CORRECTION NOTE

CORRECTION TO

ON THE ASYMPTOTIC THEORY OF FIXED-SIZE SEQUENTIAL CONFIDENCE BOUNDS FOR LINEAR REGRESSION PARAMETERS[1]

By Leon Jay Gleser

The Johns Hopkins University

In the above titled article (*Ann. Math. Statist.* **36** 463–467) the following corrections should be made in order to correct formulas and rectify several omissions:

(1) On page 463, the assumption that the ϵ_i in Equation (2.1) have 0 means was inadvertently omitted and should be inserted. In the fourth line under Equation (2.1) replace $1 - \alpha$ by α.

(2) On page 464 Equation (2.4) should read

$$\hat{\sigma}^2(n) = n^{-1} Y_n (I_n - X_n'(X_n X_n')^{-1} X_n) Y_n'. \tag{2.4}$$

(3) Equation (3.6) should read:

$$\lim_{n\to\infty} P\{n(\hat{\beta}(n) - \beta)(\hat{\beta}(n) - \beta)' \leqq d^2\} = P\{T(\lambda_1, \cdots, \lambda_p) \leqq d^2/\sigma^2\} \tag{3.6}$$

and in the first line of the proof of Corollary 3.3 we should have $X_n X_n'$ replaced by $(X_n X_n')^{\frac{1}{2}}$.

(4) On page 466, line 3, replace n^{-1} by n.

(5) On page 467, in Remarks 1 and 2 the references to Equation (4.1) should instead be references to Equation (4.2). In line 1 of the proof of Theorem 4.1, the n^{-1} before $\hat{\sigma}^2(n)$ should be deleted.

I am indebted to Professor R. A. Wijsman's review [4] of my paper for pointing out most of these errors and omissions. He also has drawn my attention to the inadequacy of two of my proofs—namely, the proof of (4.4) in Theorem 4.1 and the proof of Theorem 3.4.

As Wijsman points out [4], I have not given a clear indication as to how the results of Anscombe [1] are to be applied to prove (4.4). Rather than give my original proof, I will follow the line of attack suggested by Wijsman [4]. We prove first:

Lemma 1. *Let* $z_1, z_2, \cdots$ *be* i.i.d. *with zero means and unit variance. Let* $a_1, a_2, \cdots$ *be any sequence of numbers such that* $n^{-1}\sum_{i=1}^n a_i^2 \to 1$ *and* $n^{-1} \max_{i\le n} a_i^2 \to 0$ *as* $n \to \infty$. *Define* $u_n = n^{-\frac{1}{2}}\sum_{i=1}^n a_i z_i$ *and let* $N = N(t)$ *be a positive integer-*

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valued random variable such that $N(t)/t \to 1$ in probability as $t \to \infty$. Then u_N has a limiting standard normal distribution as $t \to \infty$.

PROOF. This lemma is an immediate corollary of Theorem 2 of Mogyoródi [3] (my thanks to Professor J. Gastwirth for suggesting this reference to replace my own bulky proof).

Now from Assumptions 3.1 and 3.2 in my paper and Lemma 1, we have that

$$\lim_{d\to 0} \mathcal{L}(N^{-\frac{1}{2}}Z_N X_N' c') = \mathfrak{N}(0, c\Sigma c')$$

for all $1 \times p$ vectors c, $cc' = 1$. Thus

$$\lim_{d\to 0} \mathcal{L}(N^{-\frac{1}{2}}Z_N X_N') = \mathfrak{N}(0, \Sigma),$$

which in turn implies that Equation (4.4) in Theorem 4.1 (of my paper) holds.

The proof of Theorem 3.4 in my paper is in error—the mistake lies in assuming that $U_n \to 0$ as $n \to \infty$ implies that $Z_n U_n' U_n Z_n = Z_n Z_n' o(1)$. However, as a result of my attempts to provide a correct proof for this theoıem, I have been able to prove an even stronger version of the theorem. The basis for this stronger theorem is the following lemma which has recently been proved by Chow [2].

LEMMA 2. *Let $z_1, z_2, \cdots$ be independent, identically distributed random variables with means $Ez_i = 0$ and variances $Ez_i^2 = \sigma^2$, $0 < \sigma^2 < \infty$. Let b_{nm} be any array of real numbers $m \leqq n$, $n = 1, 2, \cdots$, satisfyıng*

$$\lim_{n\to\infty} \textstyle\sum_{m=1}^{n} b_{nm}^2 = 1.$$

Then

$$n^{-\frac{1}{2}} \textstyle\sum_{m=1}^{n} b_{nm} z_m \to 0 \quad \text{a.s.} \qquad \textit{as } n \to \infty.$$

From this lemma we obtain the following strengthened version of Theorem 3.4:

THEOREM 3.4′. *The classical estimate $\hat{\sigma}^2(n)$ of σ^2 in the univariate linear regression problem is strongly consistent,* i.e. $\hat{\sigma}^2(n) \to \sigma^2$ a.s. *as $n \to \infty$.*

PROOF. Since for $Z_n = Y_n - EY_n$ we have $\hat{\sigma}^2(n) = n^{-1}Z_n(I - U_n'U_n)Z_n'$, and since by the strong law of large numbers $n^{-1}Z_nZ_n' \to \sigma^2$ a.s. as $n \to \infty$, we need only show that

$$\text{(i)} \qquad n^{-1}Z_n U_n' U_n Z_n' \to 0 \quad \text{a.s.}$$

But letting $U_n = (u_{n\cdot ij})$, $Z_n = (z_1, \cdots, z_n)$,

$$n^{-1}Z_n U_n' U_n Z_n = \textstyle\sum_{i=1}^{p} (n^{-\frac{1}{2}} \sum_{j=1}^{n} u_{n\cdot ij} z_j)^2.$$

Now since $U_n = (X_n X_n')^{-\frac{1}{2}} X_n$, it follows that $U_n U_n' = I_p$ or

$$\textstyle\sum_{j=1}^{n} u_{n\cdot ij}^2 = 1, \qquad \text{all } i, \text{ all } n.$$

Applying Lemma 2, we obtain

$$n^{-\frac{1}{2}} \textstyle\sum_{j=1}^{n} u_{n\cdot ij} z_j \to 0 \quad \text{a.s.,} \qquad i = 1, \cdots p,$$

from which the desired result (i) follows.

REFERENCES

[1] Anscombe, F. J. (1952). Large sample theory of sequential estimation. *Proc. Cambridge Philos. Soc.* **48** 600–607.

[2] Chow, Y. S. Some convergence theorems for independent random variables. Mimeograph Series No. 63, Department of Statistics, Purdue University.

[3] Mogyoródi, J. (1961). On limiting distributions for sums of a random number of independent random variables. *Publ. Math. Inst. Hungar. Acad. Sci.* **6** 365–371.

[4] Wijsman, R. A. (1965). Review #2643 in *Math. Reviews* **30** 508–509.