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CONSTRUCTION OF ROOM SQUARES

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1. Introduction. The particular design which has since become known as a Room square or a Room design was first introduced by T. G. Room in a brief note [4]. We shall introduce a notation slightly different from that used by Room. We take the $2n$ symbols $1, 2, 3, \cdots, 2n-1, \infty$. Then a Room design of order $2n$ consists of a square of side $2n-1$ with each compartment of the square either being blank or containing an unordered pair of the symbols $1, 2, 3, \cdots, 2n-1, \infty$. Furthermore, each row and each column of the square contains $n-1$ empty cells and n cells containing a pair of symbols; the totality of symbols appearing in each row and in each column is just the total number, $2n$, of symbols. We further require that each of the $n(2n-1)$ possible distinct pairs of symbols shall occur exactly once in a cell of the square.

We shall also find it useful to refer to a Room design of order $2n$ as a Room square of side $2n-1$. Room squares may thus exist only for odd integers $k = 2n-1$.

Room's initial note pointed out that a Room square existed trivially for $k = 1$, did not exist for $k = 3$ or 5, did exist for $k = 7$. Archbold and Johnson [2] gave a construction for $k = 7, 31, 127, \cdots$, that is, for any k which is less by unity than an odd power of 2; they also pointed out the applications of Room squares in statistical design and sketched the appropriate analysis of variance for such a design. Archbold [1] gave a different construction, based on difference sets, which produced squares of side $k = 7, 11, 19, 23$; again this method failed for $k = 15$, just as the earlier method had. Bruck [3] pointed out the connection between Room squares and quasigroups, gave an elegant new construction for the squares given by Archbold and Johnson [2], and proved that if squares of sides a and b existed, then one could use a type of Kronecker product to get a square of side ab; in particular, this gave a square of side 49. Finally, Weisner [5] constructed a square of side 9.

The preceding background sketch shows that relatively few Room squares are known. For instance, the only values of k less than 100 for which squares have been constructed are $k = 7, 9, 11, 19, 23, 31, 49, 63, 77, 81, 99$. It is the aim of the present article to outline a method whereby it appears that Room squares of any side k (k odd, $k > 5$) may be constructed; the actual construction has been carried out for all odd numbers k from 7 to 47 inclusive. Empirical evidence suggests that there exists a very large number of Room squares of given side k.

2. Construction of a cyclic Room square of side 11. We shall illustrate the method employed by discussing the case $k = 11$ in detail. We shall restrict ourselves to the construction of cyclic Room squares. These are squares in which the

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entry in cell $(i+1, j+1)$ is found by taking the entry in cell (i, j) and adding 1 modulo 11, where one employs the convention that $\infty + 1 = \infty$. Also, the cells are numbered modulo 11; thus, if one has cell (4, 11), then cell (4 + 1, 11 + 1) is cell (5, 1).

The first step is to write down six pairs of numbers with the following properties. They include all the 11 numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, t, e, and the symbol ∞. The six differences in the pairs are congruent modulo 11 to 1, 2, 3, 4, 5, ∞. We do not consider differences greater than 5, since the pairs in the cells of a Room square are not ordered. Alternatively, we may consider the absolute values of the differences $x - y$, where x and y occur in the same cell of the Room square

Such a set of pairs can be written down in many ways. Suppose that we select the set 14, 27, 35, 6t, 89, ∞ e; the differences for these pairs are respectively 3, 5, 2, 4, 1, ∞. We at once obtain the obvious

LEMMA 1. *The* 66 *pairs obtained by adding modulo* 11 *to these six initial pairs are all distinct, and hence are just the* 66 *different pairs obtainable from the* 12 *given symbols.*

For instance, the given set generates 25, 38, 46, 7e, 9t, ∞ 1; etc. If we use the six pairs just found as the first row of a cyclic Room square, then we get all succeeding rows by successive additions of 1 modulo 11. However, we can not just place these entries at random in the first row, since we also have to guarantee that the columns also possess the Room property. It is clear, because of the cyclic generation of the square, that if one column has the requisite property, then so have all the other columns; let us look at the last column (the eleventh).

It is convenient to keep ∞ upon the main diagonal; so we place (∞, e) in position (1, 1) and add diagonally; this puts the Room square in what may be called standard form; the eleventh column then consists of the following entries:

$$(\infty, e) + 10 = (\infty, t); \quad (1, 4) + x; \quad (2, 7) + y;$$
$$(3, 5) + z; \quad (6, t) + u; \quad (8, 9) + v,$$

where x, y, z, u, v, are distinct numbers chosen from among the integers 0, 1, 2, $\cdots$, 9. In order to determine a suitable set of five such numbers, we make the following table.

