Erratum to: Coalescence estimates for the corner growth model with exponential weights

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Abstract

We fix a mistake in the previously published paper Electron. J. Probab. 25: 1–31 (2020). The corrected version of the paper can also be found at arXiv:1911.03792.

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We made a mistake in the proof of Theorem 4.1, and we gratefully thank Manan Bhatia for pointing this out to us. In the original paper, the mistake appeared in (4.9), and we provide correct proof for it here.

Recall the beginning part of the proof from the original paper, and we continue from equation (4.8), with a weaker estimate:

$$\tilde{P}\{\exists z \text{ outside } [0, v_N] \text{ such that } \|Z_{0 \to z}\| < \lfloor arN^{2/3} \rfloor \} \leq Cr^{-3}. \quad (4.8)$$

Here $\tilde{P}$ is the modified environment defined above (4.5), and $Z_{0 \to z}$ is the exit time for the geodesic, which is defined in the text between (3.4) and (3.5). Note the upper bound above is weaker than the one stated in (4.8) of the original paper, but it is enough for showing (4.5).

We treat the case $1 \leq Z_{0 \to z} < \lfloor arN^{2/3} \rfloor$ of (4.8). The same arguments give the analogous bound for the case $-\lfloor arN^{2/3} \rfloor < Z \leq -1$. Start by perturbing the endpoint $v_N = (\lfloor N(1-\rho)^2 \rfloor, \lfloor N\rho^2 \rfloor)$ to a new point $w_N$ as was done in Lemma 4.2:

$$w_N = v_N - \frac{1}{10}(1-\rho)arN^{2/3}e_1.$$  

Break up the northeast boundary of $[0, v_N]$ into two regions $\mathcal{L}$ and $\mathcal{D}$ as in the diagram on the right of Figure 4.3. Recall the parameter $\lambda = \rho + \frac{\rho^2}{N^{2/3}}$ defined at the beginning of the proof, and note that the $(- (1-\lambda)^2, -\lambda^2)$-directed ray started from $w_N$ still goes through

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the interval \([arN^{2/3}, brN^{2/3}]\) on the \(e_1\)-axis. We now require \(0 < a < \frac{1}{10}(1 - \rho) < 10 \frac{2}{\omega} < b\) for \(a, b\) in order to apply Lemma 4.2 directly in the later part of the proof.

First consider geodesics that hit \(D\). In the remainder of this erratum, we will show

\[
\tilde{P}\{ \exists z \in D : 1 \leq Z^{0 \to z} < \lfloor arN^{2/3} \rfloor \} \leq Cr^{-3},
\]

and this replaces the estimate (4.9) in the original paper.

Let \(\sigma^{0 \to x}_1\) denote the exit time of the optimal path among those \(0 \to x\) paths whose first step is \(e_1\). Then we have

\[
\tilde{P}\{ \exists z \in D : 1 \leq Z^{0 \to z} < \lfloor arN^{2/3} \rfloor \} \leq \tilde{P}\{ \exists z \in D : \sigma^{0 \to z}_1 < \lfloor arN^{2/3} \rfloor \}
\]

\[
\leq \tilde{P}\{ \sigma^{0 \to wN}_1 < \lfloor arN^{2/3} \rfloor \}.
\]

The second inequality comes from the uniqueness of maximizing paths: the maximizing path to \(w_N\) cannot go to the right of a maximizing path to \(D\).

The task is to bound \(\tilde{P}\{ \sigma^{0 \to wN}_1 < \lfloor arN^{2/3} \rfloor \}\). Define an environment with \(P^\lambda\) distribution by multiplying the \(P^\rho\) boundary weights by \(\frac{1 - \rho}{1 - \lambda}\) on the \(e_1\)-axis and by \(\tilde{\xi}\) on the \(e_2\)-axis. We have now three coupled weight configurations with marginal distributions \(\tilde{P}, P^\rho\) and \(P^\lambda\). Denote their joint distribution by \(P\). Let \(G, G^\rho\) and \(G^\lambda\) denote the last-passage values under these three environments. Additionally, let \(\tilde{G}_{0, w_N}(I)\) denote the last-passage value restricted to paths that exit through the set \(I\).

To obtain

\[
\tilde{P}\{ \sigma^{0 \to wN}_1 < \lfloor arN^{2/3} \rfloor \} \leq Cr^{-3}
\]

we show

\[
P\{ \tilde{G}_{0, w_N}(\lfloor e_1, \lfloor arN^{2/3} - 1 \rfloor e_1 \rfloor) < \tilde{G}_{0, w_N}(\lfloor \lfloor arN^{2/3} \rfloor e_1, \lfloor brN^{2/3} \rfloor e_1 \rfloor) \} \geq 1 - Cr^{-3}.
\]

By Lemma 4.2 there exists an event \(A_1\) with \(P(A_1) \geq 1 - e^{-Cr^3}\) such that on this event the geodesic of \(\tilde{G}_{0, w_N}\) exits inside \(\lfloor \lfloor arN^{2/3} \rfloor e_1, \lfloor brN^{2/3} \rfloor e_1 \rfloor\). The following equality holds on \(A_1\):

\[
\tilde{G}_{0, w_N}(\lfloor \lfloor arN^{2/3} \rfloor e_1, \lfloor brN^{2/3} \rfloor e_1 \rfloor) + \sum_{k=1}^{\lfloor arN^{2/3} - 1 \rfloor} \left( \frac{1 - \rho}{1 - \lambda} - 1 \right) \omega_{ke_1} = G^\lambda_{0, w_N}.
\]

