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Efficient use of a two-stage randomized response procedure

Sally Abdelfatah[a], Reda Mazloum[a] and Sarjinder Singh[b]

[a]*Cairo University*
[b]*Texas A&M University–Kingsville*

Abstract. Towards the search for improving the randomized response models used to estimate a population proportion bearing a sensitive characteristic, a new model, based on the use of a two-stage randomized response procedure, is proposed. The condition under which the proposed estimator is more efficient than the Odumade and Singh [*Comm. Statist. Theory Methods* **38** (2009) 439–446] estimator has been obtained. An empirical study has also been performed to examine the relative efficiency of the proposed estimator with respect to the Odumade and Singh [*Comm. Statist. Theory Methods* **38** (2009) 439–446] estimator. Moreover, the proposed estimator can be easily adjusted to be more efficient than the Warner [*J. Amer. Statist. Assoc.* **60** (1965) 63–69], Mangat and Singh [*Biometrika* **77** (1990) 439–442], Mangat [*J. R. Stat. Soc. Ser. B Stat. Methodol.* **56** (1994) 93–95] and Odumade and Singh [*Comm. Statist. Theory Methods* **38** (2009) 439–446] estimators.

1 Introduction

An interviewing technique known as the randomized response technique was first introduced by Warner (1965). Such technique provided a way to maintain the respondents' privacy in an attempt to increase the response likelihood and facilitate more truthful responses on sensitive questions.

Warner (1965) supposed that every person in the population belongs to either a sensitive group (A) or a nonsensitive group (A^c). For estimating the population proportion π belonging to the sensitive group (A), a simple random sample with replacement (SRSWR) of n persons is drawn from the population and each respondent is provided with a random device in order to choose one of two statements of the form:

I am a member of group A "selected with probability P_0"
I am not a member of group A "selected with probability $(1 - P_0)$"

Such a random device can be an identical spinner with a face marked so that the spinner points to the letter A with probability P_0 and to the letter A^c with probability $(1 - P_0)$. Then, in each interview, the interviewee is asked to spin the spinner unobserved by the interviewer and report only whether or not the spinner

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points to the letter representing the group to which the interviewee belongs. That is, the interviewee is required only to say "Yes" or "No" according to whether or not the spinner points to the correct group; he/she does not report the group to which the spinner points.

The maximum likelihood estimator of π is:

$$\hat{\pi}_w = \frac{(n'/n) - (1 - P_0)}{2P_0 - 1}, \qquad P_0 \neq 0.5, \tag{1.1}$$

where n' is the number of "Yes" answers obtained from the n respondents.

$\hat{\pi}_w$ is an unbiased estimator of π, with variance given by:

$$V(\hat{\pi}_w) = \frac{\pi(1-\pi)}{n} + \frac{P_0(1-P_0)}{n(2P_0-1)^2}, \qquad P_0 \neq 0.5. \tag{1.2}$$

Mangat and Singh (1990) developed a two-stage randomized response procedure which requires the use of two randomization devices in an attempt to propose a new procedure that is more efficient than Warner's model (1965). In this method, each interviewee in the SRSWR of n respondents is provided with two random devices. The random device R_1 consists of two statements, namely (i) "I belong to the sensitive group" and (ii) "Go to random device R_2," represented with probabilities T_0 and $(1 - T_0)$, respectively. The random device R_2 which uses two statements, namely (i) "I belong to the sensitive group" and (ii) "I don't belong to the sensitive group," with known probabilities P_0 and $(1 - P_0)$, respectively, is exactly the same as used by Warner.

The interviewee is to use R_2 only if directed by the outcome of R_1. In case of outcomes R_1 (i), R_2 (i) or R_2 (ii), the respondent is required to answer "Yes" or "No" according to the statement and the actual status he possesses.