	8 9	6 t	3 5	2 7	1 4
0	8 9	—	3 5	2 7	1 4
1	—	7 e	4 6	3 8	2 5
2	—	8 1	5 7	4 9	3 6
3	e 1	9 2	6 8	—	4 7
4	1 2	—	7 9	6 e	5 8
5	2 3	e 4	—	7 1	6 9
6	3 4	1 5	9 e	8 2	—
7	4 5	2 6	—	9 3	8 e
8	5 6	3 7	e 2	—	9 1
9	6 7	4 8	1 3	e 5	—

The entries in column 11 must be five pairs chosen from this array under the restrictions that:

(a) the five pairs give all the ten symbols 1, 2, $\cdots$, 9, e (note that ∞ and t already occur in column 11); (b) there must be one entry chosen from each of the five columns of this array; (c) there can not be more than one entry from each of the rows.

It is easy to find a solution. Suppose that we select the pair (6, 7) from the first column; this cuts our remaining choices down to the following array (since both 6 and 7 now appear).

—	—	35	—	14
—	—	—	38	25
—	81	—	49	—
—	92	—	—	—
—	—	—	—	58
—	e4	—	—	—
—	15	9e	82	—
—	—	—	93	8e
—	—	e2	—	91
67	—	—	—	—

From the third column, we must either take 35 or an entry containing e. If we try 35, then we can exclude seven more entries, namely, 15, 9e, e2, 38, 93, 25, 58. This leaves only two choices from column four; if we try 49, there is left no permissible choice from column 2. So we must select 82 from column 4; this choice forces us to take e 4 from column 2 and 91 from column 5, and we thus have a solution. We need to take the following choices of x, y, z, u, v.

$$(8, 9) + 9; \quad (6, t) + 5; \quad (3, 5) + 0; \quad (2, 7) + 6; \quad (1, 4) + 8;$$

to give a final column of (6, 7), (e, 4), (3, 5), (8, 2), (9, 1). The adders are just the amounts needed to transport the given pair to the eleventh column. Hence the first row of the Room square is simply

$$\infty\ e\ 89\ 14 — 27\ 6t — — — — 35.$$

Since the square is cyclic, we need only state the first row; that determines the rest of the square by diagonal addition modulo 11.

Actually, the above square was worked out to illustrate the method; the square originally obtained by the same method was a square with first row

$$\infty\ e\ 35 — 89\ 27\ 14 — — — 6t —.$$

This square, of course, has the same entries in each row (a necessity of the method, once the first row is prescribed), but the pattern of blanks is different.

3. Application to other values of k. The method illustrated in detail in the last section depends upon two things. One of these is that we must be able to find a "starter", that is, a set of pairs for the first row. Such a starter must consist of n pairs such that the absolute differences formed from the pairs are the numbers $1, 2, 3, \cdots, n-1, \infty$. We immediately find

Lemma 2. *The $2n - 1$ rows formed from such a starter by addition of successive 1's modulo $2n - 1$ have the property that we obtain $n(2n - 1)$ distinct pairs, and each row obtained contains exactly $2n$ distinct symbols.*

It is clear that such a starter can always be written down, and usually there are many such. For example ($k = 7$), if we standardize the first row by fixing the entry $(\infty, 5)$ in position (1, 1), then there are only 15 possible ways of dividing up the symbols 1, 2, 3, 4, 6, 7, into 3 pairs. Of these 15 possibilities, only three satisfy the conditions for a starter. These are:

12, 37, 46; 16, 23, 47; 17, 24, 36.

If we select another standardization by placing $(\infty, 7)$ in position (1, 1), these three starters become (add 2)

34, 52, 61; 31, 45, 62; 32, 46, 51.

To show that a starter always exists, we note

Lemma 3. *The set of pairs given by $(n - 1, n)$, $(n - 2, n + 1)$, $(n - 3, n + 2)$, $(n - 4, n + 3)$, $\cdots$, $(1, 2n - 2)$, together with $(\infty, 2n - 1)$ has the property that the absolute differences formed are the numbers* $1, 3, 5, 7, \cdots, 6, 4, 2, \infty$.

For example, when $k = 13$, this lemma gives the starter as (6, 7), (5, 8), (4, 9), (3, 10), (2, 11), (1, 12), $(\infty, 13)$ with successive differences as 1, 3, 5, 6, 4, 2, ∞.

