The Annals of Applied Statistics
2019, Vol. 13, No. 2, 1319–1328
https://doi.org/10.1214/19-AOAS1248
Main article: https://doi.org/10.1214/18-AOAS1180

CORRECTION
STATISTICAL MODELING AND ANALYSIS OF TRACE ELEMENT CONCENTRATIONS IN FORENSIC GLASS EVIDENCE

BY KAREN D. H. PAN[1] AND KAREN KAFADAR[2]

University of Virginia

1. Paper figures. The abcissa on certain figures in "Statistical Modeling and Analysis of Trace Element Concentrations in Forensic Glass Evidence" [Pan and Kafadar (2018)] and the corresponding supplement have been corrected. Estimated match rates are around 20–30% lower than originally stated (when the true relative difference in concentrations is less than 15% in all elements); however, the main results and conclusions of the paper remain unchanged. Two samples that come from batches whose mean log concentrations differ by $\delta = 0.1$ (roughly 10%) in all 17 elements would not be "considered distinguishable" [ASTM International (2016), Section 11.1.7] 62.22–65.41% of the time using the covariance matrix estimate from the German data, and 77.18–78.14% of the time using the estimate from the Canadian data set. Affected figures and tables below are labeled corresponding to Pan and Kafadar (2018). Section 2 contains corrected supplemental figures and tables.

TABLE 3
Canadian and German data simulation match rates at various δ

(δ)	0.05	0.1	0.15	0.2	0.25	0.3
			(a) Canadian data match rates			
t_3	0.991	0.781	0.103	0.024	0.006	0.003
t_6	0.997	0.773	0.018	0.000	0.000	0.000
t_{10}	0.998	0.775	0.006	0.000	0.000	0.000
G	0.999	0.772	0.000	0.000	0.000	0.000
			(b) German data match rates			
t_3	0.968	0.622	0.016	0.003	0.001	0.001
t_6	0.988	0.638	0.001	0.000	0.000	0.000
t_{10}	0.989	0.648	0.000	0.000	0.000	0.000
G	0.990	0.654	0.000	0.000	0.000	0.000

Received February 2019.

[1]Supported in part by National Institute of Standards & Technology via subcontract from Iowa State University.

[2]Supported by Isaac Newton Institute for Mathematical Sciences, *Probability and Statistics in Forensic Science*, EPSRC Grant EP/K032208/1.

TABLE 4
Match rates by sample size at $n = 4$ (G = Gaussian, $t_3 = t$ with df = 3) for various δ

(δ)	0.05	0.1	0.15	0.2	0.25	0.3
3 G	0.829	0.354	0.006	0.000	0.000	0.000
3 t_3	0.754	0.370	0.051	0.013	0.005	0.003
9 G	0.999	0.768	0.000	0.000	0.000	0.000
9 t_3	0.991	0.779	0.105	0.023	0.007	0.003
12 G	1.000	0.852	0.000	0.000	0.000	0.000
12 t_3	0.997	0.856	0.115	0.025	0.007	0.003

TABLE 5
n-SD approach match rates where $\delta = 0.1, 0.2$ for certain values of n. Values are multiplied by 100

(n)	1.00	1.50	2.00	2.50	3.00	3.50	4.00
			(a) $r = 3$				
$\delta = 0.1$ G	0.000	0.000	0.045	0.638	4.437	15.883	35.335
$\delta = 0.1$ t_3	0.091	0.426	1.456	4.002	9.847	21.156	36.788
$\delta = 0.2$ G	0.000	0.000	0.000	0.000	0.000	0.001	0.007
$\delta = 0.2$ t_3	0.010	0.033	0.080	0.197	0.408	0.765	1.281
			(b) $r = 6$				
$\delta = 0.1$ G	0.000	0.000	0.007	0.211	4.227	26.491	62.176
$\delta = 0.1$ t_3	0.100	0.611	2.320	6.766	17.616	38.684	64.383
$\delta = 0.2$ G	0.000	0.000	0.000	0.000	0.000	0.000	0.001
$\delta = 0.2$ t_3	0.008	0.037	0.095	0.237	0.527	1.073	2.024