The maximum likelihood estimator of π is:

$$\hat{\pi}_{\text{ms}} = \frac{(n'/n) - (1-T_0)(1-P_0)}{2P_0 - 1 + 2T_0(1-P_0)}. \tag{1.3}$$

$\hat{\pi}_{\text{ms}}$ is an unbiased estimator of π, with variance given by:

$$V(\hat{\pi}_{\text{ms}}) = \frac{\pi(1-\pi)}{n} + \frac{(1-T_0)(1-P_0)[1-(1-T_0)(1-P_0)]}{n[2P_0 - 1 + 2T_0(1-P_0)]^2}. \tag{1.4}$$

Mangat (1994) developed a randomized response procedure which in addition of being more efficient than both Warner (1965) and Mangat and Singh (1990) models, it has the benefit of simplicity over that of Mangat and Singh (1990). In this procedure, each of n respondents assumed to be selected by equal probabilities with replacement sampling, is instructed to say "Yes" if he/she has the attribute A. If the respondent doesn't have the attribute A, then he/she is required to use the Warner randomization device consisting of two statements: (i) I am a member of group A "selected with probability P_0" and (ii) I am not a member of group A "selected with probability $(1 - P_0)$." Since the "Yes" answer may come from

Table 1 *Classification of the responses from Deck* (1) *and Deck* (2)

Responses from Deck (1)	Responses from Deck (2)	
	Yes	No
Yes	n_{11}	n_{10}
No	n_{01}	n_{00}

respondents in both group A and group not-A, the confidentiality of the person reporting "Yes" will not be violated.

The maximum likelihood estimator of π is:

$$\hat{\pi}_m = \frac{(n'/n) - 1 + P_0}{P_0}. \tag{1.5}$$

$\hat{\pi}_m$ is an unbiased estimator of π, with variance given by:

$$V(\hat{\pi}_m) = \frac{\pi(1-\pi)}{n} + \frac{(1-\pi)(1-P_0)}{nP_0}. \tag{1.6}$$

Odumade and Singh (OS) (2009) suggested the use of two decks of cards in a randomized response model where each of the decks includes the two statements used in the Warner (1965) model. Each respondent in a SRSWR of n respondents is provided with two decks of cards. Deck (1) includes the two statements: (a) I belong to group A and (b) I don't belong to group A, with probabilities P and $(1-P)$, respectively. Deck (2) includes the two statements as in Deck (1) with probabilities T and $(1-T)$, respectively. Each respondent is requested to draw two cards simultaneously, one card from each deck of cards, and read the statements in order. The respondent first matches his/her status with the statement written on the card taken from Deck (1), and then he/she matches his/her status with the statement written on the card taken from Deck (2). According to this procedure, the responses from the n respondents can be classified into a 2×2 contingency table as shown in Table 1.

An unbiased estimator of the population proportion π is given by:

$$\hat{\pi}_{\text{os}} = \frac{1}{2} + \frac{(P+T-1)(n_{11}/n - n_{00}/n) + (P-T)(n_{10}/n - n_{01}/n)}{2[(P+T-1)^2 + (P-T)^2]}, \tag{1.7}$$

where P and $T \neq 0.5$.

The variance of the estimator $\hat{\pi}_{\text{os}}$ is given by:

$$\begin{aligned} V(\hat{\pi}_{\text{os}}) = \big(&(P+T-1)^2[PT + (1-P)(1-T)] \\ &+ (P-T)^2[T(1-P) + P(1-T)]\big) \\ &/\big(4n[(P+T-1)^2 + (P-T)^2]^2\big) \\ &- \frac{(2\pi-1)^2}{4n}, \qquad P \text{ and } T \neq 0.5. \end{aligned} \tag{1.8}$$

From equation (1.8), it can be observed that $V(\hat{\pi}_{\mathrm{os}})$ remains the same after exchanging the values of P and T and that it is symmetric about $\pi = 0.5$.

An empirical study showed that the OS (2009) estimator is expected to perform better than both Warner (1965) and Mangat and Singh (1990) estimators if the value of $\pi \to 0$ or $\pi \to 1$. But it remains more efficient than the Mangat (1994) estimator if the value of $\pi \to 0$. Thus, the OS model can be more safely used if the proportion of the sensitive attribute is rare in the population.

A new randomized response model, based on the use of a two-stage randomized response procedure, along with the condition under which the proposed estimator is more efficient than the Odumade and Singh (2009) estimator are presented in Section 2. Section 3 will show that the proposed estimator is more efficient than the Warner (1965), Mangat and Singh (1990), Mangat (1994) and Odumade and Singh (2009) estimators.

2 The proposed model

The proposed model is a modified form of the Odumade and Singh (2009) model where the two-stage randomized response procedure suggested by Mangat and Singh (1990) is used in an attempt to obtain a more efficient estimator of π. According to the proposed procedure, each interviewee in a SRSWR of n respondents is provided with four decks of cards as shown in Figure 1.

