BOOK REVIEW

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RICHARD E. BARLOW AND FRANK PROSCHAN (with contributions by LARRY C. HUNTER), *Mathematical Theory of Reliability*. John Wiley and Sons, Inc., New York, 1965. \$11.00. XIV + 256 pp.

Review by J. J. McCALL

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This is an excellent book. The spirit of modern reliability theory has been captured and presented in a compact and coherent manner. The need for such a book has been apparent for several years; it is surprising the need has been satisfied so successfully. The methodology utilized throughout is that of applied probability theory. With the exception of an appendix, the statistical properties of reliability problems are not investigated.

Increasing failure rate (IFR) (decreasing failure rate (DFR)) is a concept that recurs throughout the book and in many ways constitutes the unifying theme. The definition of failure rate,

$$(F(t+x) - F(t))/(1 - F(t)), \tag{1}$$

where F denotes the failure distribution, differs slightly from the usual definition

$$f(t)/(1 - F(t)), \tag{2}$$

where f denotes the density function. Of course, (2) is easily derived from (1) if (1) is divided by x and $x \to 0$. "A distribution is IFR (DFR) in t if and only if (1) is increasing (decreasing) in t for $x > 0$, $t \geqq 0$, such that $F(t) < 1$." The authors show that this definition of IFR (DFR) distributions is equivalent to stating that $1 - F(x + y)$ is totally positive of order 2 in x and y for $x + y \geqq 0$. The mathematics of totally positive functions has been developed elsewhere (a brief review is presented in an appendix) and is ingeniously exploited in this monograph.

In the past the exponential distribution, which is the boundary between IFR and DFR distributions, has played a prominent role in reliability analysis. The analytical simplicity of this distribution together with its frequent occurrence account for its popularity. However in some cases this popularity has been unwarranted and the introduction and analysis of distributions with monotone failure rate should cause reliability engineers to re-assess the role of the exponential distribution. Nevertheless, this re-assessment should be careful not to underemphasize the practical importance of exponential distributions. While indeed many equipments possess monotone failure rates, the deviation from exponentiality is often not serious and exponential policies remain good approximations.

The authors neglect the importance of non-monotonic failure rates. Many equipments have failure rates that initially decline, remain constant over an interval and then increase. Operating characteristics and optimal policies for these equipments are, however, not known and the authors should not be reprimanded for their silence. This remains a fertile area for future research.

Much of the monograph is devoted to maintenance policies. Operating characteristics are obtained for a variety of these policies. The characteristics include: the probability that a unit will operate for t hours or more during a specified time interval, the probability distribution of the number of failures during a given period of time and the expected number of failures during a given time interval. Renewal theory plays an important role in these calculations and the authors present a succinct and lucid discussion of the relevant renewal theoretic results. The main use for this analysis is the comparison of alternative maintenance policies. The authors illustrate this by comparing age and block replacement policies. A block replacement policy replaces components as they fail and also replaces *all* components at periodic intervals without regard to their respective ages. An age replacement policy applies a preventive maintenance policy to each component, i.e., each component is replaced at failure or at time T, whichever occurs first. It is shown that if the replacement interval is T for both policies and the failure distributions are identical and IFR, more unfailed components are removed under the block policy, but the expected number of failures over any interval of time is less for the block policy. Useful bounds on the operating characteristics of these and other policies are also obtained. Calculation of these bounds depends critically on the properties of IFR distributions.

An obvious difficulty with the above comparison of block and age replacement policies is that replacement costs are not considered. When they are it is apparent that the interval T will not be the same for both policies. This difficulty is overcome in the chapter that discusses optimal maintenance policies, that is, policies that optimize a particular criterion function. Optimal age replacement policies are derived for infinite and finite time spans; optimal block replacement policies are also derived. These policies are obtained for a single component. Clearly block policies are desirable because of economics of scale in replacement, that is, it is cheaper to replace all components together rather than each separately. This feature of block replacement policies is not examined here, but is an important topic for future research. Optimal sequential replacement policies and optimal inspection policies are treated concisely and rigorously.

Multi-state maintenance policies are analyzed. Each state corresponds to a particular level of deterioration. Markov chains and semi-Markov processes are used to describe the stochastic behavior of a system. A terse but very clear explanation of these Markovian models precedes the analysis. Various operating characteristics are calculated for models with different configurations of machines, service facilities (repairmen) and spares. The analysis makes use of birth and death processes and embedded semi-Markov processes. The discussion of these models points out another interesting area for further research. In particular, preventive maintenance is not permitted in these models—the machines are repaired at failure. Designing optimal maintenance policies for several machines must cope with the problem of queuing.

The problem of optimal redundancy is treated in a slightly different fashion from the other topics. Over thirty pages are devoted to this problem; the pace is more leisurely than in the other chapters and there are more numerical examples. Since this is an extremely important practical problem, I hope that reliability engineers will survive to this point. From a pedogogical perspective, this topic should have been discussed earlier.

The final chapter presents a theoretical analysis of multi-component structures. The work of Moore and Shannon regarding the use of relatively unreliable components to achieve arbitrarily high reliabilities is summarized. Several generalizations are also investigated.

Undoubtedly many practicing reliability engineers will find this book too theorem-oriented and will surrender early in their reading. Since this book is so rich in potential applications, this is regrettable, but unavoidable. However, graduate students in reliability or operations research who evince similar grumblings should move to a different school or change their field of interest.

In brief then, the monograph is a review of the recent journal literature—much of it by the authors—on probabilistic models in reliability. Many novel results are also included and an ingenious use of total positivity unifies topics that appear unrelated. The book contains an admirable mix of theorems and examples. It should serve quite well as a supplementary text to a course in applied probability or as the basic text for a graduate course in reliability. Readers of this journal should find the book stimulating. Serious students of reliability and operations research should find it invaluable.