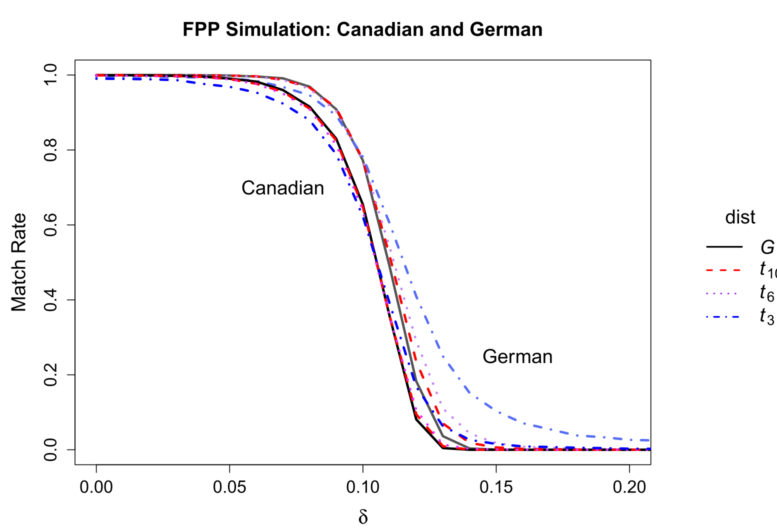


FIG. 2. *Match rates from Canadian and German simulations for data from four different distributions. δ gives the approximate relative change in means.*

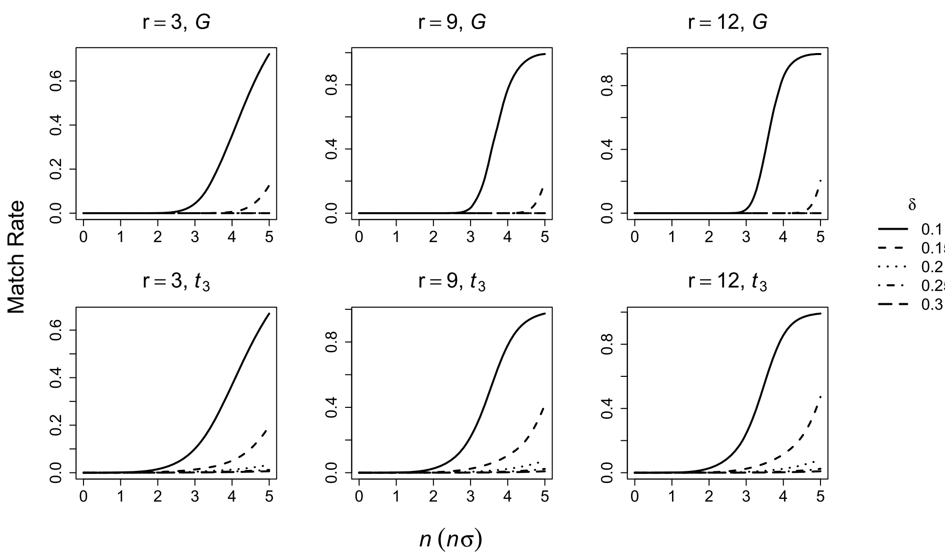


FIG. 3. *Simulation match rates using the Canadian covariance matrix by sample size and distribution for five δ values with $n = 0, \dots, 5$.*

Since the time of publication, we have had access to data from Iowa State University [Park and Carriquiry (2018, 2019)] which includes measurements on 24 fragments from each of 48 panes of glass from two glass manufacturers [Park and