Each respondent is requested to draw two cards simultaneously; one card from each of the two decks "Deck (1) and Deck (2)" and read the statements in order. The respondent is requested to draw a card from Deck (3) only if directed by the outcome of Deck (1) and he/she is also requested to draw a card from Deck (4) only if directed by the outcome of Deck (2). Deck (3) and Deck (4) are exactly the same decks used by Odumade and Singh (2009).

The respondent first matches his/her actual status with the statement written on the card drawn from Deck (1) or Deck (3), and then he/she matches his/her actual

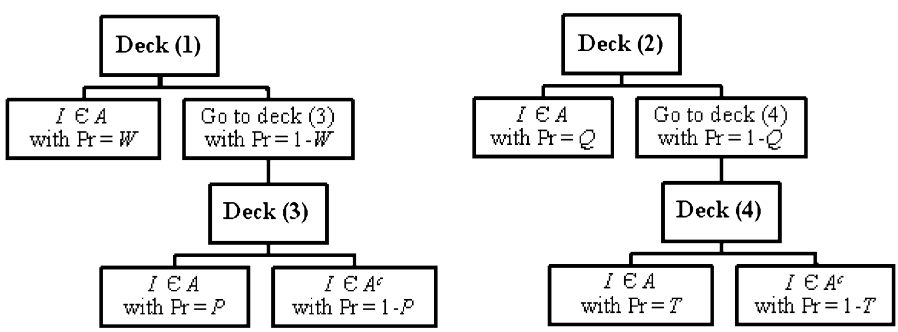

Figure 1 *Statements used in the four decks of cards.*

status with the statement written on the card drawn from Deck (2) or Deck (4). The whole procedure is done completely by the respondent, away from the interviewer.

Consider the following four situations in which the selected respondent belongs to group A and his/her response is (Yes, Yes):

(a) The respondent draws the first card with the statement "$I \in A$" with probability W from Deck (1) and the second card with the statement "$I \in A$" with probability Q from Deck (2).

(b) The respondent draws the first card with the statement "*Go to Deck* (3)" with probability $(1 - W)$ from Deck (1) and a card with the statement "$I \in A$" with probability P from Deck (3) and the second card with the statement "$I \in A$" with probability Q from Deck (2).

(c) The respondent draws the first card with the statement "$I \in A$" with probability W from Deck (1) and the second card with the statement "*Go to Deck* (4)" with probability $(1 - Q)$ from Deck (2) and a card with the statement "$I \in A$" with probability T from Deck (4).

(d) The respondent draws the first card with the statement "*Go to Deck* (3)" with probability $(1 - W)$ from Deck (1) and a card with the statement "$I \in A$" with probability P from Deck (3) and the second card with the statement "*Go to Deck* (4)" with probability $(1 - Q)$ from Deck (2) and a card with the statement "$I \in A$" with probability T from Deck (4).

Consider another situation in which the selected respondent belongs to group A^c and his/her response is also (Yes, Yes): If the respondent draws the first card with the statement "*Go to Deck* (3)" with probability $(1 - W)$ from Deck (1) and a card with the statement "$I \in A^c$" with probability $(1 - P)$ from Deck (3) and the second card with the statement "*Go to Deck* (4)" with probability $(1 - Q)$ from Deck (2) and a card with the statement "$I \in A^c$" with probability $(1 - T)$ from Deck (4).

As shown the response (Yes, Yes) can be obtained from respondents either belonging to group A or A^c and hence the confidentiality of the person reporting (Yes, Yes) will not be violated.

The probability of getting a (Yes, Yes) response (θ_{11}) is given by:

$$\begin{aligned}
\theta_{11} &= P(\text{Yes, Yes}) \\
&= WQ\pi + W(1-Q)T\pi \\
&\quad + (1-W)PQ\pi + (1-W)P(1-Q)T\pi \\
&\quad + (1-W)(1-P)(1-Q)(1-T)(1-\pi) \\
&= [(1-W)P + (1-Q)T + Q + W - 1]\pi \\
&\quad + (1-W)(1-P)(1-Q)(1-T).
\end{aligned} \tag{2.1}$$

Table 2 *Classification of the responses from the four decks of cards*

Responses from decks (1 or 3)	Responses from decks (2 or 4)		
	Yes	No	$\sum$
Yes	n_{11}	n_{10}	n_{1+}
No	n_{01}	n_{00}	n_{0+}
$\sum$	n_{+1}	n_{+0}	n

