy to improve its reliability rather than the usual question of what is the system's reliability. The emphasis of Barlow's paper, "Maintenance and Replacement Policies," and Hunter's paper, "Optimum Checking Procedures," is directed to the question of how to alter not the system itself but rather its operation, checkouts, maintenance, and replacement of components in order to improve the expected fractional amount of time the system is operating in an infinite time horizon. These authors primarily summarize the results of their own research on these questions, giving only passing reference to contributions of others.

Also included after each paper is a record of the discussion of the paper which ensued at the seminar.

In summary, the main value of the book is in its role as an annotated bibliography and partial summary of some of the current work in reliability theory.