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Donsker-type theorems for correlated geometric fractional Brownian motions and related processes

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Abstract

We prove a Donsker-type theorem for vector processes of functionals of correlated Wiener integrals. This includes the case of correlated geometric fractional Brownian motions of arbitrary Hurst parameters in $(0, 1)$ driven by the same Brownian motion. Starting from a Donsker-type approximation of Wiener integrals of Volterra type by disturbed binary random walks, the continuous and discrete Wiener chaos representation in terms of Wick calculus is effective. The main result is the compatibility of these continuous and discrete stochastic calculi via these multivariate limit theorems.

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1 Introduction

A fractional Brownian motion $B^H = (B^H_t)_{t\geq 0}$ with Hurst parameter $H \in (0, 1)$ is a continuous zero mean Gaussian process with covariance function $\mathbb{E}[B^H_t B^H_s] = 1/2\left(t^{2H} + s^{2H} - |t-s|^{2H}\right)$, $s, t \geq 0$. It is the unique Gaussian H-self-similar process with stationary increments. The process $B^{1/2}$ is a standard Brownian motion, but B^H is not a semimartingale for $H \neq 1/2$. The corresponding fractional Gaussian noise $(B^H_{n+1} - B^H_n)_{n\in\mathbb{N}}$ for $H > 1/2$ exhibits long-range dependence and is commonly used in modeling phenomena in economy, finance, physics or neuroscience (see e.g. the monographs [8] and [21] and the references therein). There is a powerful representation as a Wiener integral of Volterra type

$$B^H_t = I(z_H)_t := \int_0^t z_H(t, s)dB_s, \tag{1.1}$$

for some kernel $z_H(t, \cdot) \in L^2([0, t])$, $t \geq 0$ and Brownian motion B (cf. [23, 5.2] or [24, Section 5.1.3]). Exemplary, suppose three fractional Brownian motions with the Hurst parameters $H_1, H_2, H_3 \in (0, 1)$ driven by the same Brownian motion according to (1.1). The goal of this article is a Donsker-type approximation of processes with highly correlated components like

$$\left(B^{H_1}_t, \exp(B^{H_2}_t), \sin(B^{H_3}_t)\right)_{t\geq 0}. \tag{1.2}$$

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The functionals involved can be represented in terms of geometric fractional Brownian motions $\exp(B_t^H - t^{2H}/2)_{t\geq 0}$ and reformulations of the Wiener chaos expansion. This Donsker theorems extend the Fractional Donsker theorem in [28, 25] and the results in [5] to multivariate functional type Donsker theorems.

More generally, we consider Wiener integrals of Volterra type as in (1.1), denoted by $I(f)$, and the corresponding stochastic exponentials as

$$\exp\left(I(f) - \frac{1}{2}\int_0^\infty f^2(s)ds\right). \tag{1.3}$$

As a primary result we obtain a Donsker theorem for vector processes of such stochastic exponentials and related functionals.

In particular, we are interested in approximating sequences which rely on disturbed random walks converging to the Wiener integrals $I(f)$ as in [28, 25] and an appropriate discrete stochastic calculus which is justified by these convergence results, cf. [5, 6].

In contrast to related multivariate invariance principles based on discrete chaos as in [2, 3, 4], we consider elements with an infinite chaos expansion as in (1.3).

We note that the convergence of finite-dimensional distributions is for example a consequence of a functional limit theorem in [26]. Here the main effort is assigned to the tightness of such general processes. This is handled by checking a well-known tightness criterion and some combinatorial reformulations of L^p-norms of discrete counterparts of functionals as (1.3) applied on correlated Volterra-type discrete Wiener integrals.

The article is organized as follows. In Section 2 we give a brief description of the class of functionals extending the stochastic exponentials in (1.3). Section 3 is devoted to the analogue in a disturbed random walk setting. In Section 4 we state and prove the main result. The technical lemmas are postponed to Section 5.

2 Wick-analytic functionals

We suppose a Brownian motion $(B_t)_{t\geq 0}$ on the probability space $(\Omega, \mathcal{F}, P)$, where the σ-field $\mathcal{F}$ is generated by the Brownian motion and completed by null sets. Therefore the stochastic calculus is based on the Gaussian Hilbert space $\{I(f) : f \in L^2([0,\infty))\} \subset L^2(\Omega)$, where $I(f) = \int_0^\infty f(s)dB_s$ denotes the Wiener integral. We denote the norm and inner product on $L^2([0,\infty))$ by $\|\cdot\|$ and $\langle\cdot,\cdot\rangle$. Due to the totality of the stochastic exponentials

$$\exp\left(I(f) - \|f\|^2/2\right)\ , \quad f \in L^2([0,\infty)),$$

in $L^2(\Omega)$, (see e.g. [16, Corollary 3.40]), for every $X \in L^2(\Omega, \mathcal{F}, P)$ and $f \in L^2([0,1])$, the *S-transform* of X at f is defined as

$$(SX)(f) := \mathbb{E}[X \exp\left(I(f) - \|f\|^2/2\right)].$$

The S-transform $(S\cdot)(\cdot)$ is a continuous and injective function on $L^2(\Omega, \mathcal{F}, P)$ (see e.g. [16, Chapter 16] for more details). As an example, for $f, g \in L^2([0,1])$, we have $(S\ \exp\left(I(f) - \|f\|^2/2\right))(g) = \exp\left(\langle f, g\rangle\right)$. In particular the characterization of random variables via the S-transform can be used to introduce the Skorokhod integral, an extension of the Itô integral to nonadapted integrands (cf. e.g. [16, Section 16.4]). For more information on the S-transform and Skorokhod integral we refer to [16], [18] or [24]. Similarly, the S-transform can be used to define the Skorokhod integral with respect to fractional Brownian motion, see e.g. [21]. We recall that for the Hermite polynomials

$$h_\alpha^k(x) = (-\alpha)^k \exp\left(\frac{x^2}{2\alpha}\right)\frac{d^k}{dx^k}\exp\left(\frac{-x^2}{2\alpha}\right)$$

and every $k \in \mathbb{N}$, the k-th Wiener chaos $H^{:k:}$ is the L^2-completion of $\{h^k_{\|f\|^2}(I(f)) : f \in L^2([0,1])\}$ in $L^2(\Omega)$ and these subspaces are orthogonal and fulfill $L^2(\Omega, \mathcal{F}, P) = \bigoplus_{k\geq 0} H^{:k:}$. Thus, for the projections

$$\pi_k : L^2(\Omega) \to H^{:k:},$$

for every random variable $X \in L^2(\Omega)$, we denote the *Wiener chaos decomposition* as

$$X = \sum_{k=0}^{\infty} \pi_k(X).$$

We refer to [16, 15] for further details and a reformulation in terms of multiple Wiener integrals.

