ON THE NON-EXISTENCE OF A FIXED SAMPLE ESTIMATOR OF THE MEAN OF A LOG-NORMAL DISTRIBUTION HAVING A PRESCRIBED PROPORTIONAL CLOSENESS

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In Section 2 of [2] a heuristic argument is given for the non-existence of a fixed sample procedure which can guarantee the prescribed closeness condition. This argument is however unsatisfactory, and we give here a rigorous proof. We first remark that the problem of estimating the mean, ξ, of a log-normal distribution in a manner which guarantees a prescribed closeness condition is equivalent to the problem of estimating $\mu + \sigma^2/2$, in the case the observations have a $\mathfrak{N}(\mu, \sigma^2)$ distribution law, with an interval estimator of a fixed width, 2δ say. Generalizing, we wish to prove that there is no fixed sample interval estimator procedure for $\mu + f(\sigma)$, $f(\sigma)$ being any finite real-valued function of σ, which can guarantee a prescribed confidence level for a system of intervals of a fixed width. Let $Y_1, \cdots, Y_n$ be iid random variables, having a $\mathfrak{N}(\mu, \sigma^2)$ distribution law. Let $\psi(Y_1, \cdots, Y_n)$ designate a midpoint statistic for a system of confidence interwals for $\mu + f(\sigma)$, of width 2δ. We show that for every ψ,

$$\inf_{\mu,\sigma} P_{\mu,\sigma}\{|\psi(Y_1, \cdots, Y_n) - \mu - f(\sigma)| < \delta\} = 0 \tag{1}$$

Indeed, for a given value of σ, the minimax mid-point statistic for fixed width interval estimator of $\mu + f(\sigma)$ is $\bar{Y}_n + f(\sigma)$, where $\bar{Y}_n = n^{-1} \sum_{i=1}^{n} Y_i$ (see J. Wolfowitz [1]). Hence,

$$\begin{aligned} &\inf_{\mu,\sigma} P_{\mu,\sigma}\{|\psi(Y_1, \cdots, Y_n) - \mu - f(\sigma)| < \delta\} \\ &\leqq \lim_{\sigma\to\infty} \sup_{\psi} \inf_{\mu} P_{\mu,\sigma}\{|\psi(Y_1, \cdots, Y_n) - \mu - f(\sigma)| < \delta\} \\ &= \lim_{\sigma\to\infty} P\{|U| < \delta(n)^{\frac{1}{2}}/\sigma\} = 0, \end{aligned} \tag{2}$$

where U is a random variable having a $\mathfrak{N}(0, 1)$ distribution law.

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