

REJOINER: FITTING A FOLDED NORMAL DISTRIBUTION WITHOUT EM

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I am grateful to Professors [Jung, Foskey and Marron](#) for the trouble they have gone to in response to my letter, which related to only a minor part of their 2011 paper. I am of course pleased that they have found that their methods and mine produce almost identical numerical results for the 15 sets of data used in [Jung, Foskey and Marron \(2011\)](#), and for a shifted version thereof; I note the differences in running times.

Whether such differences matter will depend on the context; actual running times of approximately 0.1 seconds and 0.01 seconds are both essentially instantaneous, although the difference could matter if it is necessary to perform such computations on a much larger scale. And I would consider efficiency to include not only computer time but also human time (which is expensive). If a few hours of human time can produce a reduction of 0.1 seconds in computing time, that may or may not be worthwhile. A related point is that one would expect a ‘carefully designed’ algorithm (statistical or otherwise), well adapted to the specifics of the problem at hand, to be more efficient in computer time than an existing general-purpose optimizer which has not been designed for a narrowly defined problem.

Although such judgments are likely to be a matter of opinion, it now seems clear that a problem previously described as ‘not straightforward’ is indeed straightforward, and can be solved quickly by either of two easy-to-implement techniques for finding MLEs. I believe that [Jung, Foskey and Marron](#) have made a valuable contribution to the question of when an EM algorithm is advantageous and when it is not, a question that to me seems neglected. In particular, they hit the nail on the head when they refer to a case in which the optimization at the M step does not have a closed-form solution ([Eltzner, Huckemann and Mardia \(2018\)](#)); in such a case the potential advantages of an EM algorithm seem very limited.

The discussions of the ‘linear vs nonlinear’ dimension reduction problem and the folding of multivariate distributions contain much useful information and identify problems well worth further investigation.

I thank the Editor-in-Chief, Prof. Kafadar, for enthusiastically encouraging this exchange, and Prof. Linda Haines for her suggestions. And I thank [Jung, Foskey and Marron](#) for their extremely constructive response.

REFERENCES

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