Erratum: Nonlinear filtering for reflecting diffusions in random environments via nonparametric estimation*

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Abstract

This is an erratum to EJP paper number 18, volume 9, Nonlinear filtering for reflecting diffusions in random environments via nonparametric estimation.

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Equation (2.2) in [4] is incorrect, which puts the proof of Theorem 2 [4, Appendix] in doubt. The theorem is true as stated. In the following, we will revise the places in [4, Appendix] where equation (2.2) is used.

As in [4], we let $p^0(t, x, y)$ be the transition density function of $X^t_0$. Theorem 3.1, Theorem 3.4 and Lemma 4.3 in [2] imply that

$$
p^0(t, x, y) \leq c_1 t^{-d/2} \exp\left(-\frac{|x - y|^2}{c_2 t}\right), \quad \forall t > 0, x, y \in \overline{D},
$$

(0.1)

and

$$
p^0(t, x, y) \geq c_3 t^{-d/2}, \quad \forall t > 0, x, y \in \overline{D} \text{ such that } |x - y| \leq \varepsilon \sqrt{t},
$$

(0.2)

where $c_1, c_2, c_3, \varepsilon > 0$ are constants independent of $x, y, t$.

We denote by $p(t, x, y)$ the transition density function of $X_t$ and $\mathcal{E}$. It is known that (0.1) and (0.2) are quasi-isometry stable (cf. the remark before Section 1.2 and the remark after Theorem 1.2 in [3]). For any $M > 0$, there exist constants $c_1(M), c_2(M), c_3(M), \varepsilon(M) > 0$ independent of $x, y, t$ such that if $\|W\|_\infty \leq M$ then

$$
p(t, x, y) \leq c_1(M) t^{-d/2} \exp\left(-\frac{|x - y|^2}{c_2(M) t}\right), \quad \forall t > 0, x, y \in \overline{D},
$$

(0.3)

and

$$
p(t, x, y) \geq c_3(M) t^{-d/2}, \quad \forall t > 0, x, y \in \overline{D} \text{ such that } |x - y| \leq \varepsilon(M) \sqrt{t}.
$$

(0.4)

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It is known that (0.3) and (0.4) imply that (cf. [1, Corollary 4.2]) for any $M > 0$, there exist constants $c_4(M), 0 < \alpha(M) < 1$ independent of $x, x', t$, such that if $\|W\|_{\infty} \leq M$ and $f \in B_b(D)$ satisfy $\|f\|_{\infty} \leq M$ then

$$\left| \int_D p(t, x, y) \mu(dy) - \int_D p(t, x', y) \mu(dy) \right| \leq c_4(M) |x - x'|^{\alpha(M)}, \quad \forall t > 0, x, x' \in \overline{D}. \quad (0.5)$$

Hence $X_t$ is a strong Feller diffusion.

We define on $L^2(D; dx)$ the symmetric bilinear form

$$A^W(u, v) = \frac{1}{2} \int_D \sum_{i,j=1}^d a_{ij}(x) \frac{\partial^2 u W(x)}{\partial x_i \partial x_j}(x) e^{-W(x)} dx, \quad u, v \in D(A^W),$$

$$D(A^W) = \{ u \in L^2(D; dx) : u e^{W/2} \in H^{1,2}(D) \}.$$ 

Let $W_n \in B_b(\overline{D})$, $n \in \mathbb{N}$, satisfy $\lim_{n \to \infty} \|W_n - W\|_{\infty} = 0$. Similar to [5, Lemma, page 864], we can show that the form $A^{W_n}$ is Mosco-convergent to the form $A^W$ on $L^2(D; dx)$, equivalently, $(s_t^{W_n})_{t>0}$ converges to $(s_t^W)_{t>0}$ strongly on $L^2(D; dx)$, where $(s_t^{W_n})_{t>0}$ and $(s_t^W)_{t>0}$ denote the semigroups of $A^{W_n}$ and $A^W$, respectively. Note that for $f \in B_b(\overline{D})$, we have

$$p_t f = e^{W/2} s_t^W (e^{-W/2} f), \quad \forall t > 0.$$ 

Denote by $(p_t^n)_{t>0}$ the semigroup associated with $X^n$. Then, we obtain by Theorem 1 in [4] that $p_t^n f$ converges to $p_t f$ on $L^2(D; dx)$ for any $f \in B_b(D)$ and $t > 0$. Therefore, we obtain by (0.5) that for any sequence $\{\nu^n\}$ of probability measures on $\overline{D}$ converging weakly to some probability measure $\nu$ on $\overline{D}$, $(X^n_0, X^n_1)$ with the initial distribution $\nu^n$ converges weakly to $(X_0, X_1)$ with the initial distribution $\nu$.

References