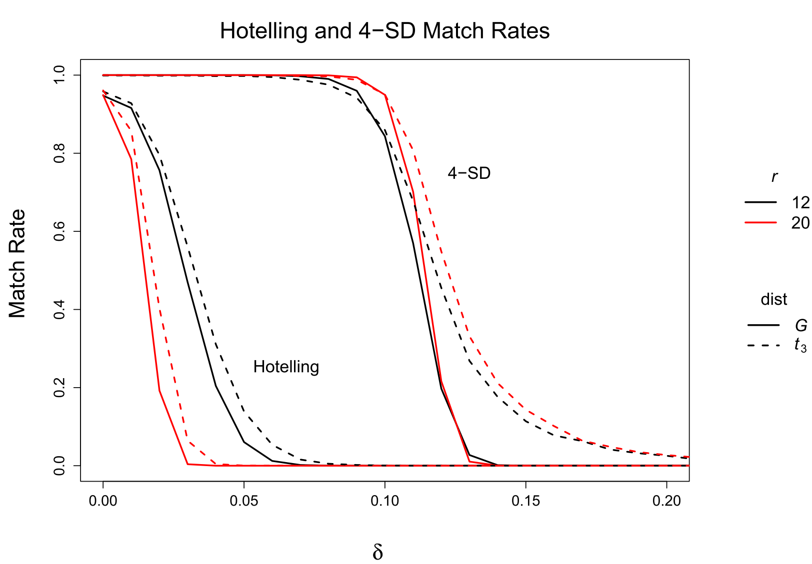


FIG. 4. *Match rates for Hotelling's T^2 vs. 4-SD approach for G and t_3 distributed data. Distribution has a much larger effect on the 4-SD approach.*

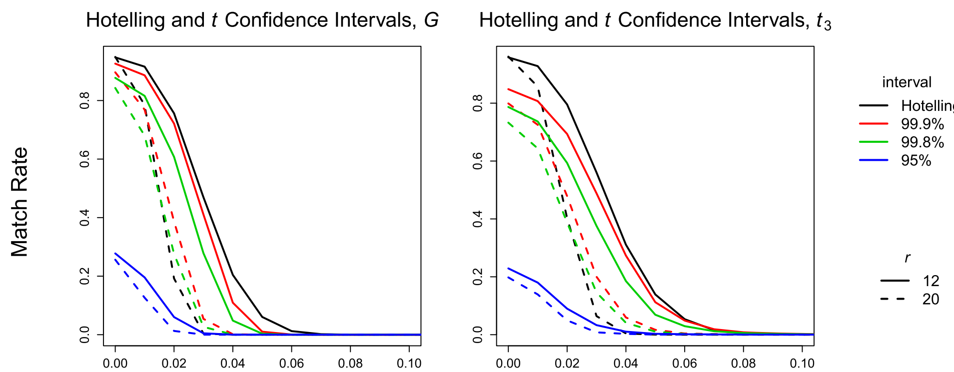


FIG. 5. *Match rates for Hotelling T^2 and t* (95%, 99.8%, 99.9%) *intervals*.

Carriquiry (2018)]. The data suggest that at least half of the elements show within-pane variation of at least 10%, suggesting $\delta = 0.1$ is a typical (or, at least, not an unreasonable) lower bound.

2. Supplement figures. The supplement contained additional match rate plots for simulations using the German and Canadian covariance matrices from Section 6.1 and 6.2. These simulations used a covariance matrix $V^* = V_e^* + V_f^*$ that takes into account both between- and within-fragment variability. All figures

