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Erratum: One dimensional annihilating particle systems as extended Pfaffian point processes.

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Abstract

This erratum corrects the statement of the main result in ECP volume 17 paper 40 ([2]) which establishes that the multi-time particle distributions for certain interacting Brownian motions on the real line are extended Pfaffian point processes.

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1 Correction

In [2] the following result is proved. Consider a system of annihilating Brownian motions on the real line, where the particles move independently except for instantaneous annihilation when they meet. Assume that the initial distribution of particles is given by a natural maximal entrance law, which can be constructed as the infinite intensity limit of Poisson initial conditions (see [1] for details). The particles, at any fixed time $t > 0$, form a simple point process on $\mathbf{R}$ and it is shown in [1] that the (Lebesgue) intensities $\rho_t(z_1, z_2, \ldots, z_n)$ are given by

$$\rho_t(z_1, z_2, \ldots, z_n) = \mathrm{Pf}\left[K_t(z_i - z_j) : 1 \le i, j \le n\right]$$

where the Pfaffian is of the $2n \times 2n$ anti-symmetric matrix constructed using the 2×2 matrix kernel

$$K_t(z) = \begin{pmatrix} K_t^{11}(z) & K_t^{12}(z) \\ K_t^{21}(z) & K_t^{22}(z) \end{pmatrix} = \begin{pmatrix} -t^{-1}F''(zt^{-1/2}) & -t^{-1/2}F'(zt^{-1/2}) \\ t^{-1/2}F'(zt^{-1/2}) & \mathrm{sgn}(z)F(|z|t^{-1/2}) \end{pmatrix}$$

and F is the Gaussian tail probability given by

$$F(z) = \frac{1}{2\pi^{1/2}} \int_z^\infty e^{-x^2/4} dx$$

with $\mathrm{sgn}(z) = 1$ for $z > 0$, $\mathrm{sgn}(z) = -1$ for $z < 0$ and $\mathrm{sgn}(0) = 0$. The first part of Theorem 1.1 in [2] is as follows.

Theorem 1.1. *Under the maximal entrance law for annihilating Brownian motions, the particle positions at times $t > 0$ form an extended Pfaffian point process, with multi-time joint intensities*

$$\rho_{t_1 t_2 \ldots t_n}(z_1, z_2, \ldots, z_n) = Pf[K(t_i, z_i; t_j, z_j) : 1 \le i, j \le n] \tag{1.1}$$

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where the space-time kernel K is defined as follows: for $t > s$ and $i, j \in \{1, 2\}$

$$K^{ij}(t, x; s, y) = G_{t-s}K_s^{ij}(y - x) - 2I_{\{i=1,j=2\}}g_{t-s}(y - x);$$

for $t < s$ and $i \neq j \in \{1, 2\}$,

$$K^{ii}(t, x; s, y) = -K^{ii}(s, y; t, x), \quad K^{ij}(t, x; s, y) = -K^{ji}(s, y; t, x);$$

and $K(t, x; t, y) = K_t(y - x)$. Here we write $(G_r)_{r\geq 0}$ for the semi-group generated by convolution with the Gaussian density $g_r(z) = (2\pi r)^{-1/2}e^{-z^2/2r}$.

Theorem 1.1 of [2] then goes on to claim that the extended Pfaffian property holds for instantly coalescing Brownian motions under the same entrance law, but this is false. The relevant part in the proof, step 7, is simply wrong for coalescing particles. We do not yet have a simple description of the multi-time distributions for coalescing Brownian motions.

References

[1] Tribe, Roger; Zaboronski, Oleg. Pfaffian formulae for one dimensional coalescing and annihilating systems. Electron. J. Probab. 16 (2011), no. 76, 2080–2103. MR-2851057

[2] Tribe, Roger; Yip, Siu Kwan; Zaboronski, Oleg. One dimensional annihilating and coalescing particle systems as extended Pfaffian point processes. Electron. Commun. Probab. 17 (2012), no. 40, 7 pp. MR-2981896