Together with the fact that

\[
\tilde{G}_{0, w_N}(\lfloor e_1, \lfloor arN^{2/3} - 1 \rfloor e_1 \rfloor) \leq G^\rho_{0, w_N},
\]

the probability in (4.11) can be lower bounded as

\[
(4.11) \geq P\left( \left\{ G^\rho_{0, w_N} < G^\lambda_{0, w_N} - \sum_{k=1}^{\lfloor arN^{2/3} - 1 \rfloor} \left( \frac{1 - \rho}{1 - \lambda} - 1 \right) \omega_{ke_1} \right\} \cap A_1 \right).
\]

Up to a \(\rho\)-dependent constant

\[
E\left[ \sum_{k=1}^{\lfloor arN^{2/3} - 1 \rfloor} \left( \frac{1 - \rho}{1 - \lambda} - 1 \right) \omega_{ke_1} \right] \sim ar^2 N^{1/3},
\]

and recall that the parameter \(a\) can be fixed arbitrarily small. On the other hand, a computation in eqn. (5.53) in the arXiv version of [1] with \(\kappa^1_N = -\frac{1}{10}(1 - \rho)rN^{2/3}\) and \(\kappa^2_N = 0\) gives

\[
E[G^\lambda_{0, w_N}] - E[G^\rho_{0, w_N}] \geq c_1 r^2 N^{1/3}
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where $c_1$ is another $\rho$-dependent constant. Hence for small $\alpha > 0$ the event inside the braces in (4.12) should occur with high probability. This we now demonstrate.

Let

$$A_2 = \{ G_{0,w_N}^\rho > E[G_{0,w_N}^\rho] + \frac{1}{2}c_1 r^2 N^{1/3} \}.$$ 

We show that $P(A_2) \geq 1 - C r^{-3}$. First we estimate the variance $\text{Var}[G_{0,w_N}^\rho]$. The first equality below is Theorem 5.6 in the arXiv version of [1]:

$$\text{Var}[G_{0,w_N}^\rho] = \frac{[1 - \rho^2 N]}{1 - \rho^2} + \frac{\rho^2 N}{(1 - \rho)^2} \geq 2 \rho^2 E \left[ \frac{Z^{0 \rightarrow w_N}}{\rho^2} \right] + \frac{2}{1 - \rho} E \left[ \omega_{k_e_1}^\rho \right]$$

$$\leq C r N^{2/3} + \frac{2}{1 - \rho} E \left[ \omega_{k_e_1}^\rho \right] \leq C r N^{2/3} + C' r^{2/3}.$$  (4.15)

Shifting the endpoint from $w_N$ back to $v_N$ inside the expectations increases the expected value because $Z^{0 \rightarrow w_N} \leq Z^{0 \rightarrow v_N}$ almost surely. This gives the inequality between the two expectations. The last expectation is of order $N^{2/3}$ as shown through Lemma 5.8 and Proposition 5.9 in the arXiv version of [1]. Now we can bound:

$$P(A_2^c) = P(\{ G_{0,w_N}^\lambda \leq E[G_{0,w_N}^\lambda] + \frac{c_1}{2} r^2 N^{1/3} \})$$

(using (4.14))

$$\leq P(\{ G_{0,w_N}^\lambda \leq E[G_{0,w_N}^\lambda] - \frac{c_1}{2} r^2 N^{1/3} \})$$

$$\leq \frac{c_2}{r^4 N^{2/3}} \text{Var}[G_{0,w_N}^\lambda]$$

(Lemma 5.7, arXiv version of [1])

$$\leq \frac{c_2}{r^4 N^{2/3}} (\text{Var}[G_{0,w_N}^\rho]) + c_3 r N^{-1/3}(1 - \rho)^2 N \leq C r^{-3}.$$ 

For the last inequality we take $r \geq C'$ from the last line of (4.15). We have the further lower bound

$$P(\{ G_{0,w_N}^\rho < E[G_{0,w_N}^\rho] + \frac{c_1}{2} r^2 N^{1/3} \}) \geq P(\{ G_{0,w_N}^\rho < E[G_{0,w_N}^\rho] + \frac{c_1}{2} r^2 N^{1/3} \} \cap A_1 \cap A_2).$$  

(4.16)

We handle the i.i.d. sum above using large deviation of i.i.d. exponential random variables. Let $I(\cdot)$ denote the Cramér rate function of the $\text{Exp}(1 - \rho)$ distribution. Then

$$P\left\{ \sum_{k=1}^{\lfloor ar N^{2/3} \rfloor - 1} \omega_{k_e_1} > \frac{c_1}{4} r^2 N^{1/3} \right\} \leq e^{-ar N^{2/3} I(c_5/a)} \leq e^{-c_6 r N^{2/3}}$$

where $c_5$ is a certain constant, and for small enough $a > 0$, $I(c_5/a) \geq c_0/a$. Thus the event

$$A_3 = \left\{ \sum_{k=1}^{\lfloor ar N^{2/3} \rfloor - 1} \omega_{k_e_1} \leq \frac{c_1}{4} r^2 N^{1/3} \right\}$$

satisfies $P(A_3) \geq 1 - e^{-c_6 r N^{2/3}}$. Continuing the lower bound,

$$P(\{ G_{0,w_N}^\rho < E[G_{0,w_N}^\rho] + \frac{c_1}{4} r^2 N^{1/3} \} \cap A_1 \cap A_2 \cap A_3).$$  

(4.17)

The variance bound from (4.15) gives

$$P\left\{ G_{0,w_N}^\rho - E[G_{0,w_N}^\rho] \geq \frac{c_1}{4} r^2 N^{1/3} \right\} \leq \frac{c_2}{r^4 N^{2/3}} \text{Var}[G_{0,w_N}^\rho] \leq C r^{-3}.$$ 

All four events inside the probability in (4.17) have probability at least $1 - C r^{-3}$, and this verifies (4.9).
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References