PATRICK BILLINGSLEY, *Ergodic Theory and Information*. John Wiley and Sons, Inc., New York, 1965. $8.50. xiii + 189 pp.

Review by ARSHAG HAJIAN

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This is an interesting book on the limited number of topics from ergodic theory and information theory which it treats. The book is written in a very loose fashion, and the general tone is that of an expository presentation. It seems that the author has chosen a few topics from the general subjects of ergodic theory and information theory and presented them in a quite free and conversational style. In this sense the book is useful to a beginner in the field. Many of the so called "well known" and simple examples, results, and observations which do not find their way in the literature ordinarily, can be found scattered throughout in the book. It tries in a successful way to motivate many notions and techniques in the field by a great number of examples and interesting side discussions. As a supplement to the literature in the subject the book serves a useful purpose. If the reader is aware of the restricted number of topics discussed and supplements this by the existing literature he will find the book to be quite useful besides being enjoyable to read. The material presented is well treated, and great pains are taken throughout to make the book self contained.

In Chapter 1 a measure preserving transformation T defined on a finite measure space $(\Omega, \mathfrak{F}, \mathcal{P})$ is introduced, and a fair number of examples are discussed. These examples, even though most of them quite trivial, are used effectively to exhibit and motivate the standard definitions that are made in the subject. Notions of ergodicity and mixing for a measure preserving transformation are introduced in a natural manner. The Ergodic Theorem is motivated first by showing a simple case, and then both the Mean Ergodic and Individual Ergodic Theorems are proven. The proofs presented are the usual, and by now the classical ones, except that the author tries to show to the reader the significance of some of the steps that enter into the proofs. The chapter is concluded with some interesting and useful applications of the Ergodic Theorem to various other topics. The discussion on continued fractions and diophantine approximations is particularly interesting. The connection of ergodic theory to these topics is presented in an interesting style.

Chapter 2 introduces the notion of isomorphism between measure preserving transformations. The interrelation among the various concepts of isomorphism, conjugacy, and spectral isomorphism is discussed in great detail by many examples and comments. Finally the notion of entropy is introduced as an isomorphism invariant. The Kolmogorov-Sinai Theorem is then proven and used in calculating the entropy of the shift transformations. The chapter concludes with some extensions to Kolmogorov shifts and a number of interesting unsolved problems connected with the concept of entropy in ergodic theory. Most of these

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problems are the outcome of the extensive work done by the Russian school in the late fifties and early sixties on the important role of entropy in ergodic theory.

Chapter 3 is an exposition of conditional probability and expectation. Some of the basic properties and elementary facts of convergence of conditional expectations are discussed. This chapter is intended mainly to present the necessary material used in the proofs of the main theorems where notions of conditional probability and martingale convergence theorems are utilized.

In Chapter 4 the notion of conditional entropy is generalized with respect to an arbitrary σ-subfield. A detailed proof of the Shannon-McMillan-Breiman Theorem on the convergence of conditional entropy is given. The chapter ends with an interesting interrelation between Hausdorff dimension and entropy.

The last chapter is on coding theorems of information theory. The author gives some of the main theorems in the subject; the presentation is somewhat scanty and does not seem to have much connection with the rest of the book.