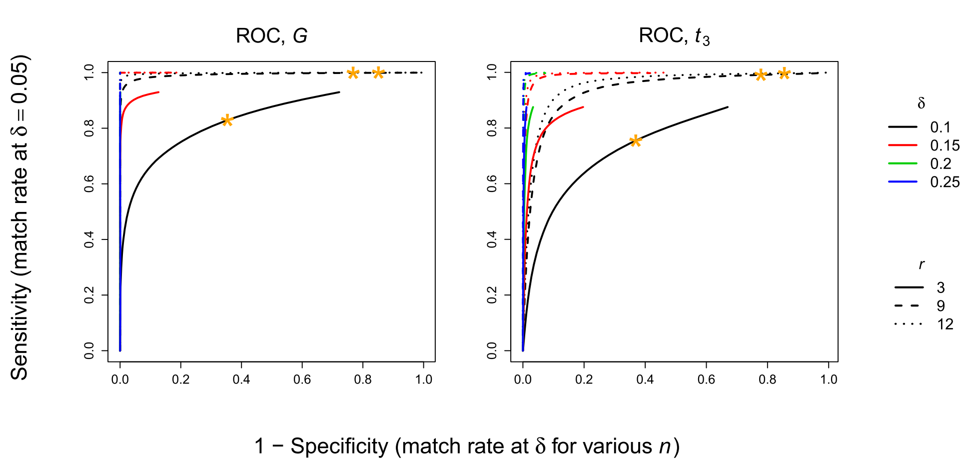


FIG. 6. *ROC curves for n-SD simulations using Canadian covariance matrix for four δ values with $n = 0, \dots, 5$. The three orange stars denote the $n = 4$ point on each $\delta = 0.1$ curve for the three replicate levels.*

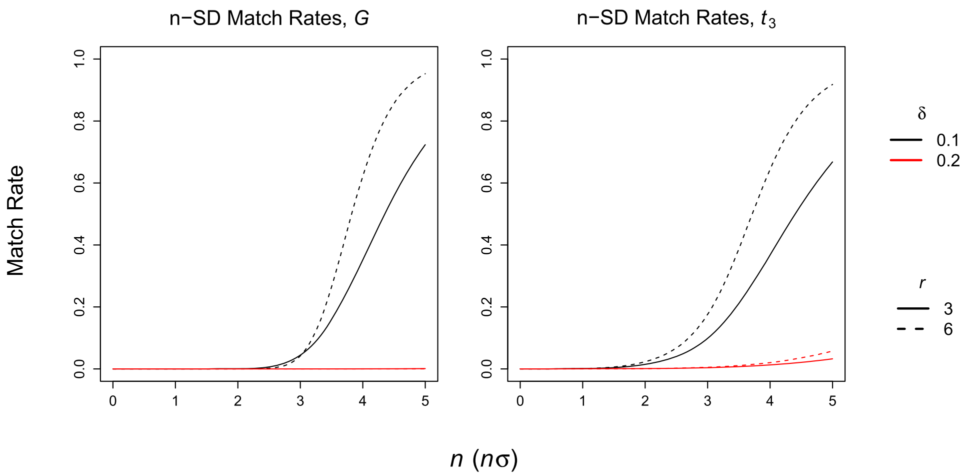


FIG. 7. *n-SD approach match rates where $\delta = 0.1, 0.2$ for $n = 0, \ldots, 5$. Assuming Gaussian data with* 17 *elements and $\delta = 0.1$, to ensure a match rate of* 5%, $n = 3.04$ *should be used. These multipliers decrease under the t_3 assumption to* 2.62 *and* 2.36 ($r = 3, 6$, *respectively*).