The S-transform is closely related to a product imitating uncorrelated random variables as $\mathbb{E}[X \diamond Y] = \mathbb{E}[X]\mathbb{E}[Y]$, which is implicitly contained in the Skorokhod integral and a fundamental tool in stochastic analysis. Due to the injectivity of the S-transform, the *Wick product* can be introduced via

$$\forall g \in L^2([0,1]) \;:\; S(X \diamond Y)(g) = (SX)(g)(SY)(g)$$

on a dense subset in $L^2(\Omega) \times L^2(\Omega)$. For more details on Wick product we refer to [16, 15, 18]. For example, for a Wiener Integral $I(f)$, Hermite polynomials play the role of monomials in standard calculus as

$$I(f)^{\diamond k} = h^k_{\|f\|^2}(I(f)).$$

Therefore the stochastic exponential is also knows as Wick exponential:

$$\exp\left(I(f) - \|f\|^2/2\right) =: \exp^{\diamond}(I(f)) = \sum_{k=0}^{\infty} \frac{1}{k!} I(f)^{\diamond k}. \tag{2.1}$$

The Wick exponential $\exp^{\diamond}(I(f)_t)_{t\geq 0}$ is the unique solution of the Doléans-Dade equation

$$Y_t = f(t) Y_t dB_t,\; Y_0 = 1,$$

(cf. [19, Section 8.7]). Following [15, 10], we denote the Wiener chaos decomposition in terms of Wick products as the *Wick-analytic representation*. In particular, for fixed $f_1, \dots, f_K \in L^2([0,\infty))$, $g : \mathbb{R}^K \to \mathbb{R}$ and a square integrable left hand side, there exist $a_{l_1,\dots,l_K} \in \mathbb{R}, l_1, \dots, l_K \geq 0$, such that

$$g(I(f_1), \dots, I(f_K)) = \sum_{l_1,\dots,l_K \geq 0} a_{l_1,\dots,l_K} I(f_1)^{\diamond l_1} \diamond \cdots \diamond I(f_K)^{\diamond l_K}. \tag{2.2}$$

This is a reformulation of the the Wiener chaos decomposition in terms of generalized Hermite polynomials, see e.g. [12, 1].

Definition 2.1. *We define the class of* Wick-analytic functionals *as*

$$F^{\diamond} := \sum_{k=0}^{\infty} a_{1,k} I(f_1)^{\diamond k} \diamond \cdots \diamond \sum_{k=0}^{\infty} a_{K,k} I(f_K)^{\diamond k}, \;\; \max_{i\leq K} \sup_{k\geq 0} \sqrt[k]{k! |a_{i,k}|} =: C < \infty. \tag{2.3}$$

Remark 2.2. These Wick analytic functionals are very close to the finite chaos elements: All moments are finite and all (finite) Wick products of Wick-analytic functionals exist in $L^p(\Omega)$ for all $p \in \mathbb{N}$ (see Proposition 9 in [22]). Moreover, the analytic representation $G(I(f_1), \dots, I(f_K)) = F^{\diamond}$ for fixed $f_1, \dots, f_K \in L^2([0,\infty))$ fulfills $G \in C^{\infty}(\mathbb{R}^K_+, \mathbb{R})$ (see Proposition 10 in [22]). One advantage of the Wick-analytic reformulation is the characterization of Skorokhod integrands which allow exact simulation [22, Theorem 17].

3 Discrete stochastic calculus

As a discrete counterpart of B we consider, for every $n \in \mathbb{N}$, a random walk approximation

$$B_t^n := \frac{1}{\sqrt{n}} \sum_{i=1}^{\lfloor nt \rfloor} \xi_i^n \ , \ t \geq 0,$$

on some probability space $(\Omega_n, \mathcal{F}_n, P_n)$, where $\xi_1^n, \xi_2^n, \dots$ is a sequence of i.i.d. random variables with $P_n(\xi_1^n = \pm 1) = 1/2$ (i.e. a Rademacher sequence). The counterpart of the Gaussian Hilbert space is $\{I^n(f^n) : f \in L_n^2(\mathbb{N})\} \subset L^2(\Omega_n)$, where

$$I^n(f^n) := \frac{1}{\sqrt{n}} \sum_{i=1}^{\infty} f^n(i) \xi_i^n$$

is the discrete Wiener integral and $L_n^2(\mathbb{N}) := \{f^n : \mathbb{N} \to \mathbb{R} : \frac{1}{n} \sum_{i=1}^{\infty} (f^n(i))^2 < \infty\}$. As a counterpart, due to the discrete analogue of the Doléans-Dade equation

$$Y_i^n = Y_{i-1}^n \left(1 + \frac{1}{\sqrt{n}} f^n(i) \xi_i^n\right), \ Y_0^n = 1,$$

the discrete Wick exponential is given by

$$\exp^{\diamond_n}(I^n(f^n)) := \prod_{i=1}^{\infty} \left(1 + \frac{1}{\sqrt{n}} f^n(i) \xi_i^n\right)$$

(cf. [6]). A representation in terms of Hermite polynomials is not possible anymore, but there exists a discrete Hermite recursion formula for discrete Wick products of discrete Wiener integrals, cf. [26, Lemma 3.2].

Remark 3.1. Analogously to the continuous setting, a discrete S-transform with similar properties can be defined via

$$(S^n X^n)(f^n) := \mathbb{E}[X^n \exp^{\diamond_n}(I^n(f^n))]$$

and used to introduce discrete Malliavin operators, in particular a discrete Skorokhod integral (see [7]).