In the same way, the rest of the probabilities are given by:

$$\begin{aligned}\theta_{10} &= P(\text{Yes, No})\\ &= [W - Q + P(1-W) - T(1-Q)]\pi \\ &\quad + (1-W)(1-P)[Q + (1-Q)T],\end{aligned} \tag{2.2}$$

$$\begin{aligned}\theta_{01} &= P(\text{No, Yes})\\ &= [Q - W + T(1-Q) - P(1-W)]\pi \\ &\quad + (1-Q)(1-T)[W + (1-W)P],\end{aligned} \tag{2.3}$$

$$\begin{aligned}\theta_{00} &= P(\text{No, No})\\ &= [1 - W - Q - P(1-W) - T(1-Q)]\pi \\ &\quad + WQ + W(1-Q)T + (1-W)PQ + (1-W)P(1-Q)T.\end{aligned} \tag{2.4}$$

The responses from the n respondents can be classified into a 2×2 contingency table as shown in Table 2.

In order to estimate the unknown population proportion π of the respondents belonging to group A, let n_{11}/n, n_{10}/n, n_{01}/n and n_{00}/n be the observed proportions of (Yes, Yes), (Yes, No), (No, Yes) and (No, No) responses and they are unbiased estimators for θ_{11}, θ_{10}, θ_{01} and θ_{00}, respectively, where $\sum_{i=0}^{1}\sum_{j=0}^{1}\theta_{ij} = 1$.

We define the squared distance between the observed proportions and the true proportions as:

$$D = \frac{1}{2}\sum_{i=0}^{1}\sum_{j=0}^{1}(\theta_{ij} - n_{ij}/n)^2,$$

$$\begin{aligned}D &= \frac{1}{2}\bigg\{[(1-W)P + (1-Q)T + Q + W - 1]\pi \\ &\qquad + (1-W)(1-P)(1-Q)(1-T) - \frac{n_{11}}{n}\bigg\}^2 \\ &\quad + \frac{1}{2}\bigg\{[W - Q + P(1-W) - T(1-Q)]\pi\end{aligned} \tag{2.5}$$

$$
\begin{aligned}
&\qquad + (1-W)(1-P)[Q+(1-Q)T] - \frac{n_{10}}{n}\bigg\}^2 \\
&+ \frac{1}{2}\bigg\{[Q-W+T(1-Q)-P(1-W)]\pi \\
&\qquad + (1-Q)(1-T)[W+(1-W)P] - \frac{n_{01}}{n}\bigg\}^2 \\
&+ \frac{1}{2}\bigg\{[1-W-Q-P(1-W)-T(1-Q)]\pi + WQ + W(1-Q)T \\
&\qquad + (1-W)PQ + (1-W)P(1-Q)T - \frac{n_{00}}{n}\bigg\}^2.
\end{aligned}
$$

We want to choose π that minimizes the squared distance D in (2.5). We have

$$
\begin{aligned}
\frac{\partial D}{\partial \pi} &= [(1-W)P+(1-Q)T+Q+W-1]^2\pi \\
&\quad - \frac{n_{11}}{n}[(1-W)P+(1-Q)T+Q+W-1] \\
&\quad + [(1-W)P+(1-Q)T+Q+W-1] \\
&\qquad \times (1-W)(1-P)(1-Q)(1-T) \\
&\quad + [W-Q+P(1-W)-T(1-Q)]^2\pi \\
&\quad - \frac{n_{10}}{n}[W-Q+P(1-W)-T(1-Q)] \\
&\quad + [W-Q+P(1-W)-T(1-Q)] \\
&\qquad \times (1-W)(1-P)[Q+(1-Q)T] \\
&\quad + [Q-W+T(1-Q)-P(1-W)]^2\pi \\
&\quad - \frac{n_{01}}{n}[Q-W+T(1-Q)-P(1-W)] \\
&\quad + [Q-W+T(1-Q)-P(1-W)](1-Q)(1-T)[W+(1-W)P] \\
&\quad + [1-W-Q-P(1-W)-T(1-Q)]^2\pi \\
&\quad - \frac{n_{00}}{n}[1-W-Q-P(1-W)-T(1-Q)] \\
&\quad + [1-W-Q-P(1-W)-T(1-Q)] \\
&\qquad \times [WQ+W(1-Q)T+(1-W)PQ+(1-W)P(1-Q)T].
\end{aligned} \tag{2.6}
$$