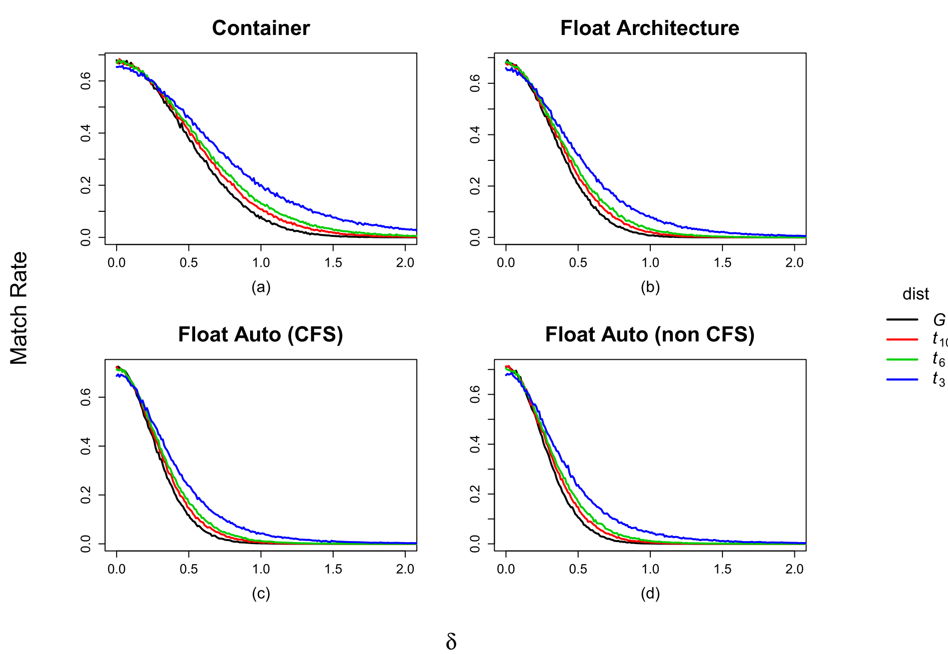
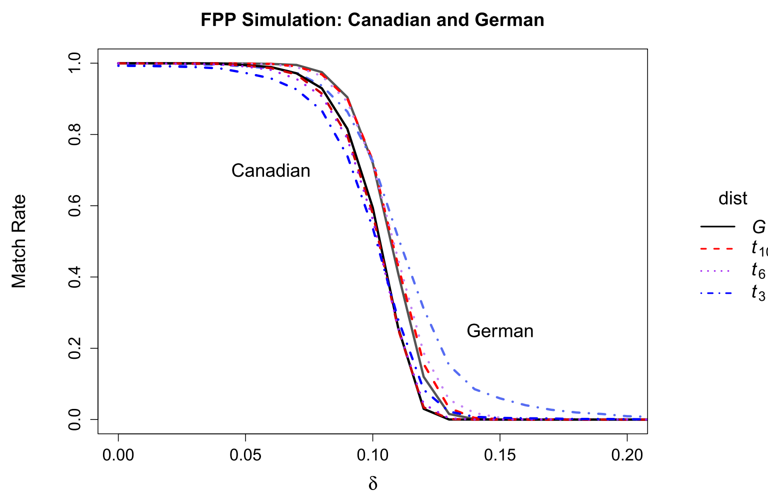


FIG. 8. *Match rates for FIU* (*ICP-MS*) *data for four different glass categories. The number of Wishart degrees of freedom by category are*: *Container* (75), *Float Architecture* (75), *Float Autowindow non CFS* (75), *Float Autowindow CFS* (25), *Headlamp* (25). *The match rates for Headlamp are not shown*, *but are similar to those in Figure* 8(c).

below consider only measurement variability, V_e^*. An overall decrease in match rates can be seen.

2.1. *German and Canadian covariance simulations.*



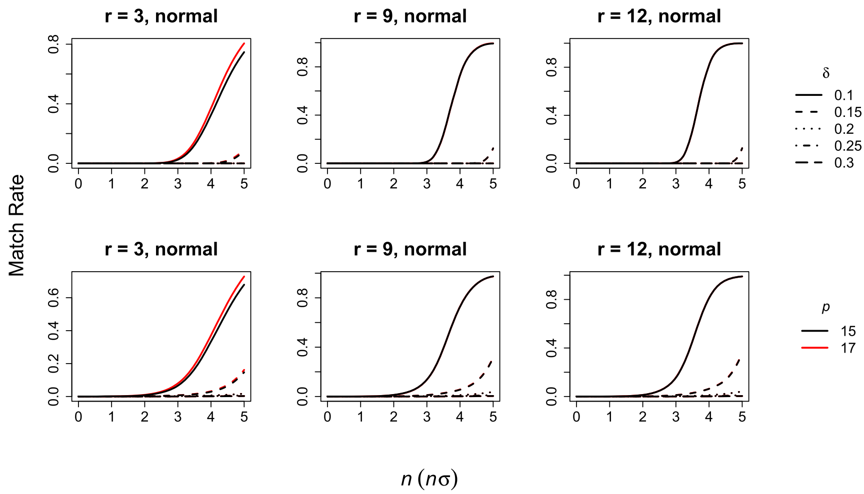
(Analogous to paper Figure 2) *Match rates from Canadian and German simulations for data from four different distributions. δ gives the approximate relative change in means. The match rates are considerably lower as δ increases in comparison to $V^* = V_e^* + V_f^*$.*

