# CORRECTION NOTE

## CORRECTION TO

## "ON THE BLOCK STRUCTURE OF CERTAIN PARTIALLY BALANCED INCOMPLETE BLOCK DESIGNS"

By S. M. Shah

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My attention has been drawn by Professor W. H. Clatworthy, Department of Mathematics Statistics, State University of New York at Buffalo, to the fact that the Theorems 2.1, 2.4, 3.1, 3.3, 3.4, 4.1, 4.4, 5.1 and 5.4 of my paper published in *Ann. Math. Statist.* **37** 1016–1020 are vacuous in the sense that the designs about which they are concerned do not exist. I wish to express my thanks to him for this.

# BOOK REVIEWS

*Correspondence concerning reviews should be addressed to the Book Review Editor, Professor James F. Hannan, Department of Statistics, Michigan State University, East Lansing, Michigan 48823.*

J. Neveu, *Mathematical Foundations of the Calculus of Probability*. Translated by Amiel Feinstein. Holden-Day, Inc., San Francisco, 1966. xiii + 233 pp. $9.50.

Review by R. M. Blumenthal

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In a relatively small amount of space this book presents the part of measure theory most relevant to study and research in modern probability theory, and also presents the most important parts of ergodic theory and martingale theory. The presentation is elegant, but with a reasonable allowance for human frailty.

Anyone intending to learn out of this book must be able to read graduate-level mathematics. Also it will be helpful if he has some intuitive background from an elementary course in probability or statistics. If he is this well-equipped, he will find the study of this book an enjoyable and profitable experience. Having mastered the material (supplemented by a bit of extra work on characteristic functions) he will be prepared for reading most of the current research papers in probability as well as many of those in mathematical statistics.

The book is suitable for use as a course textbook or for individual study. There are problems at the end of each section, usually containing important information. The problems are not short or easy, but the most interesting ones are broken up into short steps, so that most serious readers will be able, ultimately, to solve them.

In Chapter 1 the author introduces events and their probabilities axiomatically. He proves various theorems on the extension of a measure on an algebra or semi-algebra to a measure on a larger $\sigma$-algebra. One ends up with the extension theorem stated in the form most useful for application to product spaces. Distribution functions on $R$ and (in an exercise) on $R^n$ are discussed.

Chapter 2 is a short course on measurable functions and integration, arranged for maximum usefulness to probabilists. It contains a very nice section on measures defined on the Baire sets or Borel sets of a topological space including the theorem of Daniell and F. Riesz on the representation of certain linear functionals. This is a topic that no longer should be omitted from measure theory courses for probabilists.

Chapter 3 begins with a careful construction of the probability measure on a product space $\Omega_1 \times \Omega_2$ arising from a probability $P$ on $\Omega_1$ and a transition function $P(\omega_1, A_2)$, $\omega_1 \varepsilon \Omega_1$, $A_2 \subset \Omega_2$. Fubini's theorem emerges as a special case. Then follows a rather general form of Kolmogorov's theorem on the existence of a stochastic process (of function space type) having prescribed finite dimensional distributions. The Gaussian case is outlined in an extensive exercise. Next comes a nice discussion of separability and measurability of stochastic processes and criteria for sample function regularity of separable processes. There is, for example, a detailed outline of the theorem that the sample functions of separable Brownian motion are Lipschitz of order $\frac{1}{2} - \epsilon$. The chapter closes with a tightly written section on stopping times. All the facts necessary for further developments are presented, but there is not enough background to permit discussion of the important examples from continuous parameter Markov processes.

Chapter 4 starts with a brisk, but thorough, survey of absolute continuity and the Banach space aspects of $L_p$. Then comes conditional expectation and independence. The rest of the chapter is devoted to martingale theory and applications. The main emphasis is on the fact that a submartingale remains one under optional sampling. All the important convergence theorems for submartingale sequences and the sample function properties of continuous parameter submartingales are presented, and the proofs are constructed so as to feature optional sampling. Many important topics (symmetric dependence, derivatives) are treated in the exercises. The general theory is then applied to the discussion of almost sure convergence for (normalized) sums of independent random variables.

Chapter 5 is on ergodic theory. The author begins with a careful construction (using Ionescu Tulcea's theorem) of a Markov process (discrete time, arbitrary state space) with prescribed initial distribution and transition function. Then follows a concise but complete development of the convergence and decomposition theorems when Doeblin's classical condition is satisfied. The rest of the chapter contains an extensive treatment of the pointwise ergodic theorem for positive endomorphisms of $L_1$ of norm $\leqq 1$. This of course includes the decomposition of the state space into conservative and dissipative parts, the Chacon-Ornstein theorem on convergence of operator averages, and many supplements to these results.