The discrete Wick product is introduced via

$$\exp^{\diamond_n}(I^n(f^n)) \diamond_n \exp^{\diamond_n}(I^n(g^n)) = \exp^{\diamond_n}(I^n(f^n + g^n)), \tag{3.1}$$

where $I^n(f^n)$ and $I^n(g^n)$ are two, possibly correlated, discrete Wiener integrals. Then, (3.1) extends bilinearly to a dense subset of $L^2(\Omega_n, \mathcal{F}_n, P_n) \times L^2(\Omega_n, \mathcal{F}_n, P_n)$ and is equivalent to the characterization in terms of the canonical basis

$$\{\Xi_A^n := \prod_{i \in A} \xi_i^n, \quad A \subseteq \mathbb{N}, |A| < \infty\}$$

as introduced in [14] via

$$\Xi_A^n \diamond_n \Xi_B^n := \Xi_{A \cup B}^n \mathbb{1}_{A \cap B = \emptyset}. \tag{3.2}$$

For example, we have the simple discrete Wiener chaos expansion

$$I^n(f^n)^{\diamond_n N} = N! \sum_{A \subset \mathbb{N}, |A| = N} \left(\prod_{i \in A} \frac{1}{\sqrt{n}} f^n(i)\right) \Xi_A^n, \tag{3.3}$$

(cf. [26, Example 3.1]).

Remark 3.2. In contrast to the continuous setting, the discrete Wick product does not fulfill

$$(S^n(X^n \diamond_n Y^n))(f^n) = (S^n X^n)(f^n)(S^n Y^n)(f^n)$$

in general, as illustrated by $X^n = I^n(g^n)$ and $Y^n = I^n(h^n)$ for $g^n, h^n \in L_n^2(\mathbb{N})$. Moreover, the discrete Wick product has zero divisors, but the continuous Wick product is free of zero divisors even in more general spaces (cf. [13]).

For more information on the discrete calculus we refer to [6, 11] or the monographs [27, 29]. In particular, the discrete counterpart of (2.1) is true for all $f^n \in L_n^2(\mathbb{N})$ as

$$\exp^{\diamond_n}(I^n(f^n)) = \sum_{k=0}^{\infty} \frac{1}{k!} I^n(f^n)^{\diamond_n k}.$$

Definition 3.3. *We define the class of* discrete Wick-analytic functionals *as ($n \in \mathbb{N}$ fixed)*

$$F^{\diamond_n} := \sum_{k=0}^{\infty} a_{1,k}^n I^n(f_1^n)^{\diamond_n k} \diamond_n \cdots \diamond_n \sum_{k=0}^{\infty} a_{K,k}^n I^n(f_K^n)^{\diamond_n k}, \quad \max_{i \le K} \sup_{k \ge 0} \sqrt[k]{k! |a_{i,k}^n|} =: C < \infty.$$

4 The main result

We denote a function $f(t,s)_{t,s\ge 0}$ with $f(t,\cdot) \in L^2([0,\infty))$ for all t and $f(t,s) = 0$ for $t \le s$ as an *integrand of Volterra type*. Analogously, the *discrete integrand of Volterra type* is given by $f^n(l,i)_{l,i\in\mathbb{N}}$ such that $f^n(l,\cdot) \in L_n^2(\mathbb{N})$ for all $l \in \mathbb{N}$ with $f^n(l,i) = 0$ for $l < i$.

We specified the conditions on the continuous and discrete integrands for a Donsker theorem for Volterra type Wiener integrals in Theorem 3 of [25]. This can be reformulated for the weak convergence

$$(I^n(f_1^n), \ldots, I^n(f_m^n)) \xrightarrow{d} (I(f_1), \ldots, I(f_m))$$

as the following variant of the multivariate central limit theorem:

Proposition 4.1. *Suppose the constants $\alpha > 0$ and $L > 0$, the Volterra integrands $f_1, \ldots, f_m$, and $f_1^n, \ldots, f_m^n$ satisfy for all $j, j' \in \{1, \ldots, m\}$ and $s, t, t_1, \ldots, t_k \in [0,1]$, $k \in \mathbb{N}$, the following conditions of infinite smallness, convergence of the variance and tightness:*

$$\lim_{n\to\infty} \max_{l \le k} \max_{i \le \lfloor nt_l \rfloor} \frac{1}{\sqrt{n}} |f_j^n(\lfloor nt_l \rfloor, i)| = 0, \tag{4.1}$$

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{\infty} f_j^n(\lfloor nt \rfloor, i) f_{j'}^n(\lfloor ns \rfloor, i) = \int_0^{\infty} f_j(t,u) f_{j'}(s,u) du, \tag{4.2}$$

$$\forall n \in \mathbb{N} \ : \quad \frac{1}{n} \sum_{i=1}^{\infty} \left(f_j^n(\lfloor nt \rfloor, i) - f_j^n(\lfloor ns \rfloor, i) \right)^2 \le L \left| \frac{\lfloor nt \rfloor}{n} - \frac{\lfloor ns \rfloor}{n} \right|^{\alpha}. \tag{4.3}$$

Then, for every $\Gamma \in \mathbb{R}^m$ (with inner product $\langle \cdot, \cdot \rangle$),

$$I^n\left(\langle \Gamma, (f_1^n, \ldots, f_m^n) \rangle\right) \xrightarrow{d} I\left(\langle \Gamma, (f_1, \ldots, f_m) \rangle\right)$$

in the Skorokhod space $D([0,1],\mathbb{R})$ as n tends to infinity. We notice that the normalizing term $1/\sqrt{n}$ is implicitly contained in the discrete Wiener integral.

Remark 4.2. Our main result is that this weak convergence is compatible with the continuous and discrete Wick-analytic functionals. This is not trivial due to the differences of the calculi and the different correlations of discrete and continuous Wiener integrals.