The particular starter given by Lemma 3 has the property that each pair has a constant sum (13 in the illustration). Another way of obtaining a starter for the case when k is a prime can easily be described. Suppose that a is a primitive element in the Galois field on k symbols; then the field can be written as 0, a, a^2, a^3, $\cdots$, $a^{k-1} = 1$. We obtain

Lemma 4. *The pairs* (a, a^n), (a^2, a^{n+1}), (a^3, a^{n+2}), $\cdots$, (a^{n-1}, a^{2n-2}), $(\infty, 2n - 1)$ *have the properties required by a starter.*

Actually, since $a^{n-1} = -1$, we see that these pairs are just the pairs $(a, -a)$, $(a^2, -a^2)$, $(a^3, -a^3)$, etc; they have the property that the two entries in a pair are of constant sum, and are actually just the starter obtained in Lemma 3, but written in a different order. For instance, for $k = 17$, this method gives the starter (using 3 as primitive element) (3, 14), (9, 8), (10, 7), (13, 4), (5, 12), (15, 2), (11, 6), (16, 1), $(\infty, 17)$. If a^t is another primitive element, then the pairs obtained from it are just $(a^t, -a^t)$, $(a^{2t}, -a^{2t})$, $\cdots$, etc. These all have the form $(j, -j)$, and so we have

Lemma 5. *The set of starter pairs obtained in Lemma 4 is independent of the particular primitive element a selected.*

Having seen that we can always select a starter, we now turn to the second problem. We can not write the pairs of a starter in any random position in the first row. Indeed, if we put the entry (a, b) in position u in the first row, then $k - u$ is the amount which must be added (modulo k) to obtain the element generated in the last column of the square. So we need to find a set of adders for any starter.

For example, consider the starter $(\infty, 17)$, (3, 14), (9, 8), $\cdots$, given as an example for $k = 17$ immediately after Lemma 4. We agree to the standardization whereby $(\infty, 17)$ appears in the first row and first column; we then find that we can define the following adder:

$$(3, 14) + 3; \quad (9, 8) + 12; \quad (10, 7) + 5; \quad (13, 4) + 1;$$
$$(5, 12) + 6; \quad (15, 2) + 11; \quad (11, 6) + 13; \quad (16, 1) + 9.$$

The adder has the property that all the integers used are distinct; also, all the resulting pairs obtained are distinct; and the element 16 does not occur since $(\infty, 16)$ is known to be the entry in position (k, k).

From the adder, we at once deduce that (3, 14) appears in the 14th position of the first row; (9, 8) appears in the fifth position of the first row; (10, 7) appears in the twelfth position of the first row; (13, 4) appears in the sixteenth position of the first row; etc. The elements generated in the last column are then (6, 17) in the third position of the last column, (4, 3) in the twelfth position of the last column, etc.

We thus have illustrated the use of a starter and an adder to find a cyclic Room square of side 17. A starter is always predictable (of course, there are many other possibilities besides the patterned starters we have described); the adder can be found by easy trial as in the last section. Hand application of this method produced the following Room squares (those for 13, 15, 17, are new, and it might be mentioned that the methods of [1] and [2] both broke down for $k = 15$). In all cases, we write only the first row of the square.

$k = 7$ — ∞ 6 — — 14 — 75 23

$k = 9$ — The use of a patterned starter produces no square; this point will be discussed again in a later section when we consider the number of Room squares obtained from the particular patterned starter we have mentioned. However, another starter produces ∞ 9 58 37 — — — — 46 12

$k = 11$ — ∞ 5 — 91 — — — 46 82 *te* — 37 (using $t = 10$, $e = 11$)

$k = 13$ — ∞ 12 2 9 — 3 8 — — — 1 10 5 6 13 11 — 4 7 —

$k = 15$ — ∞ 3 5 10 — — — 13 1 — — 9 15 7 8 — 2 6 12 14 — 4 11 (notice that here we did not use a patterned starter)

$k = 17$ — ∞ 17 — — 11 6 9 8 15 2 — 16 1 — — 5 12 10 7 — 3 14 — 13 4 —

Once one has reached this size of square, hand application of the methods so far described becomes a bit tedious. Consequently, in the next section, we shall describe the adaptation of the method for computer application, and will give the first rows of cyclic Room squares for all orders up to and including 47.