Setting $\frac{\partial D}{\partial \pi} = 0$, we obtain the following estimator ($\hat{\pi}_s$) of the population proportion π

$$
\hat{\pi}_s = \frac{1}{2} + \frac{(n_{11}/n - n_{00}/n)B + (n_{10}/n - n_{01}/n)C}{2(B^2+C^2)}. \tag{2.7}
$$

Where,

$$\begin{aligned} B &= (1-W)P + (1-Q)T + W + Q - 1, \\ C &= W - Q + (1-W)P - (1-Q)T. \end{aligned} \tag{2.8}$$

Theorem 1. *The estimator $\hat{\pi}_s$ given by equation (2.7) is an unbiased estimator of the population proportion π.*

Proof. It follows from the fact that the observed proportions of (Yes, Yes), (Yes, No), (No, Yes) and (No, No) responses are unbiased estimators for the true proportions of such response (θ_{ij}), $i = 0, 1$; $j = 0, 1$.

That is: $\mathrm{E}(n_{ij}/n) = (\theta_{ij})$ for all $i = 0, 1$; $j = 0, 1$. □

Theorem 2. *The variance of the estimator $\hat{\pi}_s$ is given by*:

$$V(\hat{\pi}_s) = \frac{B^2(E+F) + C^2(G+H)}{4n(B^2+C^2)^2} - \frac{(2\pi-1)^2}{4n}, \tag{2.9}$$

where B and C are as defined in (2.8) and

$$\begin{aligned} E &= WQ + W(1-Q)T + (1-W)PQ + (1-W)P(1-Q)T, \\ F &= (1-W)(1-P)(1-Q)(1-T), \\ G &= (1-Q)(1-T)[W + (1-W)P], \\ H &= (1-W)(1-P)[Q + (1-Q)T]. \end{aligned}$$

Proof. Note that:

$$\begin{aligned} V(\hat{\pi}_s) = \bigg(& B^2 * V\left(\frac{n_{11}}{n} - \frac{n_{00}}{n}\right) + C^2 * V\left(\frac{n_{10}}{n} - \frac{n_{01}}{n}\right) \\ & + 2BC * \mathrm{Cov}\left(\frac{n_{11}-n_{00}}{n}, \frac{n_{10}-n_{01}}{n}\right)\bigg) \Big/ \left(4(B^2+C^2)^2\right). \end{aligned} \tag{2.10}$$

Using the following results from the standard multinomial distribution in (2.10), we can prove Theorem 2.

$$\begin{aligned} V(n_{11}/n) &= \theta_{11}(1-\theta_{11})/n, & \mathrm{Cov}(n_{11}/n, n_{10}/n) &= -\theta_{11}\theta_{10}/n, \\ V(n_{10}/n) &= \theta_{10}(1-\theta_{10})/n, & \mathrm{Cov}(n_{10}/n, n_{01}/n) &= -\theta_{10}\theta_{01}/n, \\ V(n_{01}/n) &= \theta_{01}(1-\theta_{01})/n, & \mathrm{Cov}(n_{11}/n, n_{01}/n) &= -\theta_{11}\theta_{01}/n, \\ V(n_{00}/n) &= \theta_{00}(1-\theta_{00})/n, & \mathrm{Cov}(n_{10}/n, n_{00}/n) &= -\theta_{10}\theta_{00}/n, \\ \mathrm{Cov}(n_{01}/n, n_{00}/n) &= -\theta_{01}\theta_{00}/n, & \mathrm{Cov}(n_{11}/n, n_{00}/n) &= -\theta_{11}\theta_{00}/n. \end{aligned}$$

□

It is obvious from equation (2.9) that the variance of $\hat{\pi}_s$ remains the same after exchanging the values of P and T and exchanging the values of W and Q. It can also be observed that $V(\hat{\pi}_s)$ is symmetric about $\pi = 0.5$.