Remark 4.3. In the following we restrict ourselves on the time horizon $[0,1]$. The proof on $[0,T]$ for some $T>1$ is analogous. Due to the tightness in Proposition 4.1 and the Kolmogorov-Chentsov theorem, we obtain the Hölder-continuity of the paths of the Gaussian processes $I(f_i)$. Then, due to Remark 2.2, the limit processes in the following main theorem 4.4 have continuous paths as well. Hence, by [20], it is possible to conclude the tightness and weak convergence in the Skorokhod space $D([0,\infty),\mathbb{R})$.

We denote the abbreviations for the continuous (discrete) Wick-analytic functionals involved

$$F_j^{\diamond} := \sum_{k=0}^{\infty} a_{j,k}(I(f_j))^{\diamond k}, \qquad F_j^{\diamond_n} := \sum_{k=0}^{\infty} a_{j,k}^n (I^n(f_j^n))^{\diamond_n k}, \qquad F_{A,t}^{\diamond_{(n)}} := \left(\diamond_{(n)}\right)_{j\in A} F_j^{\diamond_{(n)}},$$

(from Definitions 2.1 and 3.3). To simplify the notations, we assume

$$C := \sup_{k,n\in\mathbb{N}, j=1,\dots,m} \left\{ \sqrt[k]{k!|a_{j,k}^n|},\, \sqrt[k]{k!|a_{j,k}|} \right\} < \infty, \quad \forall j: \lim_{n\to\infty} a_{j,k}^n = a_{j,k}. \tag{4.4}$$

In the following we use a total order on the power set $\mathcal{P}(\{1,\dots,m\}) = \{A_1,\dots,A_{2^m}\}$ to define vectors $\Gamma \in \mathbb{R}^{\mathcal{P}(\{1,\dots,m\})}$ (with inner product $\langle\cdot,\cdot\rangle$) and the vector processes $\left(F_{A_1,t}^{\diamond_{(n)}},\dots,F_{A_{2^m},t}^{\diamond_{(n)}}\right)$.

Theorem 4.4. *Suppose the Volterra integrands $f_1,\dots,f_m$, and $f_1^n,\dots,f_m^n$ satisfy the assumptions* (4.1)-(4.3) *in Proposition 4.1 and the Wick-analytic functionals satisfy Condition* (4.4)*. Then, for every vector of constants $\Gamma \in \mathbb{R}^{\mathcal{P}(\{1,\dots,m\})}$,*

$$\left\langle \Gamma, \left(F_{A_1,t}^{\diamond_n},\dots,F_{A_{2^m},t}^{\diamond_n}\right)\right\rangle_{t\in[0,1]} \xrightarrow{d} \left\langle \Gamma, \left(F_{A_1,t}^{\diamond},\dots,F_{A_{2^m},t}^{\diamond}\right)\right\rangle_{t\in[0,1]}$$

in the Skorokhod space $D([0,1],\mathbb{R})$ as n tends to infinity.

Remark 4.5. (i) According to the Cramér-Wold device and [17, Theorem 16.16], the assertion is equivalent to the weak convergence of the vector processes

$$\left(F_{A_1,t}^{\diamond_n},\dots,F_{A_{2^m},t}^{\diamond_n}\right)_{t\in[0,1]} \xrightarrow{d} \left(F_{A_1,t}^{\diamond},\dots,F_{A_{2^m},t}^{\diamond}\right)_{t\in[0,1]}.$$

(ii) The assumptions (4.1)-(4.3) for the kernel (1.1) of the fractional Brownian motion with $H\in(0,1)$ are checked in [25] ((4.2) for the kernels follows as [25, Remark 3 (ii)]). In this case it is $L=1$ and $\alpha=2H$. Similarly to the Wick exponential (2.1), there is the representation

$$\sin^{\diamond}(I(f)) := \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{(2k-1)!}(I(f))^{\diamond(2k-1)} = \exp(\|f\|^2/2)\sin(I(f))$$

(cf. p. 107 in [15]). As an example, for arbitrary $H_1,H_2,H_3\in(0,1)$, the process with correlated components

$$(\exp^{\diamond}(B^{H_1}+B^{H_2}), (B^{H_1}+B^{H_3})^{\diamond 10}, \sin^{\diamond}(B^{H_1}+B^{H_3})),$$

is the weak limit of the sequence of processes

$$(\exp^{\diamond_n}(B^{n,H_1}+B^{n,H_2}), (B^{n,H_1}+B^{n,H_3})^{\diamond_n 10}, \sin^{\diamond_n}(B^{n,H_1}+B^{n,H_3})),$$

where $B_t^{n,H} := I^n(z_H^n(\lfloor nt\rfloor,\cdot))_{t\geq 0}$ is a disturbed random walk (discrete Wiener integral of Volterra type) in a fractional Donsker theorem and $z_H^n(\lfloor nt\rfloor, i) := \int_{(i-1)/n}^{i/n} z_H(\lfloor nt\rfloor/n, s)ds$ is the discrete Volterra type integrand (see [25, Theorem 4]). The exponentials involved are geometric fractional Brownian motion and discrete geometric fractional Brownian motion, cf. [5]. Similarly, thanks to the continuous mapping theorem, we conclude that the sequence of processes

$$\left(B_t^{n,H_1}, \exp(-t^{2H_2}/2)\exp^{\diamond_n}(B_t^{n,H_2}), \exp(-t^{2H_3}/2)\sin^{\diamond_n}(B_t^{n,H_3})\right)_{t\geq 0}$$

converges weakly to the process (1.2) as n tends to infinity.

The proof of Theorem 4.4 relies on the following estimate. The technical proof is postponed to Section 5.

Lemma 4.6. *Suppose the Volterra integrands $f_1, \dots, f_m$, and $f_1^n, \dots, f_m^n$ satisfy* (4.1)-(4.3) *in Proposition 4.1 and the Wick-analytic functionals satisfy Condition* (4.4). *Then, for all $K \in \mathbb{N}$ and $A \subseteq \{1, \dots, m\}$ there exists a constant $c = c(K, C, L, |A|)$ such that*

$$\forall s, t \in [0,1]: \quad \mathbb{E}\left[\left(F_{A,t}^{\diamond_n} - F_{A,s}^{\diamond_n}\right)^{2K}\right] \leq c\left|\frac{\lfloor nt\rfloor}{n} - \frac{\lfloor ns\rfloor}{n}\right|^{K\alpha}. \tag{4.5}$$

Moreover we make use of the following variant of a well-known tightness criterion Theorem 15.6 in [9]. See e.g. Remark 1 in [25] for the connection.