4. Computer construction of Room squares. Suppose that r is the number which is paired with ∞ in the starting row. Then the sum of all the other $2n - 2 = k - 1$ numbers (omitting ∞ and r) is simply $\frac{1}{2}k(k + 1) - r$. In the patterned starters which we have described in the last section, all pairs have the same internal sum modulo k; for instance, at the end of the last section, the pairs in row 1 have sum 10 for $k = 11$, sum 11 for $k = 13$, sum 14 for $k = 15$, sum 17 for $k = 17$. In general, this sum is equal to

$$[\tfrac{1}{2}k(k + 1) - r]/\tfrac{1}{2}(k - 1) \equiv 2r \bmod k.$$

Now let T_i be the pair (x, y) with internal difference of i; we at once find that we can write T_i as $(r - \frac{1}{2}i, r + \frac{1}{2}i)$.

As mentioned before, these pairs are taken modulo k, and we now require a sequence of integers a_i such that the column image of T_i (that is, the last column of the square) is suitable. This image is $T_i' = (r - \frac{1}{2}i + a_i, r + \frac{1}{2}i + a_i)$ modulo k. Clearly we must impose the conditions $a_i \neq a_j$; $T_i' \cap T_j' = 0$ for $i \neq j$; these impose the restriction

$$a_i \neq a_j \pm \tfrac{1}{2}(i \pm j) \quad \text{modulo} \quad k.$$

This results in an algorithm which is awkward for hand computation but which is easily programmed in Fortran. The results obtained are given below. The value of k in each case is one greater than the number of symbols in the patterned starter.

PATTERNED STARTER
(1 2) (4 6) (7 3)

POSSIBLE COLUMN IS
(1 2) (5 7) (3 6)

The above print-out for $k = 7$ must be interpreted in the following way: the pairs (1 2), (4 6), and (7 3) appear in the first row; they must appear in those positions which will, on diagonal addition, place them respectively in the positions giving (1 2), (5 7), and (3 6) in the last column. Of course, the pair (∞ 5) appears in the first position in the first row. Hence the other pairs appear respectively in positions $7 - 0 = 7, 7 - 1 = 6$, and $7 - 3 = 4$ of the first row. A similar interpretation applies to all succeeding print-outs.

PATTERNED STARTER
(9 1) (4 6) (8 2) (3 7)
NO POSSIBLE COLUMN

PATTERNED STARTER
(10 11) (4 6) (9 1) (3 7) (8 2)
POSSIBLE COLUMN IS
(1 2) (8 10) (6 9) (3 7) (11 5)

PATTERNED STARTER
(11 12) (4 6) (10 13) (3 7) (9 1) (2 8)
POSSIBLE COLUMN IS
(12 13) (6 8) (2 5) (7 11) (9 1) (10 3)

PATTERNED STARTER
(12 13) (4 6) (11 14) (3 7) (10 15) (2 8) (9 1)
POSSIBLE COLUMN IS
(13 14) (9 11) (3 6) (12 1) (5 10) (2 8) (15 7)

PATTERNED STARTER
(13 14) (4 6) (12 15) (3 7) (11 16) (2 8) (10 17) (1 9)
POSSIBLE COLUMN IS
(14 15) (6 8) (10 13) (16 3) (2 7) (11 17) (5 12) (1 9)

PATTERNED STARTER
(14 15) (4 6) (13 16) (3 7) (12 17) (2 8) (11 18) (1 9) (10 19)
POSSIBLE COLUMN IS
(15 16) (6 8) (17 1) (9 13) (5 10) (12 18) (19 7) (14 3) (2 11)

PATTERNED STARTER
(15 16) (4 6) (14 17) (3 7) (13 18) (2 8) (12 19) (1 9) (11 20) (21 10)

POSSIBLE COLUMN IS
(16 17) (6 8) (18 21) (11 15) (19 3) (7 13) (5 12) (1 9) (14 2) (10 20)

PATTERNED STARTER
(16 17) (4 6) (15 18) (3 7) (14 19) (2 8) (13 20) (1 9) (12 21) (23 10) (11 22)
POSSIBLE COLUMN IS
(17 18) (6 8) (19 22) (10 14) (20 2) (15 21) (5 12) (1 9) (7 16) (3 13) (23 11)

PATTERNED STARTER
(17 18) (4 6) (16 19) (3 7) (15 20) (2 8) (14 21) (1 9) (13 22) (25 10) (12 23) (24 11)
POSSIBLE COLUMN IS
(18 19) (6 8) (20 23) (9 13) (11 16) (21 2) (3 10) (17 25) (5 14) (22 7) (15 1) (12 24)

