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LETTER TO THE EDITOR: SOME COMMENTS ON

ON CONSTRUCTION OF THE SMALLEST ONE-SIDED CONFIDENCE INTERVAL FOR THE DIFFERENCE OF TWO PROPORTIONS

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1. Introduction. Wang (2010) has rediscovered a general technique due to Buehler [(1957), Section 7] which was justified by Jobe and David [(1992), Appendix A1] and then in more generality by Lloyd and Kabaila (2003). The Buehler $1-\alpha$ upper confidence limit for a scalar parameter of interest, based on a designated statistic L, is $u(L)$ where u is that nondecreasing function which makes $u(L)$ as small as possible subject to the constraint that the infimal coverage is $1-\alpha$. Because Buehler illustrated the application of his result to the reliability of a parallel system, his work was virtually unknown outside the reliability literature for over 40 years. We believe that Buehler confidence limits have many important statistical and computational properties. The purpose of this letter is to point the reader to some of the literature on these properties.

2. Important statistical and computational properties of Buehler confidence limits. In the reliability literature on Buehler confidence limits, partly for computational reasons, the ordering induced on the sample space was usually based on an estimator L of the parameter of interest. It turns out that this ordering typically leads to confidence limits that do not have large sample efficiency; see Kabaila (2001) and Kabaila and Lloyd (2003). To obtain large sample efficiency, a Buehler $1-\alpha$ confidence limit needs to be based on an ordering induced on the sample space by L an approximate $1-\alpha$ confidence limit. However, as noted by Kabaila and Lloyd (2004), such Buehler confidence limits do not satisfy the nesting property. In the same paper, we suggested a method of resolving the tension between large sample efficiency and the satisfaction of the nesting property. Of course, one seeks to obtain not only good large sample performance, but also good finite sample performance. Kabaila and Lloyd (2002, 2005, 2006) examine some of the factors that influence the finite sample performance of Buehler confidence limits. Proposition 2 of Wang (2010) is a rediscovery of one result in the

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latter paper. Buehler confidence limits also have computational advantages which are examined by Kabaila (2005).

To summarize, Buehler confidence limits are exact, relatively easily computed and possess an attractive finite-sample optimality property. These advantages usually come at the cost of either some loss of large sample efficiency or nonsatisfaction of the nesting property. Nonetheless, Buehler confidence limits play an important part in statistical practice; see, for instance, Lloyd and Moldovan (2007a, 2007b).

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