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Comment on "A Review of Self-Exciting Spatio-Temporal Point Process and Their Applications" by Alex Reinhart

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I believe that Dr. Reinhart has written an excellent review on the methodologies, techniques and applications related to spatio-temporal self-exciting processes that have developed during recent years. Here, I would like to mention the following points to complement this article:

1. *On the diagnostics related to the clustering model*. All the methods that are explained in this article give the goodness-of-fit of the entire model, globally to the data or in a local window of the observation space–time range. To check whether the formulation for each individual component is appropriate for fitting the data or not, the stochastic reconstruction techniques (Zhuang, Ogata and Vere-Jones, 2004), based on which the diagnostics of each model components can be easily constructed, can be utilized. Zhuang, Ogata and Vere-Jones (2004) and Zhuang (2006) also used this method to test some hypotheses that are related to earthquake clusters but not formulated in the model. The method is helpful for finding the clues of formulating better models.

2. *On estimating background rate.* Not only separable clustering structure discussed in this review paper, but also complex and nesting background components can be reconstructed. Recently, Zhuang and Mateu (2018) developed a semiparametric estimation method to obtain simultaneously the clustering structure and the background in the occurrence rate of crimes, where the latter includes two periodic components, a daily and a weekly. The estimation procedure can be outlined as following.

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Consider a model with a conditional intensity function

$$\lambda(t,x) = \mu_{\mathrm{t}}(t)\mu_{\mathrm{d}}(t)\mu_{\mathrm{w}}(t)\mu_{\mathrm{b}}(x) + \iint_{(-\infty,t)\times S} g(t-s, x-u)\,\mathrm{d}N(s,u),$$

where $\mu_{\mathrm{t}}(t)$, $\mu_{\mathrm{d}}(t)$ and $\mu_{\mathrm{w}}(t)$ represent the trend term, the daily periodicity and the weekly periodicity in the temporal components of the background rate, respectively, $\mu_{\mathrm{b}}(x)$ represents the spatial homogeneity of the background rate, and $g(t-s, x-u)$ represents the subprocess triggered by an event previously occurring at location u and time s.

Given a realization of the point process $\{(t_i, x_i) : i = 1, 2, \ldots, n\}$ in a time-space range $[T_1, T_2] \times S$, where t_i and x_i denote the occurrence time and location, respectively, the long-term trend term $\mu_{\mathrm{t}}(t)$ in the background component can be reconstructed in the following way. Let

$$w^{(\mathrm{t})}(t,x) = \mu_{\mathrm{t}}(t)\mu_{\mathrm{b}}(x)/\lambda(t,x).$$

Then, assuming that μ_{t} is smooth enough,

$$\begin{aligned}\sum_i w^{(\mathrm{t})}(t_i,x_i)\mathbf{1}\big(t_i \in [t-\Delta_t, t+\Delta_t]\big) &\approx \int_{T_1}^{T_2}\iint_S w^{(\mathrm{t})}(s,x)\lambda(s,x) \\ &\quad\cdot \mathbf{1}\big(s\in[t-\Delta_t,t+\Delta_t]\big)\,\mathrm{d}s\,\mathrm{d}x \\ &= \int_{t-\Delta_t}^{t+\Delta_t}\mu_{\mathrm{t}}(s)\,\mathrm{d}s \iint_S \mu_{\mathrm{b}}(x)\,\mathrm{d}x \\ &\propto \int_{t-\Delta_t}^{t+\Delta_t}\mu_{\mathrm{t}}(s)\,\mathrm{d}s \\ &\approx 2\mu_{\mathrm{t}}(t)\Delta_t,\end{aligned}$$

where Δ_t is a small positive number. That is,

$$\hat{\mu}_{\mathrm{t}}(t) \propto \sum_i w_i^{(\mathrm{t})}\mathbf{1}\big(t_i\in[t-\Delta_t,t+\Delta_t]\big),$$

where

$$w_i^{(\mathrm{t})} = \mu_{\mathrm{t}}(t_i)\mu_{\mathrm{b}}(x_i)/\lambda(t_i, x_i).$$

Similarly, the other components in the background rate can be reconstructed as

$$\hat{\mu}_{\mathrm{d}}(t) \propto \sum_i w_i^{(\mathrm{d})} I\Big(t_i \in \bigcup_{k\in\mathbb{Z}} [t + k - \Delta_t, t + k + \Delta_t]\Big), \quad t \in [0, 1],$$

$$\hat{\mu}_{\mathrm{w}}(t) \propto \sum_i w_i^{(\mathrm{w})} I\Big(t_i \in \bigcup_{k\in\mathbb{Z}} [t + 7k - \Delta_t, t + 7k + \Delta_t]\Big), \quad t \in [0, 7]$$

and

$$\hat{\mu}_{\mathrm{b}}(x) \propto \sum_i \varphi_i \mathbf{1}\big(x_i \in [x - \Delta_x, x + \Delta_x]\big),$$

where

$$w_i^{(\mathrm{d})} = \mu_{\mathrm{d}}(t_i)\mu_{\mathrm{b}}(x_i)/\lambda(t_i, x_i),$$
$$w_i^{(\mathrm{w})} = \mu_{\mathrm{w}}(t_i)\mu_{\mathrm{b}}(x_i)/\lambda(t_i, x_i),$$
$$\varphi_i = \mu_{\mathrm{t}}(t_i)\mu_{\mathrm{d}}(t_i)\mu_{\mathrm{w}}(t_i)\mu_{\mathrm{b}}(x_i)/\lambda(t_i, x_i),$$

and Δ_x is a small positive number. In the above, φ_i has the same meaning as the background probability $\Pr\{u_i = 0\}$ defined in (10), and $w_i^{(\mathrm{t})}$, $w_i^{(\mathrm{d})}$, $w_i^{(\mathrm{w})}$ are rescaled weights for each event for the purpose of estimating each component. The estimation of the clustering term g is already solved by the conventional reconstruction method.

3. *On earthquake modeling*. More physics can be built into the model. An interesting development is by Guo, Zhuang and Zhou (2015) and Guo et al. (2017), where, instead of regarding large earthquakes as having point sources, the finite-source ETAS model treats their sources as ruptures that extend in space. Each earthquake rupture consists of many small patches, and each patch triggers its own aftershocks isotropically and independently as a usual mainshock. The superposition of triggering effects from all the patches produce an anisotropic pattern of the aftershock locations, mainly distributing along the rupturing fault. A similar EM-type iterative algorithm is designed to invert the unobserved fault geometry and other components simultaneously.

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