Lemma 4.7. *Suppose X_n, $n \in \mathbb{N}$, are processes with paths in the Skorokhod space $D([0,1], \mathbb{R})$ and X is a process with paths in $C([0,1], \mathbb{R})$. We suppose furthermore the following conditions:*

1. *The sequence X_n converges weakly in finite-dimensions to X, i.e.*

$$\forall t_1, \dots, t_k \in [0,1]: \ (X_{t_1}^n, \dots, X_{t_k}^n) \xrightarrow{d} (X_{t_1}, \dots, X_{t_k}).$$

2. *There exist constants $\beta > 1$ and $K, L > 0$ such that for all $s \leq t$ in $[0,1]$, $n \in \mathbb{N}$,*

$$\mathbb{E}\left[|X_t^n - X_s^n|^K\right] \leq L\left|\frac{\lfloor nt\rfloor}{n} - \frac{\lfloor ns\rfloor}{n}\right|^{\beta}.$$

Then X^n converges weakly to X in the Skorokhod space $D([0,1], \mathbb{R})$.

Proof of Theorem 4.4. In [26, Theorem 4.1] we derived a Wick functional limit theorem which gives the convergence of the finite-dimensional distributions. Suppose $s, t \in [0,1]$ and $\Gamma = (\gamma_{A_1}, \dots, \gamma_{A_{2^m}}) \in \mathbb{R}^{\mathcal{P}(\{1,\dots,m\})}$. Thanks to the Hölder inequality and Lemma 4.6, we obtain

$$\begin{aligned}
&\mathbb{E}\left[\left(\left\langle\Gamma, \left(F_{A_1,t}^{\diamond_n}, \dots, F_{A_{2^m},t}^{\diamond_n}\right) - \left(F_{A_1,s}^{\diamond_n}, \dots, F_{A_{2^m},s}^{\diamond_n}\right)\right\rangle\right)^{2K}\right] \\
&= \mathbb{E}\left[\left(\sum_{A\subseteq\{1,\dots,m\}} \gamma_A\left(F_{A,t}^{\diamond_n} - F_{A,s}^{\diamond_n}\right)\right)^{2K}\right] \\
&\leq \left(\sum_{A\subseteq\{1,\dots,m\}} \gamma_A^{\frac{2K}{2K-1}}\right)^{2K-1} \sum_{\emptyset\neq A\subseteq\{1,\dots,m\}} \mathbb{E}\left[\left(F_{A,t}^{\diamond_n} - F_{A,s}^{\diamond_n}\right)^{2K}\right] \leq L\left|\frac{\lfloor nt\rfloor}{n} - \frac{\lfloor ns\rfloor}{n}\right|^{K\alpha}
\end{aligned}$$

for some constant $L = L(K, C, L, \Gamma) > 0$. Thus, for every $\alpha > 0$ we find some $K \in \mathbb{N}$ with $K\alpha > 1$ and it suffices to apply the tightness criterion in Lemma 4.7. $\square$

5 Proof of Lemma 4.6

Firstly we note some expansion formulae and an inequality. We denote by $\dot\bigcup$ the disjoint union.

Suppose $n, K \in \mathbb{N}$, $a_j, b_j, c_j, a_j^{(i)}, b_j^{(i)} \in \mathbb{R}$ and $A, A_i, D \subseteq \{1, \ldots, n\}$ for all $i, j \in \mathbb{N}$. We make use of the shorthand notations for products as

$$a_A := \prod_{j\in A} a_j,\ a_A^{(i)} := \prod_{j\in A} a_j^{(i)}.$$

The following formula is clear by expansion:

$$\prod_{j\in D}\left(a_j^{(1)} + a_j^{(2)} + \ldots + a_j^{(K)}\right) = \sum_{\dot\bigcup_{i=1,\ldots,K} A_i = D} a_{A_1}^{(1)} \cdots a_{A_K}^{(K)}. \tag{5.1}$$

Via the expansion $\prod_{i\in A} a_i - \prod_{i\in A} b_i = \sum_{i\in A}(a_i - b_i)\prod_{k\in A, k<i} a_i \prod_{k\in A, k>i} b_i$ and the Cauchy-Schwarz inequality, we observe

$$\left(\prod_{i\in A} a_i - \prod_{i\in A} b_i\right)^2 \le |A| \sum_{i\in A}(a_i - b_i)^2 \prod_{k\in A, k<i} a_i^2 \prod_{k\in A, k>i} b_i^2. \tag{5.2}$$

The generalized Cauchy-Schwarz inequality follows easily by induction:

$$\sum_{j=1}^n a_j^{(1)} \ldots a_j^{(K)} \le \prod_{i=1}^K \left(\sum_{j=1}^n (a_j^{(i)})^2\right)^{1/2}. \tag{5.3}$$

Proposition 5.1.

$$\sum_{D\subseteq\{1,\ldots,n\}} \sum_{A\subseteq D} (a_A - b_A)^2 c_{D\setminus A}^2 \le 2\exp\left(\sum_{j=1}^n (a_j^2 + b_j^2 + c_j^2)\right) \sum_{j=1}^n (a_j - b_j)^2. \tag{5.4}$$