PATTERNED STARTER
(18 19) (4 6) (17 20) (3 7) (16 21) (2 8) (15 22) (1 9) (14 23) (27 10) (13 24) (26 11) (12 25)
POSSIBLE COLUMN IS
(19 20) (6 8) (21 24) (9 13) (2 7) (12 18) (10 17) (22 3) (23 5) (15 25) (16 27) (26 11) (1 14)

PATTERNED STARTER
(19 20) (4 6) (18 21) (3 7) (17 22) (2 8) (16 23) (1 9) (15 24) (29 10) (14 25) (28 11) (13 26) (27 12)
POSSIBLE COLUMN IS
(20 21) (6 8) (22 25) (9 13) (24 29) (5 11) (16 23) (28 7) (10 19) (17 27) (1 12) (14 26) (2 16) (18 3)

PATTERNED STARTER
(20 21) (4 6) (19 22) (3 7) (18 23) (2 8) (17 24) (1 9) (16 25) (31 10) (15 26) (30 11) (14 27) (29 12) (13 28)
POSSIBLE COLUMN IS
(21 22) (6 8) (23 26) (9 13) (25 30) (10 16) (7 14) (11 19) (27 5) (24 3) (20 31) (17 29) (2 15) (18 1) (28 12)

PATTERNED STARTER
(21 22) (4 6) (20 23) (3 7) (19 24) (2 8) (18 25) (1 9) (17 26) (33 10) (16 27) (32 11) (15 28) (31 12) (14 29) (30 13)
POSSIBLE COLUMN IS
(22 23) (6 8) (24 27) (9 13) (26 31) (5 11) (28 2) (10 18) (3 12) (15 25) (21 32) (17 29) (7 20) (19 33) (1 16) (14 30)

PATTERNED STARTER
(22 23) (4 6) (21 24) (3 7) (20 25) (2 8) (19 26) (1 9) (18 27) (35 10) (17 28) (34 11) (16 29) (33 12) (15 30) (32 13) (14 31)
POSSIBLE COLUMN IS
(23 24) (6 8) (25 28) (9 13) (27 32) (5 11) (30 2) (18 26) (7 16) (10 20) (1 12) (17 29) (21 34) (19 33) (35 15) (22 3) (14 31)

PATTERNED STARTER
(23 24) (4 6) (22 25) (3 7) (21 26) (2 8) (20 27) (1 9) (19 28) (37 10) (18 29) (36 11) (17 30) (35 12) (16 31) (34 13) (15 32) (33 14)
POSSIBLE COLUMN IS
(24 25) (6 8) (26 29) (9 13) (28 33) (5 11) (30 37) (27 35) (14 23) (12 22) (36 10) (19 31) (7 20) (3 17) (1 16) (18 34) (15 32) (21 2)

PATTERNED STARTER
(24 25) (4 6) (23 26) (3 7) (22 27) (2 8) (21 28) (1 9) (20 29) (39 10) (19 30) (38 11) (18 31) (37 12) (17 32) (36 13) (16 33) (35 14) (15 34)

