

## A NOTE ON “BAYESIAN NONPARAMETRIC ESTIMATORS DERIVED FROM CONDITIONAL GIBBS STRUCTURES”

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The present note aims at clarifying some possibly confusing notation used in [1], even if its correct interpretation should be clear from context and from the proofs contained therein. In particular, the expression

$$(*) \quad \mathbb{P}[K_n = k, \mathbf{N}_n = (n_1, \dots, n_{K_n})]$$

displayed in (3) of [1] has been used to indicate the probability of observing a specific realization of a random partition of the integers  $[n] = \{1, \dots, n\}$  into  $k$  blocks of sizes  $(n_1, \dots, n_k) \in \Delta_{n,k}$ . However, the notation in (\*) may be actually misleading since its natural interpretation is as the probability of all partitions of  $[n] = \{1, \dots, n\}$  into  $k$  blocks of sizes  $(n_1, \dots, n_k) \in \Delta_{n,k}$ . For this reason, (3) in [1] should be rewritten as

$$(3) \quad \Pi_k^{(n)}(n_1, \dots, n_k) = \frac{\theta^k}{(\theta)_n} \prod_{j=1}^k (n_j - 1)!,$$

where  $\Pi_k^{(n)}$  is the *exchangeable partition probability function* notation introduced in Section 2. Hence, if  $\Pi_n$  is a random element taking values in the set of all partitions of  $[n]$ , one has  $\Pi_k^{(n)}(n_1, \dots, n_k) = \mathbb{P}[\Pi_n = \pi]$  for any partition  $\pi$  of  $[n]$  into  $k$  blocks  $\{A_1, \dots, A_k\}$  with  $|A_i| = n_i$ , for  $i = 1, \dots, k$ . See also the first displayed equation on page 1522 after (4) in [1]. The probability of all partitions of  $[n]$  into  $k$  blocks with respective sizes  $n_1, \dots, n_k$  (in exchangeable order) is then equal to  $n! \Pi_k^{(n)}(n_1, \dots, n_k) / (k! n_1! \cdots n_k!)$ . The same caveat also applies to (28) and (44)–(46) in [1].

An analogous clarification concerns

$$(**) \quad \mathbb{P}[K_m^{(n)} = k, L_m^{(n)} = s, \mathbf{S}_{L_m^{(n)}} = (s_1, \dots, s_{K_m^{(n)}}) | K_n = j]$$

displayed in (9) of [1]. Indeed, (\*\*) has been used to denote the probability of a specific partition of  $[s]$  into  $k$  blocks with sizes  $(s_1, \dots, s_k)$  in  $\Delta_{s,k}$ , given any partition of  $[n]$  into  $j$  parts, thus differing from its natural meaning as the probability

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Received February 2013.

of all such partitions. Our interpretation of (\*\*) in [1] is clearly consistent with the right-hand side of (9), that is,

$$\frac{V_{n+m,j+k}}{V_{n,j}} \binom{m}{s} (n-j\sigma)_{m-s} \prod_{i=1}^k (1-\sigma)_{s_i-1}$$

and with the result displayed in Corollary 1, where equation (10) arises after multiplying the right-hand side of (9) by  $s!/(k!s_1! \cdots s_k!)$  and, then, marginalizing with respect to all  $(s_1, \dots, s_k)$  in  $\Delta_{s,k}$ . Similar remarks apply to the notation appearing in the following displayed formulas: (17), (19), equation at the end of page 1528, (21), equation right after Corollary 2 on page 1529, (22), equation at the end of page 1531, (34).

## REFERENCES

- [1] LIJOI, A., PRÜNSTER, I. and WALKER, S. G. (2008). Bayesian nonparametric estimators derived from conditional Gibbs structures. *Ann. Appl. Probab.* **18** 1519–1547. [MR2434179](#)

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