Proof. Firstly, thanks to (5.1) we observe

$$\sum_{A\subseteq\{1,\ldots,n\}} b_A^2 = \prod_{i=1}^n (1 + b_i^2). \tag{5.5}$$

The inequality

$$\sum_{A\subseteq\{1,\ldots,n\}} (a_A - b_A)^2 \le 2\left(\prod_{i=1}^n \left(1 + a_i^2 + b_i^2\right)\right) \sum_{i=1}^n (a_i - b_i)^2 \tag{5.6}$$

is clearly true for $n = 1$ and then proved by induction: W.l.o.g. let $a_{n+1}^2 \le b_{n+1}^2$. Due to $(a_{n+1}a_A - b_{n+1}b_A)^2 \le 2a_{n+1}^2(a_A - b_A)^2 + 2b_A^2(a_{n+1} - b_{n+1})^2$, (5.5) and the induction hypothesis, we obtain

$$\begin{aligned}\sum_{A\subseteq\{1,\ldots,n+1\}} (a_A - b_A)^2 &= \sum_{n+1\notin A\subseteq\{1,\ldots,n+1\}} (a_A - b_A)^2 + \sum_{n+1\in A\subseteq\{1,\ldots,n+1\}} (a_A - b_A)^2 \\ &\le (1 + 2a_{n+1}^2) \sum_{A\subseteq\{1,\ldots,n\}} (a_A - b_A)^2 + 2\prod_{i=1}^n (1 + b_i^2)(a_{n+1} - b_{n+1})^2 \\ &\le 2\left(\prod_{i=1}^{n+1}\left(1 + a_i^2 + b_i^2\right)\right)\sum_{i=1}^{n+1}(a_i - b_i)^2.\end{aligned}$$

Via interchanging sums, for functions P, Q on $\mathcal{P}(\{1,\dots,n\})$, we have

$$\sum_{\substack{D\subseteq\{1,\dots,n\},\\ A\subseteq D}} P(A)Q(D\setminus A) = \sum_{\substack{A,B\in\{1,\dots,n\}\\ A\cap B=\emptyset}} P(A)Q(B) = \sum_{\substack{A\subseteq\{1,\dots,n\},\\ B\subseteq\{1,\dots,n\}\setminus A}} P(A)Q(B). \tag{5.7}$$

Hence, via (5.7), (5.5) and (5.6),

$$\begin{aligned}\sum_{D\subseteq\{1,\dots,n\}}\sum_{A\subseteq D}(a_A-b_A)^2 c_{D\setminus A}^2 &= \sum_{A\subseteq\{1,\dots,n\}}(a_A-b_A)^2\sum_{B\subseteq\{1,\dots,n\}\setminus A} c_B^2\\ \le \sum_{A\subseteq\{1,\dots,n\}}(a_A-b_A)^2\prod_{j=1}^n(1+c_j^2) &\le 2\left(\prod_{j=1}^n(1+c_j^2)\left(1+a_i^2+b_i^2\right)\right)\sum_{i=1}^n(a_i-b_i)^2.\end{aligned}$$

Thanks to $(1+x)\le\exp(x)$ we conclude (5.4). $\square$

Proof of Lemma 4.6. Due to (3.2), we only have nonvanishing discrete Wick products on disjoint sets as

$$\Xi^n_{B_1}\diamond_n\cdots\diamond_n\Xi^n_{B_k} = \Xi^n_{\dot\bigcup_{i=1,\dots,k}B_i}. \tag{5.8}$$

To simplify the products to common factors we define for shorthand

$$\mathbf{f}^n_{i,u,l} := \frac{1}{\sqrt{n}}f^n_i(\lfloor nu\rfloor, l), \qquad \mathbf{f}^n_{i,u,A} := \prod_{l\in A}\mathbf{f}^n_{i,u,l}.$$

Thus, via (3.3) and (5.8), the discrete Wiener chaos expansion of the discrete Wick product of Wick-analytic functionals is given by

$$\begin{aligned}F^{\diamond_n}_{A,u} &= (\diamond_n)_{i\in A}\left(\sum_{B\subseteq\{1,\dots,n\}}|B|!a^n_{i,|B|}\mathbf{f}^n_{i,u,B}\Xi^n_B\right)\\ &= \sum_{D\subseteq\{1,\dots,n\}}\sum_{\dot\bigcup_{i\in A}B_i=D}\big(\prod_{i\in A}|B_i|!a^n_{i,|B_i|}\big)\left(\prod_{i\in A}\mathbf{f}^n_{i,u,B_i}\right)\Xi^n_D.\end{aligned}$$

Since the set $\{\Xi^n_A, A\subset\{1,\dots,n\}\}$ is orthonormal, we conclude the L^2-norm

$$\mathbb{E}\left[\left(F^{\diamond_n}_{A,u}\right)^2\right] = \sum_{D\subseteq\{1,\dots,n\}}\left(\sum_{\dot\bigcup_{i\in A}B_i=D}\big(\prod_{i\in A}|B_i|!a^n_{i,|B_i|}\big)\left(\prod_{i\in A}\mathbf{f}^n_{i,u,B_i}\right)\right)^2.$$

Analogously, for all integers $K\ge 1$, the orthogonality yields

$$\begin{aligned}&\mathbb{E}\left[\left(F^{\diamond_n}_{A,t}-F^{\diamond_n}_{A,s}\right)^{2K}\right]\\ &= \sum_{\substack{D_1,\dots,D_{2K}\subseteq\{1,\dots,n\}\\ D_1\neq\emptyset,\dots,D_{2K}\neq\emptyset,\\ D_1\triangle D_2\triangle\cdots\triangle D_{2K}=\emptyset}}\prod_{j=1}^{2K}\left(\sum_{\dot\bigcup_{i\in A}B_i=D_j}\big(\prod_{i\in A}|B_i|!a^n_{i,|B_i|}\big)\left(\prod_{i\in A}\mathbf{f}^n_{i,t,B_i}-\prod_{i\in A}\mathbf{f}^n_{i,s,B_i}\right)\right),\end{aligned} \tag{5.9}$$

where the condition $D_1\triangle D_2\triangle\cdots\triangle D_{2K}=\emptyset$ gives exactly those terms in the expansion of $\left(F^{\diamond_n}_{A,t}-F^{\diamond_n}_{A,s}\right)^{2K}$ with nonzero expectation. The sums involved in (5.9) require a subtle reformulation. An application of standard inequalities is not obvious since (5.9) is far beyond the multiplicative form as in a Cauchy-Schwarz inequality (5.3). The reformulation of the sums is the content of Step 1 below. Then, the final upper bound inequality will be proved in Step 2.