POSSIBLE COLUMN IS
(25 26) (6 8) (27 30) (9 13) (29 34) (5 11) (31 38) (12 20) (32 2) (18 28) (10 21) (24 36) (1 14) (3 17) (7 22) (23 39) (16 33) (19 37) (35 15)
PATTERNED STARTER
(25 26) (4 6) (24 27) (3 7) (23 28) (2 8) (22 29) (1 9) (21 30) (41 10) (20 31) (40 11) (19 32) (39 12) (18 33) (38 13) (17 34) (37 14) (16 35) (36 15)
POSSIBLE COLUMN IS
(26 27) (6 8) (28 31) (9 13) (30 35) (5 11) (32 39) (10 18) (33 1) (15 25) (12 23) (36 7) (16 29) (20 34) (2 17) (24 40) (38 14) (19 37) (3 22) (21 41)
PATTERNED STARTER
(26 27) (4 6) (25 28) (3 7) (24 29) (2 8) (23 30) (1 9) (22 31) (43 10) (21 32) (42 11) (20 33) (41 12) (19 34) (40 13) (18 35) (39 14) (17 36) (38 15) (16 37)
POSSIBLE COLUMN IS
(27 28) (6 8) (29 32) (9 13) (31 36) (5 11) (33 40) (10 18) (30 39) (16 26) (14 25) (22 34) (37 7) (3 17) (43 15) (19 35) (38 12) (23 41) (2 21) (24 1) (42 20)
PATTERNED STARTER
(27 28) (4 6) (26 29) (3 7) (25 30) (2 8) (24 31) (1 9) (23 32) (45 10) (22 33) (44 11) (21 34) (43 12) (20 35) (42 13) (19 36) (41 14) (18 37) (40 15) (17 38) (39 16)
POSSIBLE COLUMN IS
(28 29) (6 8) (30 33) (9 13) (32 37) (5 11) (34 41) (10 18) (31 40) (12 22) (14 25) (23 35) (39 7) (3 17) (45 15) (27 43) (19 36) (24 42) (2 21) (26 1) (44 20) (16 38)
PATTERNED STARTER
(28 29) (4 6) (27 30) (3 7) (26 31) (2 8) (25 32) (1 9) (24 33) (47 10) (23 34) (46 11) (22 35) (45 12) (21 36) (44 13) (20 37) (43 14) (19 38) (42 15) (18 39) (41 16) (17 40)
POSSIBLE COLUMN IS
(29 30) (6 8) (31 34) (9 13) (33 38) (5 11) (35 42) (10 18) (32 41) (12 22) (17 28) (36 1) (7 20) (25 39) (47 15) (21 37) (2 19) (27 45) (44 16) (23 43) (40 14) (24 46) (3 26)

5. Concluding Remarks. We have now demonstrated that Room squares exist for all odd numbers k between 7 and 49 inclusive. The Bruck result on the existence of a square of side ab, given squares of sides a and b, now settles many other cases. And there is impressive empirical evidence that Room squares of side k exist for all odd k greater than 5.

In this connection we cite the following data. We have concentrated not on the whole class of Room squares, but only on the subclass of cyclic Room squares And the computer algorithm was written to find only a special kind of cyclic Room square, namely, the squares which can be obtained from our "patterned starter." Let us call these "patterned Room squares" or PRS. Then the algorithm mentioned produced the following results for PRS; in each case, the Room square was standardized so that the entry in position (1, 1) was ∞, 5.

Value of k	Number of PRS
7	2
9	0
11	4
13	8
15	44
17	416
19	The programme was turned off after the production of 967 PRS.

It would appear that the number of PRS goes to infinity very rapidly.

If we seek some data on the number of cyclic Room squares (CRS), we have to allow all possible starters. This is easily done. Suppose that we put $\infty\ k$ in position (1, 1). Then we can write down all possible pairings from the integers $1, 2, 3, \cdots, k - 1$. There are, for example, only 15 such pairings for $k = 7$ and 105 for $k = 9$. The general formula is easily written down as

$$\binom{k-1}{2}\binom{k-3}{2}\binom{k-5}{2} \cdots \binom{4}{2}\binom{2}{2}/\tfrac{1}{2}(k-1)! = 2s!/2^s s!$$

where $2s = k - 1$. Most of these pairings do not give starters; for example, of the fifteen pairings for $k = 7$, namely:

(1 2) (3 4) (5 6); (1 2) (3 5) (4 6); (1 2) (3 6) (4 5);
(1 3) (2 4) (5 6); (1 3) (2 5) (4 6); (1 3) (2 6) (4 5);
(1 4) (2 3) (5 6); (1 4) (2 5) (3 6); (1 4) (2 6) (3 5);
(1 5) (2 3) (4 6); (1 5) (2 4) (3 6); (1 5) (2 6) (3 4);
(1 6) (2 3) (4 5); (1 6) (2 4) (3 5); (1 6) (2 5) (3 4);

it is clear that only three of them give possible starters for CRS, namely,

(1 3) (2 6) (4 5); (1 5) (2 3) (4 6); (1 6) (2 5) (3 4).

These three starters can then be used to generate CRS; the total number of CRS for small values of k is given by the following table.

Value of k	Number of Starters	Number of CRS
7	3	6
9	9	12
11	25	80
13	133	1760

To prove that a Room square of side k exists for any odd k, one method would be to establish the existence of squares of the following sides: (a) p, where p is an odd prime greater than 5; (b) $3m$, where m is prime to 3; (c) $5n$, where n is prime to 5; (d) 9, 27, 25, 125. The result would then follow from the fact that any odd integer can be written in the form 3^a5^bc, where c is a product of odd primes not involving 3 or 5. It is hoped to discuss this matter in a later paper.

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