

## BOOK REVIEWS

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N. U. PRABHU. *Queues and Inventories*. Wiley Series in Probability and Mathematical Statistics. J. Wiley and Sons, Inc. New York, 1965. xii + 275 pp. \$14.00

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In this book Professor Prabhu, formerly of the University of Western Australia and now of Cornell, has provided us with a very clear and mathematically unified account of the stochastic process theory of queues, inventories, and dam storage. These subjects now have such a vast literature that it would not be possible to write an encyclopedic account in one volume of less than three hundred pages. The author has wisely chosen to give an account of the mathematical theory using a unified notation and approach, ignoring practical or numerical illustrations and some of the more specialised queueing problems. In spite of these restrictions a very large amount of ground is covered, and this is achieved by concentrating on the mathematics which is described in a very clear, accurate and concise manner.

The first chapter begins with a description of queueing problems in general. Using the Kolmogorov equations the system  $M/M/1$  is studied and the Laplace transform of the generating function of the transition probabilities for the queue length is obtained. The limiting distribution, the busy period, the waiting time and Champernowne's combinatorial theory are then studied. Many server queues are introduced and other extensions such as  $M/E_k/1$ ,  $E_k/M/1$ , and  $M/D/s$  analysed in the same way. Imbedded Markov chains, introduced by D. G. Kendall, are used to analyse  $M/G/1$  and  $G1/M/1$ , although the main theory of these queues is held over till Chapter 4. Finally the waiting time distributions for  $M/G/1$  and  $G1/G/1$  are studied by integral and integro-differential equations.

Chapter 2 is a systematic account of the transient behaviour of  $M/G/1$  and  $G1/M/1$ , particularly their idle and busy periods, and various special cases such as  $M/D/1$ ,  $M/M/1$  and  $M/E_k/1$ . As in the first chapter there is also a long series of exercises giving various interesting special results.

Chapter 3 reconsiders  $M/G/1$  and  $G1/M/1$  by using imbedded Markov chains, and studying the first passage time distributions. An interesting duality theorem is then proved between queueing systems in which the inter-arrival time distribution and the service time distribution are interchanged.

Chapter 4 studies the more general system  $GI/G/1$  on the basis of the Spitzer-

Pollaczek identity which is proved by using Feller's method of ladder indices. The author does not seem aware that this theorem was first proved by Pollaczek (*Comptes Rendus*, Paris, **234**, (1952) 2334–2336). Using this theorem the transforms of the busy period and busy cycle are obtained, and the waiting time. The results are then specialised to the cases  $E_k/G/1$  and  $GI/E_k/1$ . Bulk service is also studied.

The next chapter gives a relatively short introduction to inventories and collective risk theory, which are more closely related than queues to the subject of the last two chapters which is dam storage.

Chapter 6 is concerned with dam storage i.e. the theory of a store in which the input is random and the output depends on a rule of release. This chapter deals with the resulting stochastic process in the case where time is taken as discrete. The stationary distribution and the distribution of time to first emptiness can then be found by using Markov process theory either for inputs which have discrete distributions or continuous ones. Both finite and infinite storages can be studied in this way. This theory can be linked up with the theory of queues by identifying the waiting time in a queue with the amount stored in a dam.

More elaborate theory is necessary in the last chapter which deals with continuous time models since the input is now a homogeneous additive process and the equations describing such processes require a more sophisticated analysis.

This book is written in a mathematically rigorous style which is nevertheless easy to read and makes an excellent and comprehensive introduction to a thriving field of research. It should be essential reading for all concerned with random processes which arise in problems of congestion or storage.