Step 1 : Suppose a multiset of pairs in the first sum in (5.9), i.e. a family of nonempty sets $D_1, \ldots, D_{2K} \subseteq \{1, \ldots, n\}$ such that $D_1 \triangle D_2 \triangle \cdots \triangle D_{2K} = \emptyset$ (and therefore n large enough, e.g. $n \geq 2K$). Such a chosen family can be coded by intersection sets as follows: Let the index set

$$\mathcal{F} := \{\{j_1, \ldots, j_{2l}\} : 1 \leq j_1 < \ldots < j_{2l} \leq 2K, 1 \leq l \leq K\}$$

and the disjoint family of sets $U_{\{1,2\}}, U_{\{1,3\}}, \ldots, U_{\{1,2,\ldots,2K\}} \subseteq \{1, \ldots, n\}$ via

$$U_f = \bigcap_{j \in f} D_j \;\Leftrightarrow\; D_j = \bigcup_{f \in \mathcal{F}, j \in f} U_f.$$

Every intersection set U_f is covered by an even number of sets D_j which illustrates the condition $D_1 \triangle D_2 \triangle \cdots \triangle D_{2K} = \emptyset$.

The inner sum in (5.9) is reformulated by a map: Every set of partitions is generated by a unique surjective map

$$G = (G_1, \ldots, G_{2K}) : (D_1, \ldots, D_{2K}) \to A^{2K} \tag{5.10}$$

$$\text{as } \dot{\bigcup}_{i \in A} B_i = D_j = \bigcup_{f \in \mathcal{F}, j \in f} U_f \;\Leftrightarrow\; (B_i)_{i \in A} = (G_j^{-1}(i))_{i \in A}.$$

For shorthand, we denote by $\sum_U$ the sum over all disjoint families $\{U_f, f \in \mathcal{F}\}$ with nonempty $\bigcup_{f \in \mathcal{F}, j \in f} U_f = D_j$ for all $j = 1, \ldots, 2K$. The new inner sum over all surjective maps (5.10) is abbreviated via $\sum_G$. This reformulations of the first sum in (5.9) will yield a new sum which makes a multiplicative form of the summands (as in (5.3)) visible. Hence, the right hand side in (5.9) equals

$$\begin{aligned} &\sum_U \prod_{j=1}^{2K} \left(\sum_{G_j : \bigcup_{f \in \mathcal{F}, j \in f} U_f \to A} (\prod_{i \in A} |G_j^{-1}(i)|! a^n_{i, |G_j^{-1}(i)|})(\prod_{i \in A} \mathbf{f}^n_{i,t,G_j^{-1}(i)} - \prod_{i \in A} \mathbf{f}^n_{i,s,G_j^{-1}(i)}) \right) \\ &= \sum_U \sum_G \prod_{j=1}^{2K} (\prod_{i \in A} |G_j^{-1}(i)|! a^n_{i, |G_j^{-1}(i)|})(\prod_{i \in A} \mathbf{f}^n_{i,t,G_j^{-1}(i)} - \prod_{i \in A} \mathbf{f}^n_{i,s,G_j^{-1}(i)}). \end{aligned} \tag{5.11}$$

Step 2 : Thanks to this multiplicative form in (5.11) we conclude via the generalized Cauchy-Schwarz inequality in (5.3) and the condition (4.4),

$$\begin{aligned} &\sum_U \sum_G \prod_{j=1}^{2K} (\prod_{i \in A} |G_j^{-1}(i)|! a^n_{i, |G_j^{-1}(i)|})(\prod_{i \in A} \mathbf{f}^n_{i,t,G_j^{-1}(i)} - \prod_{i \in A} \mathbf{f}^n_{i,s,G_j^{-1}(i)}) \\ &\leq \prod_{j=1}^{2K} \left(\sum_U \sum_G \prod_{i \in A} \left(|G_j^{-1}(i)|! a^n_{i, |G_j^{-1}(i)|} \right)^2 \left(\prod_{i \in A} \mathbf{f}^n_{i,t,G_j^{-1}(i)} - \prod_{i \in A} \mathbf{f}^n_{i,s,G_j^{-1}(i)} \right)^2 \right)^{1/2} \\ &\leq \prod_{j=1}^{2K} \left(\sum_U \sum_G \left(\prod_{i \in A} C^{|G_j^{-1}(i)|} \mathbf{f}^n_{i,t,G_j^{-1}(i)} - \prod_{i \in A} C^{|G_j^{-1}(i)|} \mathbf{f}^n_{i,s,G_j^{-1}(i)} \right)^2 \right)^{1/2}. \end{aligned} \tag{5.12}$$

Finally, we define the shorthand elements

$$\mathbf{a}_l^{(n,i)} := C\mathbf{f}^n_{i,t,l},\; \mathbf{b}_l^{(n,i)} := C\mathbf{f}^n_{i,s,l},\; \mathbf{a}_A^{(n,i)} := \prod_{l \in A} \mathbf{a}_l^{(n,i)},\; \mathbf{b}_A^{(n,i)} := \prod_{l \in A} \mathbf{b}_l^{(n,i)}.$$

Applying (5.2) on the summands in the right hand side of (5.12) and interchanging the (nonnegative) sum, we obtain

$$\begin{aligned}
&\sum_U \sum_G \left(\prod_{i\in A} C^{|G_j^{-1}(i)|} \mathbf{f}^n_{i,t,G_j^{-1}(i)} - \prod_{i\in A} C^{|G_j^{-1}(i)|} \mathbf{f}^n_{i,s,G_j^{-1}(i)} \right)^2 \\
&\leq |A| \sum_{i\in A} \sum_U \sum_G (\mathbf{a}^{(n,i)}_{G_j^{-1}(i)} - \mathbf{b}^{(n,i)}_{G_j^{-1}(i)})^2 \prod_{k\in A, k<i} (\mathbf{a}^{(n,i)}_{G_j^{-1}(k)})^2 \prod_{k\in A,k>i} (\mathbf{b}^{(n,i)}_{G_j^{-1}(k)})^2. \qquad (5.13)
\end{aligned}$$