Theorem 3. *An unbiased estimator of the variance of $\hat{\pi}_s$ is given by*:

$$\hat{V}(\hat{\pi}_s) = \frac{1}{4(n-1)}\left[\frac{B^2(E+F)+C^2(G+H)}{(B^2+C^2)^2} - (2\hat{\pi}_s - 1)^2\right]. \tag{2.11}$$

Proof. The proof is immediate by taking the expected values on both sides of equation (2.11). □

Corollary 1. *If $W = Q = 0$, then $\hat{\pi}_s = \hat{\pi}_{\text{os}}$ and $V(\hat{\pi}_s) = V(\hat{\pi}_{\text{os}})$.*

Corollary 2. *If $W = Q = 0$ and $T = P = P_0$, then the variance of the proposed estimator $\hat{\pi}_s$ in (2.9) becomes*:

$$\begin{aligned} V(\hat{\pi}_s)_{W=Q=0,\,P=T=P_0} &= V(\hat{\pi}_{\text{os}})_{P=T=P_0} \\ &= V(\hat{\pi}_w)_{q=2} = \frac{\pi(1-\pi)}{n} + \frac{P_0(1-P_0)}{2n(2P_0-1)^2} \end{aligned} \tag{2.12}$$

which is the same variance of Warner's (1965) estimator when the same set of two related questions is administrated to each respondent twice.

2.1 Efficiency comparison

The relative efficiency (RE) of the proposed estimator $\hat{\pi}_s$ with respect to the Odumade and Singh (2009) estimator is given by:

$$\text{RE(os)} = \frac{V(\hat{\pi}_{\text{os}})}{V(\hat{\pi}_s)} \times 100\%.$$

The proposed estimator is more efficient than the Odumade and Singh (2009) estimator if $V(\hat{\pi}_s) < V(\hat{\pi}_{\text{os}})$, that is, if

$$\begin{aligned} &\frac{B^2(E+F)+C^2(G+H)}{(B^2+C^2)^2} \\ &\quad < \big((P+T-1)^2[PT+(1-P)(1-T)] \\ &\qquad + (P-T)^2[T(1-P)+P(1-T)]\big)/[(P+T-1)^2+(P-T)^2]^2. \end{aligned} \tag{2.13}$$

It is obvious that the above condition doesn't depend on π which is the parameter of interest.

3 Empirical study

For each value of π where $\pi \in \{0.1, 0.2, \ldots, 0.9\}$; the relative efficiencies are calculated for all the possible combinations (6480 cases) from the values of P, T, W and Q where each of the parameters (P, T, W and Q) takes the values: 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9. The following results were observed:

(1) The proposed estimator is more efficient than the OS estimator in about 76% of the cases.

Table 3 *Relative efficiency of the proposed estimator with respect to the OS estimator for* $P = 0.5$ (0.6), $T = 0.6$ (0.5) *and* $Q = W = 0.9$

π	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
RE(os)(%)	5256	3314	2633	2347	2270	2347	2633	3314	5256

(2) For $P \geq 0.4$ and $T \geq 0.6$, the proposed estimator is more efficient than the OS estimator for all values of W, Q and π.

(3) For $Q = 0.9$ and all values of W ($W = 0.9$ and all values of Q), the proposed estimator is more efficient than the OS estimator for all values of P and T except for the combinations $[(P = 0.1, T = 0.1), (P = 0.1, T = 0.2)]$.

(4) The RE(os) reaches its maximum at $P = 0.5$ (0.6), $T = 0.6$ (0.5), $Q = W = 0.9$ and $\pi = 0.1, 0.9$ as shown in Table 3.

As shown in Table 3, the RE(os) is symmetric around $\pi = 0.5$ "as observed from equations (1.8) and (2.9)" and it reaches its maximum as $\pi \to 0$ or $\pi \to 1$. Thus, the proposed model can be safely used whether the population proportion possessing the sensitive characteristic is rare or predominant.

Using the values for P and T that were proposed by Odumade and Singh (2009), it was found that for $P = 0.05$ and $T = 0.95$, the proposed estimator is more efficient than the OS estimator with RE about 120% for values of W around 0.5, values of Q close to 1 and values of π close to 0 or 1. In this case, the proposed estimator will not only be more efficient than the OS estimator but it will also be more efficient than the Warner (1965), Mangat and Singh (1990) and Mangat (1994) estimators.

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S. Abdelfatah
R. Mazloum
Department of Statistics
Faculty of Economics and Political Science
Cairo University
Giza
Egypt
E-mail: abdelfatah.sally@gmail.com
rmazloum@aucegypt.edu

S. Singh
Department of Mathematics
Texas A&M University–Kingsville
Kingsville, Texas
USA
E-mail: sarjinder@yahoo.com