2.2. *Canadian covariance matrix simulations.*

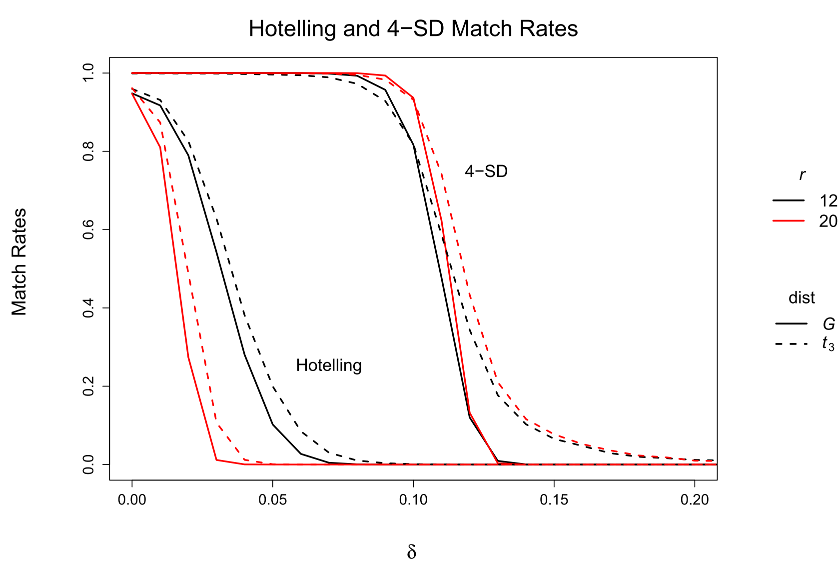
Analogous to paper Table 4

Match rates by sample size at $n = 4$ and $p = 17$ (G = Gaussian, $t_3 = t$ with df $= 3$) for various δ. Values are multiplied by 100

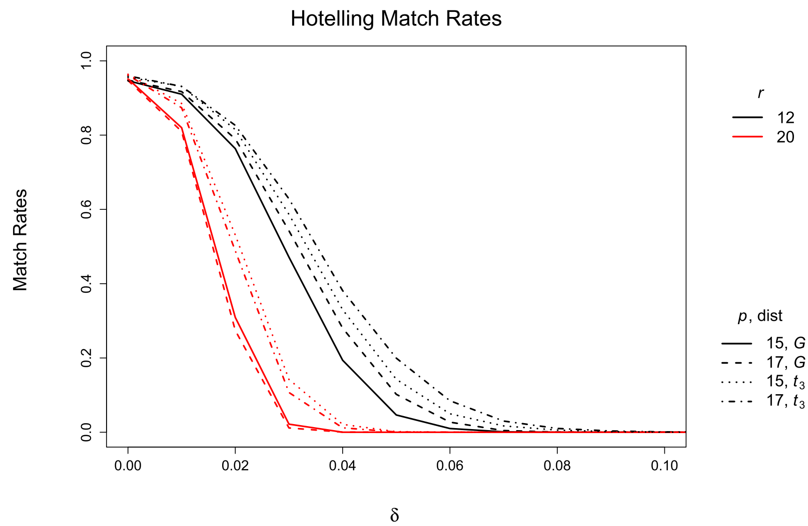
(δ)	0.1	0.15	0.2	0.25	0.3
3 G	32.509	0.152	0.001	0.000	0.000
3 t_3	33.369	2.874	0.649	0.279	0.11
9 G	72.881	0.003	0.000	0.000	0.000
9 t_3	72.682	5.487	1.078	0.359	0.139
12 G	81.833	0.002	0.000	0.000	0.000
12 t_3	81.101	6.307	1.14	0.316	0.111



