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CORRECTION NOTE

LIMIT THEOREMS FOR EMPIRICAL PROCESSES OF CLUSTER FUNCTIONALS

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We correct an error in a technical lemma of Drees and Rootzén [*Ann. Statist.* **38** (2010) 2145–2186] and discuss consequences for applications.

In Lemma 5.2(vii), it is stated that under the conditions (B1) and (B3) the length $L(Y_n)$ of the core of a cluster satisfies $\lim_{k\to\infty} \limsup_{n\to\infty} P\{L(Y_n) > k\}/(r_n v_n) = 0$. However, in general, the first inequality in the proof of this part of the lemma is not correct, and it seems likely that the assertion does not hold under the stated conditions. Note that this part of the lemma is used only in Remark 3.7(i); so none of the other results are affected.

The easiest way to correct the error is to replace condition (B3) with the corresponding condition for φ-mixing coefficients

$$\varphi_{n,k} := \sup_{1\le l\le n-k-1} \sup_{B\in\mathcal{B}^n_{n,l+k+1}, C\in\mathcal{B}^l_{n,1}} \left|P(B) - P(B|C)\right|$$

[with the convention $P(B|C) = P(B)$ if $P(C) = 0$], that is, to assume $\lim_{m\to\infty} \limsup_{n\to\infty} \varphi_{n,m} = 0$. The arguments of the proof of Lemma 5.2(vii) are rectified if $\beta_{n,k}$ is replaced with $\varphi_{n,k}$ everywhere.

However, often the following simpler condition is easier to verify:

($\widetilde{B}3$) For all $n \in \mathbb{N}$ and all $1 \le i \le r_n$ there exists $s_n(i) \ge P(X_{n,i+1} \ne 0|X_{n,1} \ne 0)$ such that $s_\infty(i) := \lim_{n\to\infty} s_n(i)$ exists and $\lim_{n\to\infty} \sum_{i=1}^{r_n} s_n(i) = \sum_{i=1}^{\infty} s_\infty(i) < \infty$.

Since, by stationarity,

$$\begin{aligned}\frac{1}{r_n v_n} P\{L(Y_n) > k\} &\le \frac{1}{r_n v_n} \sum_{i=1}^{r_n-k} \sum_{j=i+k}^{r_n} P(X_{n,j} \ne 0|X_{n,i} \ne 0) P\{X_{n,i} \ne 0\} \\ &\le \sum_{j=k}^{r_n} s_n(j),\end{aligned}$$

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the assertion of Lemma 5.2(vii) follows readily.

To check condition ($\widetilde{B}3$), typically one bounds $P(X_{n,i+1} \neq 0 | X_{n,1} \neq 0)$ by an expression of the form $s_n(i) = b_n + c_i$ with $b_n = o(1/r_n)$ and $\sum_{i=1}^{\infty} c_i < \infty$. The interchangeability of the limit and the sum is then automatically fulfilled. For example, ($\widetilde{B}3$) has been verified in Example 8.3 of Drees, Segers and Warchoł (2015) for solutions to stochastic recurrence equations.

Condition ($\widetilde{B}3$) has the additional advantage that in Remark 3.7(i) it renders condition (3.9) superfluous, that is, condition (C3) is met if ($\widetilde{B}3$) and (3.8) hold. To see this, check that, for bounded functions ϕ, ψ, using stationarity $E(g_\phi(Y_n) g_\psi(Y_n))/(r_n v_n) = \mathrm{Cov}(g_\phi(Y_n), g_\psi(Y_n))/(r_n v_n) + O(r_n v_n)$ can be represented as

$$\frac{1}{v_n} E\big(\phi(X_{n,1})\psi(X_{n,1})\big) + \sum_{k=1}^{r_n-1} \frac{1}{v_n}\left(1 - \frac{k}{r_n}\right)\Big(E\big(\phi(X_{n,1})\psi(X_{n,k+1})\big) + E\big(\psi(X_{n,1})\phi(X_{n,k+1})\big)\Big)$$

which tends to $c(g_\phi, g_\psi)$ defined in (3.10) by our assumptions and Pratt's lemma [Pratt (1960)], because the kth summand can be bounded in absolute value by $2\|\phi\|_\infty \|\psi\|_\infty s_n(k)$. Moreover, using the above representation with $\phi = \psi = 1_{E\setminus\{0\}}$, one immediately sees that ($\widetilde{B}3$) also implies condition (3.5).

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