The Annals of Mathematical Statistics
1968, Vol. 39, No. 1, 290–293

BOOK REVIEWS

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HARALD CRAMÉR AND M. R. LEADBETTER, *Stationary and Related Stochastic Processes/Sample Function Properties and Their Applications.* John Wiley and Sons, New York, 1967. vii + 348 pp. $12.50

Review by DONALD YLVISAKER

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There is a body of literature on stochastic processes, subject to sporadic growth, which pertains to second order and, more especially, normal or Gaussian processes and to their sample paths. That this area develops unevenly is attributable first to its customary location outside the main sphere of interest in probability and second, to a notable lack of tools for solving its hard problems. After presenting more standard material on the structure of such processes, this book deals primarily with their sample function behaviour. While the general level of presentation is not carried to the extent of, say, reproducing the delicate sample results known about Brownian motions (such results belonging more properly in Markov processes books), one does find a fairly complete treatment of the smoothness and crossing properties of normal sample functions. The fundamental problem of determining necessary and sufficient conditions that a stationary normal process possess continuous paths has received some attention but has seen little real progress since the 1951 paper of Hunt. An account of the relevant activity is given here. Problems revolving around the level crossings and intervals between level crossings of normal paths have fared somewhat better and the book contains many recent results in this direction. These latter problems had their mathematical genesis in the remarkable paper of Rice in 1945 and, in a more real than superficial sense, the authors give an updating of the situation after 20 years. Inasmuch as a goodly portion of the material in the book appears in this form for the first time, a noticeable gap in the literature has been filled.

The first eight chapters are devoted to mostly standard results about probability and stochastic processes in general, and second order processes in particular. This was apparently done to make the book more self-contained in view of a prospective audience of communications engineers and probabilists. The reader should be already familiar with measure theoretic probability although it is possible that the non-mathematician with some probability training can persevere if armed with experience and intuition. The authors have taken pains to write a readable book and they have succeeded. The relative novice will benefit especially from this.

In the first three chapters, the reader is carried from the empirical background of probability through a measure theoretic introduction to probability and thence to stochastic processes. This development is, on the one hand, very terse and, on the other, well organized and readable. Chapter 4 presents sample function properties of general processes. Worthy of mention is the treatment of (for instance) sample continuity. The emphasis here is on showing that there exists a process satisfying certain conditions and having continuous paths and not on showing that a separable process satisfying certain conditions has continuous paths (indeed, separability is only briefly mentioned). This is carried off quite nicely and the authors need spend little time over obscuring details.

Chapters 5 through 8 are concerned with second order processes and include such topics as the quadratic mean calculus, Hilbert space geometry and spectral representations along with a passing nod at prediction theory. There is also a proof of the individual ergodic theorem and a short survey of such things as vector processes and homogeneous random fields. The choice of material provides strong evidence of personal taste but this can hardly be faulted in such a compact presentation. The resulting segment of the book is at least as coherent as other accounts presently available.

The main business of the book begins in Chapter 9 with the analytic properties of normal sample functions. It is interesting to find here a readable proof of the continuity of sample paths subject to the correlation function r satisfying $r(h) = 1 - O(|\log|h||^{-a})$ as $h \to 0$ for some $a > 3$. Hunt's difficult paper allows $a > 1$. Chapters 10 through 13 contain the recent results on crossing problems. This entire area has benefitted greatly from the authors' active interest and participation. The principal result of Chapter 10 is the formula for the factorial moments of the number of times a stationary normal process assumes the value u in an interval of length T. Chapter 11 treats these u values as a stationary stream of events, introduces distribution functions for the lengths of time between various events, and then demonstrates that the distribution functions are, in suitable circumstances, limits of appropriate relative frequencies as $T \to \infty$. Chapter 12 gives the asymptotic Poisson character of the stream of upcrossings of the level u by a stationary normal process as $u \to \infty$, and considers also the limiting distribution of extreme values of such a process as $T \to \infty$. In Chapter 13, some of the above problems are discussed after the requirement of stationarity is discarded. The book concludes with two chapters of applications.

The book is handsome in appearance and apparently contains few misprints. Although not designed as a text book and containing no formal problems, the book can serve as a text. The student will find it to be a good exposition of this corner of probability theory. The engineer with an interest in crossing problems will find here about as many answers as are known. For the interested probabilist, it should serve as a handy reference book and, hopefully, a spur to further activity.