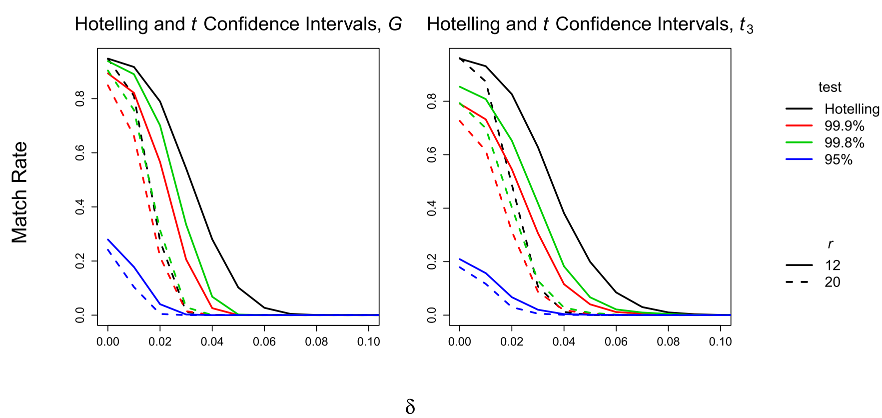
(ANALOGOUS TO PAPER FIGURE 3) *Simulation match rates using the Canadian covariance matrix by sample size and distribution for five* δ *values with* $n = 0, \ldots, 5$, *shown for* $p = 15$ *and* $p = 17$.



(ANALOGOUS TO PAPER FIGURE 4) *Match rates for Hotelling's* T^2 *vs.* 4-*SD approach for G and* t_3 *distributed data. Distribution has a larger effect on the* 4-*SD approach.*



Match rates for Hotelling's T^2 comparing $p = 15$ and $p = 17$ for G and t_3 data. Match rates from the 4-SD approach are very similar for both p and are not shown.



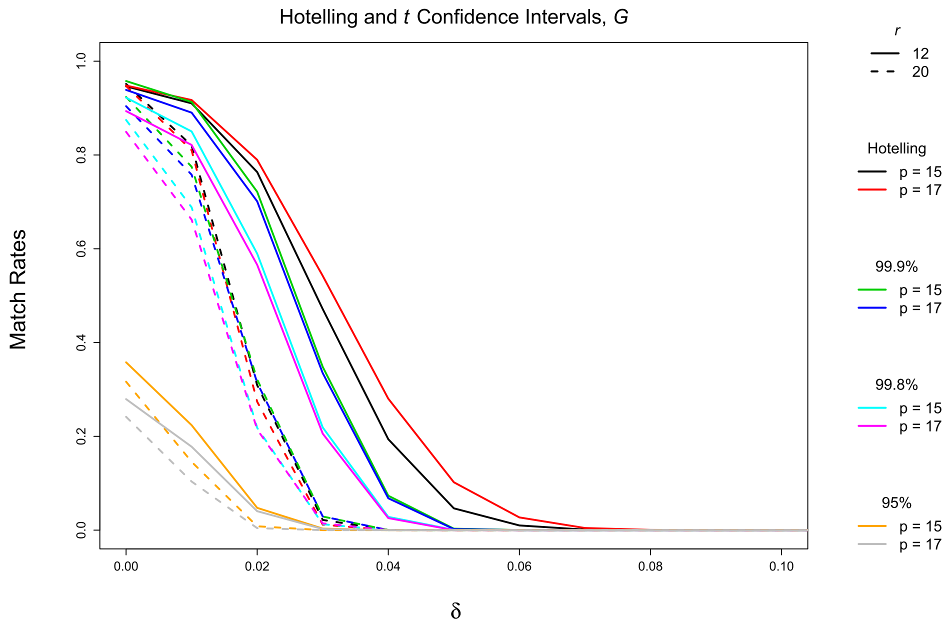
(ANALOGOUS TO PAPER FIGURE 5) *Match rates for Hotelling's T^2 and t* (95%, 99.8%, 99.9%) *confidence intervals.*

