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BOOK REVIEW

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S. VAJDA. *Patterns and Configurations in Finite Spaces. Number 22 of Griffins Statistical Monographs and Courses.* Hafner Publishing Company, New York, 1967. vii + 120 pp. $4.95.

S. VAJDA. *The Mathematics of Experimental Design. Number 23 of Griffins Statistical Monographs and Courses.* Hafner Publishing Company, New York, 1967. vii + 110 pp. $4.75.

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These two companion volumes are intended to cover only the combinatorial aspects of the construction of designs, without regard to their practical applications, analysis or indeed other usefulness.

Patterns and Configurations in Finite Spaces (**1**) contains four chapters. Chapter I, briefly, covers finite groups, permutation groups, algebra $A[s]$, finite fields, Hadamard matrices and difference sets. Chapters II and III deal with some fundamentals of finite planes and finite spaces of higher dimensions. Configurations are the subject of Chapter IV. Examples and problems with their solutions are provided throughout the text.

The Mathematics of Experimental Design (**2**) has five chapters. In Chapter I, Introduction, the concepts of finite groups, finite fields, finite projective geometries, finite Euclidean spaces and difference sets are very briefly discussed. This introductory review makes this volume almost independent of the preceding one. Chapters II and III are concerned with the construction of balanced incomplete blocks (BIB), orthogonal Latin squares and orthogonal arrays. Methods of construction of partially balanced incomplete blocks are given in Chapter IV. Chapter V discusses partially balanced incomplete block designs with two associate classes, covers group divisible designs, triangular association scheme, and Latin square type design. Again examples and problems together with their solution are given throughout the book.

The subject-matter covered by these two books is probably adequate as an introduction to the combinatorial aspects of design of experiments. The level of both books is such that a fairly broad readership could benefit from them. The mathematical knowledge assumed is very little indeed. Some knowledge of college algebra and some familiarity with the geometrical concepts, together with a sincere interest in the subject are certainly sufficient.

The major defect in these two books is the lack of precision—there are incorrect statements in definitions and theorems. The theorem of Pappus on page 32(**1**) as stated is incorrect. The classical and accepted definition of BIB designs does not rule out the possibility of identical blocks in the design. However, on page 7(**2**) the author restricts the definition of BIB to have no identical blocks. This certainly has no place in the statistical/combinatorial application of the subject. As a side remark we should mention that the same criticism applies to the reference [7]. On page 23(**2**) the author has equated the concept of Latin rectangle with Youden square. It is true that every Youden square is a Latin rectangle. However, the converse is trivially false. On page 76(**1**) he defines Youden square differently—this definition is standard.

Several concepts have been referred to repeatedly in (**1**), however, the author has never defined these concepts. For example, the definition of isomorphism of two different sets is not provided, however, it has been referred to several times on page 22(**1**). Having defined the idea of a multiplier of a difference set on page 23(**1**), the author could easily, with not much additional space, define the ismorphism concept of two difference sets. On page 40(**1**) the term "isomorphism concept of two finite projective planes" is used without being defined.

The choice and locations of examples are poor. Some places call for an example, yet none is provided. For instance, on page 20(**1**) there should be at least one example for the construction of an Hadamard matrix of order 4 or 12 by utilizing $GF(3)$ or $GF(5)$ respectively. Unfortunately the four problems provided at the end of the corresponding chapter do not solve this deficiency. In this set of four problems none is related to Hadamard matrices, whereas there are two on regular permutation groups. The reader would have been benefited if on page 42(**1**) some examples were provided concerning conics in odd and even geometry.

The monographs do not provide the reader with well-known and important results. Here are a few cases: There is no reason for the omission of the celebrated theorem of Bruck–Ryser [2], concerning a necessary condition for the existence of finite projective plane in Chapter II(**1**). This famous theorem has ruled out the existence of finite projective planes for infinitely many orders. On page 40(**1**) the author has referred to this reference only for the case $n=6$. If H_1 and H_2 are two Hadamard matrices of order n_1 and n_2 respectively, then it is well known (and important) that the Kronecker product of H_1 and H_2 is a Hadamard matrix of order n_1n_2. There is no mention of this elementary and important result in either book. Finally let $N(s)$ denote the maximal number of mutually orthogonal Latin squares of order s. Then Chowla *et al* [3] proved that $N(s) > \frac{1}{3}s^{1/91}$ for all large s. Later Rogers [6], Wang [8], and Wang [9] improved this lower bound by showing that $N(s) > s^{1/42}$, $N(s) > s^{1/28}$ and $N(s) > s^{1/24}$ respectively. However, the author on page 52(**2**) has only provided the reader with the original result obtained by Chowla *et al* [3].

The printing of neither book is satisfactory. Basic results, formulae, symbols, examples, etc., are not clearly set out. Here are a few instances. In several places the capital letter *O* has been used instead of zero; see, for example, pages 5, 11, 13, 14 and 16(**1**). In many instances the author should have put the mathematical statements in set theoretic notation, yet he failed to do so. Several such cases can be found on page 23(**1**). Because of a poor layout one cannot readily see where an example ends and the main text starts up again.

Most of the few typographical errors are in the bibliography.

The interested reader can find a better treatment of finite geometrical systems or combinatorial aspects of classical designs in references [1], [4], [5] and [7].

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