Thanks to (5.1) and

$$\mathbf{c}_l^{(n,i)} := \sqrt{\sum_{k\in A,k<i} |\mathbf{a}_l^{(n,k)}|^2 + \sum_{k\in A,k>i} |\mathbf{b}_l^{(n,k)}|^2}, \ \mathbf{c}_A^{(n,i)} := \prod_{l\in A} \mathbf{c}_l^{(n,i)},$$

for every $i \in A$,

$$\begin{aligned}
&\sum_{\dot\bigcup_{k\in A\setminus\{i\}} E_k = D} \prod_{k\in A,k<i} |\mathbf{a}^{(n,k)}_{E_k}|^2 \prod_{k\in A,k>i} |\mathbf{b}^{(n,k)}_{E_k}|^2 \\
&= \prod_{j\in D} \left(\sum_{k<i} |\mathbf{a}_j^{(n,k)}|^2 + \sum_{k>i} |\mathbf{b}_j^{(n,k)}|^2 \right) = (\mathbf{c}_D^{(n,i)})^2,
\end{aligned}$$

and therefore

$$\sum_{\dot\bigcup_{k\in A} E_k = E} (\mathbf{a}^{(n,i)}_{E_i} - \mathbf{b}^{(n,i)}_{E_i})^2 \prod_{k<i} |\mathbf{a}^{(n,k)}_{E_k}|^2 \prod_{k>i} |\mathbf{b}^{(n,k)}_{E_k}|^2 = \sum_{\emptyset \neq F \subseteq E} (\mathbf{a}^{(n,i)}_F - \mathbf{b}^{(n,i)}_F)^2 (\mathbf{c}^{(n,i)}_{E\setminus F})^2. \qquad (5.14)$$

For every fixed set $E \subseteq \{1,\dots,n\}$ every partition into $\bigcup_{f\in\mathcal{F}} U_f = E$ has less than 2^{2K} elements. Hence, we have

$$\sum_{\bigcup_{f\in\mathcal{F}} U_f = E} 1 \leq 2^{2K|E|}.$$

Similarly, for a fixed partition $\bigcup_{f\in\mathcal{F}} U_f = E$ and every $j = 1,\dots,2K$, the number of surjective maps G_j in (5.10) is less than $|A|^{|E|}$ and therefore

$$\sum_G 1 \leq |A|^{2K|E|}.$$

Thus, by these upper bounds and (5.14), the inner sums $\sum_U \sum_G$ in (5.13) are handled simultanously to

$$\begin{aligned}
&\sum_U \sum_G (\mathbf{a}^{(n,i)}_{G_j^{-1}(i)} - \mathbf{b}^{(n,i)}_{G_j^{-1}(i)})^2 \prod_{k\in A,k<i} (\mathbf{a}^{(n,i)}_{G_j^{-1}(k)})^2 \prod_{k\in A,k>i} (\mathbf{b}^{(n,i)}_{G_j^{-1}(k)})^2 \\
&\leq \sum_{\emptyset\neq E\subseteq\{1,\dots,n\}} (2|A|)^{2K|E|} \sum_{\dot\bigcup_{k\in A} E_k = E} (\mathbf{a}^{(n,i)}_{E_i} - \mathbf{b}^{(n,i)}_{E_i})^2 \prod_{k\in A,k<i} |\mathbf{a}^{(n,k)}_{E_k}|^2 \prod_{k\in A,k>i} |\mathbf{b}^{(n,k)}_{E_k}|^2 \\
&\leq \sum_{E\subseteq\{1,\dots,n\}} (2|A|)^{2K|E|} \sum_{\emptyset\neq F\subseteq E} (\mathbf{a}^{(n,i)}_F - \mathbf{b}^{(n,i)}_F)^2 (\mathbf{c}^{(n,i)}_{E\setminus F})^2.
\end{aligned}$$

An application of Proposition 5.1, the conditions (4.1)-(4.3) and $s,t \in [0,1]$ now yield

$$\sum_{\emptyset\neq E\subseteq\{1,\dots,n\}} (2|A|)^{2K|E|} \sum_{\emptyset\neq F\subseteq E} (\mathbf{a}^{(n,i)}_F - \mathbf{b}^{(n,i)}_F)^2 (\mathbf{c}^{(n,i)}_{E\setminus F})^2$$

$$
\begin{aligned}
&\le 2\exp\left((2|A|)^{2K}\sum_{j=1}^{n}(|\mathbf{a}_j^{(n,i)}|^2+|\mathbf{b}_j^{(n,i)}|^2+|\mathbf{c}_j^{(n,i)}|^2)\right)(2|A|)^{2K}\sum_{j=1}^{n}(\mathbf{a}_j^{(n,i)}-\mathbf{b}_j^{(n,i)})^2\\
&= 2\exp\left((2|A|)^{2K}C^2\sum_{i\in A}\sum_{j=1}^{n}(|\mathbf{f}_{i,t,j}^{n}|^2+|\mathbf{f}_{i,s,j}^{n}|^2)\right)(2|A|)^{2K}C^2\sum_{j=1}^{n}(\mathbf{f}_{i,t,j}^{n}-\mathbf{f}_{i,s,j}^{n})^2\\
&< \exp\left(2^{2K+1}|A|^{3K}C^2L(t^\alpha+s^\alpha)\right)(2|A|)^{2K+1}C^2L\left|\frac{\lfloor nt\rfloor}{n}-\frac{\lfloor ns\rfloor}{n}\right|^\alpha\\
&< \exp\left(2^{2K+3}|A|^{3K}C^2L\right)\left|\frac{\lfloor nt\rfloor}{n}-\frac{\lfloor ns\rfloor}{n}\right|^\alpha. \qquad (5.15)
\end{aligned}
$$

Plugging (5.11)-(5.15) into (5.9), we conclude

$$
\mathbb{E}\left[\left(F_{A,t}^{\diamond_n}-F_{A,s}^{\diamond_n}\right)^{2K}\right]\le c\left|\frac{\lfloor nt\rfloor}{n}-\frac{\lfloor ns\rfloor}{n}\right|^{K\alpha}
$$

for the constant $c:=\exp\left(K2^{2K+3}|A|^{3K}C^2L\right)$. □

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