

suggested in exercises that direct simple proofs can be given for special cases. No mention is made of stable laws. Chapter 7 contains a good discussion of conditional expectation and discrete parameter martingales. Important martingale convergence theorems are given, but unfortunately no examples are included to demonstrate how useful and powerful these theorems are. Chapter 8, the last one, contains an introduction to stochastic processes with particular emphasis on Brownian motion. The motivation leading to the notion of separability of a stochastic process is excellent, and one would wish the other chapters had some similar discussion and motivation for the theorems before the author embarked on the Lemma–Theorem–Corollary sequence. Various well-known properties of Brownian motion sample paths are given in this chapter, including the law of the iterated logarithm.

The book is remarkably free of errors and misprints. The proofs of theorems and corollaries are complete and very well arranged. The author's emphasis on rigor is clearly visible in his full treatment of the logarithm of a nonvanishing characteristic function and all its relevant properties (Chapter 4). A graduate student with a good background of measure theory should be able to follow the text without much difficulty; but he may never realize the importance of a particular theorem in the course. Exercises are interesting and given after every section in good textbook style; the usefulness of the book would definitely increase if more had been included. Considering the fact that there is perhaps no such thing as an ideal textbook this book should serve very well as a base for a one-year graduate course and is a good addition to the field of probability.

R. M. BLUMENTHAL AND R. K. GETOOR. *Markov Processes and Potential Theory*. Academic Press Inc., 1969. x + 313 pp. \$15.00.

Review by HARRY DYM  
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Blumenthal and Gettoor are highly skilled practitioners in the fine art of probabilistic potential theory. Their book has already been warmly welcomed by P. A. Meyer [*Bull. Amer. Math. Soc.* **75** (1969) 912–916] and will undoubtedly be warmly welcomed by other workers in the field. The praise is deserved. The book is carefully written, and will surely serve as a basic reference on Markov processes and potential theory for years to come.

The book is organized as follows: The first chapter introduces the theory of Markov processes, and includes a very lucid account of stopping times. Chapter 2 deals with excessive functions, exceptional sets, and the fine topology. The succeeding chapters take up in turn multiplicative functionals and subprocesses, additive functionals and their potentials, local times, processes with identical hitting distributions and dual processes.

These topics are developed comprehensively, without shirking of technical detail. Consequently the book is a valuable source of information for the specialist or the

reader with a strong interest in one of the topics under consideration. The casual reader may find the going a bit sticky in spots. [If you have seen one  $\sigma$ -algebra you have not seen them all!] There is, however, a good index of notation which makes it possible to skip around and absorb information more or less at will.

In spite of the fact that this is essentially a research monograph it does contain some excellent introductory material and numerous exercises. Still the choice of topics, and the relative technical complexity of the material treated seem to make the book most suitable for the advanced reader. That is to say the reader who already has some familiarity with Markov processes, especially with generators and the stochastic versions of such classical potential theory problems as the Dirichlet problem. The authors have omitted these topics on the grounds that they are already well covered in the treatises of Dynkin and Ito–McKean. This is certainly a reasonable point of view, especially as knowledge of these topics is not a prerequisite for mastering Blumenthal and Gettoor, which is remarkably self-contained. To my mind it is more a question of priorities. One really ought to have this sort of material under his belt before tackling Blumenthal and Gettoor. In addition to the above-mentioned sources a compact and eminently readable introduction to these basics is to be found in: *Lecture Notes on Stochastic Processes* by K. Ito. (mimeographed by the Tata Institute and unfortunately unavailable). These last remarks are somewhat beside the main point which is that Blumenthal and Gettoor have written an impressive and important book.