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Comment: Laplace and Cognitive Illusions

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Abstract. Reports in the 1970s of cognitive illusions in judgments of uncertainty had been anticipated by Laplace 150 years earlier. We discuss Miller and Gelman's remark that Laplace's anticipation of the main ideas of the heuristics and biases approach "gives us a new perspective on these ideas as more universal and less contingent on particular developments [that came much] later."

Key words and phrases: Cognitive illusions, heuristics and biases, judgment under uncertainty.

We are grateful to Miller and Gelman (MG) for showing that the reports in the 1970s of cognitive illusions in judgments of uncertainty had been anticipated by Pierre-Simon Laplace 150 years earlier. It is an honor to have walked in this giant's footsteps. In this brief comment, we elaborate on MG's remark that Laplace's anticipation of the main ideas of the heuristics and biases approach "gives us a new perspective on these ideas as more universal and less contingent on particular developments in the 1970s and later" (Miller and Gelman, 2020, p. 159–170).

WHAT IS THIS NEW PERSPECTIVE?

Laplace did not only anticipate many of the errors of judgment under uncertainty that were experimentally demonstrated much later. More importantly, he anticipated the description of at least some of these errors as "illusions of thought," akin to visual illusions (Laplace, 1995, p. 3). Perceptual illusions are a special type of error, marked by two characteristics: (1) they are (almost) universal among members of a culture; (2) they remain appealing to the observer even when the observer knows that they are errors. The most famous visual illusion (Fig. 1), named after its discoverer Franz Carl Müller-Lyer, is now known to be more or less universal in *carpentered environments*, in which right angles are ubiquitous. As you, the reader, certainly come from a carpentered environment, we can confidently predict that you see the upper line as longer than the lower line. Furthermore, we can confidently predict that the upper line will continue to still *look* longer even after you carefully measure the two and are persuaded that they are identical in length. The discredited perceptual impression persists even when the observer knows better. Some intuitive judgments show a similar kind of persistence.

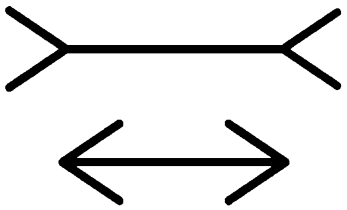

FIG. 1. *The Müller-Lyer illusion.*

A famous statement of the persistence of a cognitive illusion was offered by the late Harvard paleontologist Stephen J. Gould, who described his reaction to the notorious Linda problem. In this problem, one reads: "Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in antinuclear demonstrations" (Tversky and Kahneman, 1983, p. 297). One is then asked to rank the following possibilities according to their probability: Linda is a bank teller (B); Linda is active in the feminist movement (F); Linda is a bank teller and is active in the feminist movement (B&F). Gould humorously described his dilemma: "I know that the third statement [B&F] is least probable, yet a little homunculus in my head continues to jump up and down, shouting at me—'but she can't just be a bank teller; read the description'" (Gould, 1991, p. 469).

The recognition of an error's enduring appeal is a central characteristic of cognitive illusions. The distinctive epistemic state of feeling attracted to a belief that one knows to be false played an important role in the development of the heuristics and biases approach. Dur-

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ing the years of their collaboration in the study of judgment and decision, Amos Tversky and Daniel Kahneman's most productive mode of discovery was introspection: they searched for propositions that they found intuitively attractive although they knew them to be logically false—ergo, cognitive illusions. We believe that Laplace used the term *illusion* because he had similar experiences. His frequent use of the pronoun *us* when discussing errors and how to debias them through reason and calculation (e.g., "let us severely examine our own opinions," p. 5) further supports the conjecture that he, too, experienced the appeal of the very errors which his enlightened mind exposed.

Cognitive illusions are significant to theoretical debates about judgment in two major ways. First, and most important, illusions are neither experimental results nor theoretical constructions—they are phenomenological observations. This is the point that MG make when they say that the students of heuristics and biases began "with the same introspections that evidently guided Laplace" (p. 2). It is this similarity of simple observation that led MG to conclude that the contributions of the heuristics and biases approach "is a scientific contribution that will endure" (2019, p. 10). Experimental results can be questioned and theoretical notions can be debated, but MG implies that open-minded introspection is sometimes more compelling than either.

Perceptual illusions, such as Müller-Lyer's, can be made to go away if the lines are compared not by the naked eye, but rather by a ruler. The Linda illusion can also be made to go away, if Linda's occupational possibilities are judged not by the representativeness heuristic (i.e., by how much Linda *sounds like* a bank teller versus a feminist bank teller), but rather by the ruler of formal logic. Logically, since every feminist bank teller is a bank teller, the former cannot be more probable than the latter. People do not usually take out their physical ruler when asked which of Müller-Lyer's two lines is longer, but might do so if asked to give the two lengths in millimeters. They would then discover that the lines are of equal length. Similarly, people do not usually engage their mental ruler when given Linda's description, but might do so if asked to estimate the population frequencies of bank tellers and of feminist bank tellers. They would then realize that of course there must be fewer of the latter than of the former.

The reflective reasoning of those whom Laplace called *hommes éclairés* (enlightened men) is guided by their knowledge of the principles of logic and of probability. However, awareness of the reflective "correct" answer sometimes does little to diminish the immediate appeal of the incorrect intuition. Müller-Lyer's upper line still looks longer, and Linda still sounds like a feminist. Thus, if enlightened persons stop to introspect, they will recognize that some erroneous intuitions retain a certain seductive charm. Laplace was undoubtedly armed with an excellent cognitive ruler. This makes it all the more remarkable that he was able to set it aside in diagnosing his own susceptibility to cognitive illusions.

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