REFERENCES

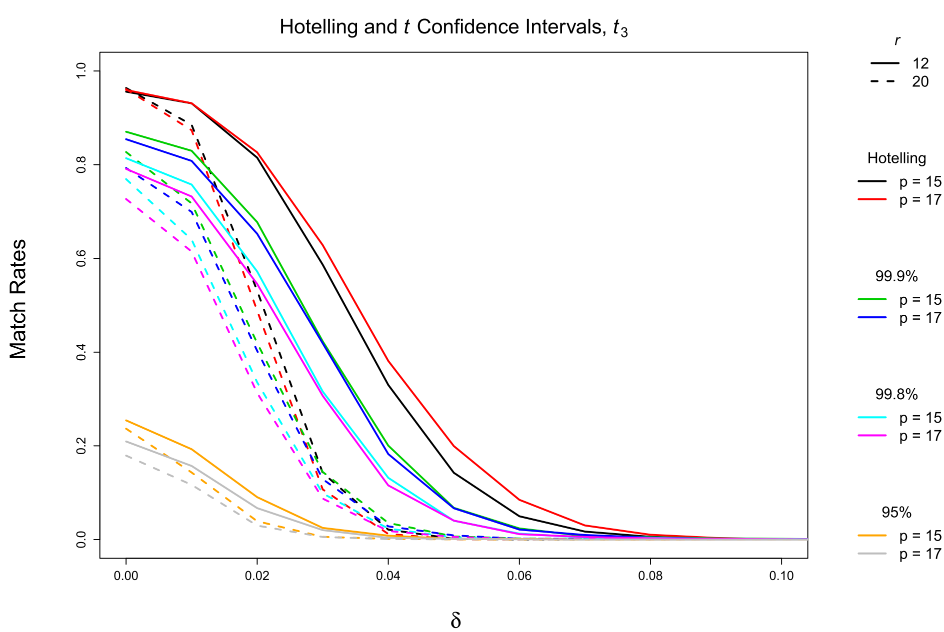
ASTM INTERNATIONAL (2016). ASTM E2927-16e1. Standard test method for determination of trace elements in soda-lime glass samples using laser ablation inductively coupled plasma mass spectrometry for forensic comparisons. Available at https://doi.org/10.1520/E2927-16E01.

PAN, K. D. H. and KAFADAR, K. (2018). Statistical modeling and analysis of trace element concentrations in forensic glass evidence. *Ann. Appl. Stat.* **12** 788–814. MR3834286

PARK, S. and CARRIQUIRY, A. (2018). Glass data description. Available at https://github.com/CSAFE-ISU/AOAS-2018-glass-manuscript.



Hotelling's T^2 and t confidence interval match rates for G data.



Hotelling's T^2 and t confidence interval match rates for t_3 data.

PARK, S. and CARRIQUIRY, A. (2019). Learning algorithms to evaluate forensic glass evidence. *Ann. Appl. Stat.* **13** 1068–1102.

DEPARTMENT OF STATISTICS
UNIVERSITY OF VIRGINIA
CHARLOTTESVILLE, VIRGINIA 22904-4135
USA
E-MAIL: kdp4be@virginia.edu
kk3ab@virginia.edu