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THE CONSERVATIVE MATRIX ON LOCALLY CONVEX SPACES*

Wu Junde, Kim Dohan and Cho Minhyung

Abstract. Let (X, τ_X) and (Y, τ_Y) be locally convex spaces, $c(X)$ and $c(Y)$ the X-valued and Y-valued convergent sequence spaces, respectively, $A_{ij} \in L(X, Y)$ and $A = (A_{ij})$ an operator-valued infinite matrix. In this paper, we characterize the matrix $A = (A_{ij})$ which transforms $c(X)$ into $c(Y)$. As its applications, we introduce the chi function χ on locally convex spaces, and show that a conservative matrix is conull if and only if $\chi(A) = 0$.

1. Introduction

let (X, τ_X) and (Y, τ_Y) be separated locally convex spaces, the topologies τ_X and τ_Y be generated by continuous seminorms $\{p : p \in D_X\}$ and $\{q : q \in D_Y\}$, respectively, $\omega(X)$ the set of all sequences $\overline{x} = (x_k)$ in X. In this sequel we consider the following sequence spaces:

$m(X) = \{\overline{x} = (x_k) \in \omega(X), \{x_k : k \in N\}$ is a bounded subset of $(X, \tau_X)\}$;

$c(X) = \{\overline{x} = (x_k) \in \omega(X)$, there exists a $x_0 \in X$ such that $\{x_k\}$ convergent to x_0 in topology $\tau_X\}$;

$c_0(X) = \{\overline{x} = (x_k) \in \omega(X)$, $\{x_k\}$ convergent to 0 in topology $\tau_X\}$;

$c_{00} = \{\overline{x} = (x_k) \in \omega(X)$, there exists a $n_0 \in N$ when $k \geq n_0, x_k = 0\}$.

The topology of $\omega(X)$ is given by the seminorms $\{\overline{p}_n; n \in N, p \in D_X\}$ where $\overline{p}_n(\overline{x}) = p(x_n)$, and the topologies of $m(X), c(X), c_0(X)$ are given by the seminorms $\{\overline{p} : p \in D_X\}$, where $\overline{p}(\overline{x}) = \sup_j p(x_j)$. It is obvious that $\lim : c(X) \to X$ is a continuous linear operator. Suppose that $A_{ij} \in L(X, Y)$ and $A = (A_{ij})$ is an infinite matrix, let ω_A denote the linear space of all sequence $\overline{x} = (x_j) \in \omega(X)$ such that for every $i \in N$, the series $\sum_j A_{ij} x_j$ is convergent and c_A the linear space of all sequence $\overline{x} = (x_j) \in \omega_A$ such that the sequence $(\sum_j A_{ij} x_j)_{i=1}^{\infty} \in c(Y)$. The

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topology of ω_A is given by the seminorms $\{\overline{p}_n : n \in N, p \in D_X\} \cup \{r_{qi} : i \in N, q \in D_Y\}$, where $r_{qi}(\overline{x}) = \sup_m q(\sum_{j=1}^{m} A_{ij}x_j)$, and the topology of c_A is given by $\{\overline{p}_n : n \in N, p \in D_X\} \cup \{r_{qi} : i \in N, q \in D_Y\} \cup \{\overline{q}_A : q \in D_y\}$, where $\overline{q}_A(\overline{x}) = \sup_i q(\sum_j A_{ij}x_j)$.

It is easily to see that if $\lambda(X)$ is any one of $\omega(X)$, $m(X)$, $c(X)$, $c_0(X)$, ω_A, c_A, let $C_k : \lambda(X) \to X$ be defined by $C_k(\overline{x}) = x_k$, then every C_k is continuous. Moreover, if $\overline{x} \in \omega(X)$, let $U_n(\overline{x}) = (x_1, x_2, ..., x_n, 0, ...)$. Then for every $\overline{x} \in c_0(X)$, $\{U_n(\overline{x})\}$ convergent to $\overline{x}$ in $c_0(X)$. Similar, for every $\overline{x} \in \omega_A$, convergence of $\{U_n(\overline{x})\}$ to $\overline{x}$ in ω_A also holds. We may prove that $A : c_A \to c(Y)$ is continuous.

An infinite matrix $A = (A_{ij})$ is said to be conservative if $c(X) \subseteq c_A$, or equivalence, the matrix A transforms $c(X)$ into $c(Y)$. When (X, τ_X) and (Y, τ_Y) are Frechet spaces, Ramanujan in ([1]) have characterized the conservative matrix A. Ramanujan's theorem has a series of important applications in summability theory ([2]). However, in order to establish the summability theory in locally convex spaces case, a crucial question is to extend this result to this class of spaces. In this paper, we study the problem. We also generalize some elementary summability theorems.

2. The General Conservative Matrix

Lemma 1 ([3]). *If infinite matrix $A = (A_{ij})$ transforms $c_0(X)$ into $c(Y)$, then for every $\overline{x} = (x_j) \in c_0(X)$, the series $\sum_j A_{ij}x_j$ converges uniformly with respect to $i \in N$.*

Lemma 2. *If (X, τ_X) is a barrelled space and $A = (A_{ij})$ transforms $c_0(X)$ into $c(Y)$, then for every bounded subset M of (X, τ_X) and $q \in D_Y$, there exists $K_M > 0$ such that for any $i, m \in N$ and $x_j \in M$,*

$$q\left(\sum_{j=1}^{m} A_{ij}x_j\right) \qquad K_M.$$

Proof. If not, we can choose a bounded subset M of (X, τ_X) such that for every $K > 0$ there exist $i_0, m_0 \in N$ and $x_j^{(0)} \in M$ satisfying that

$$q\left(\sum_{j=1}^{m_0} A_{i_0 j}x_j^{(0)}\right) > K. \tag{1}$$

Note that the matrix A transforms $c_0(X)$ into $c(Y)$ and (X, τ_X) is a barrelled space, it is easily to see that for every $j \in N$, $\{A_{ij}\}_{i=1}^{\infty}$ is equicontinuous. Thus, $\{A_{ij}(x) : x \in M, i \in N\}$ is a bounded subset of (Y, τ_Y).

At first, we show that for every $i \in N$, there exists $K_i > 0$ such that for any $m \in N$ and $x_j \in M$,

$$q\left(\sum_{j=1}^{m} A_{ij}x_j\right) \quad K_i. \tag{2}$$

If not, for every $K > 0$ there exist $m_0 \in N$ and $x_j^{(0)} \in M$, $j = 1, 2, ..., m_0$, such that

$$q\left(\sum_{j=1}^{m_0} A_{ij}x_j^{(0)}\right) \geq K.$$

Let $K = 1$, there exists $m_1 \in N$ and $x_j^{(1)} \in M, j = 1, 2, ..., m_1$, such that

$$q\left(\sum_{j=1}^{m_1} A_{ij}x_j^{(1)}\right) > 1.$$

For $\sum_{j=1}^{m_1} \sup\{q(A_{ij}x) : x \in M, i \in N\} + 2$, there exist $m_2 \in N$ and $x_j^{(2)} \in M$, $j = 1, 2, ..., m_2$, such that

$$q\left(\sum_{j=1}^{m_2} A_{ij}x_j^{(2)}\right) > \sum_{j=1}^{m_1} \sup\{q(A_{ij}x) : x \in M, i \in N\} + 2.$$

Thus, we have

$$q\left(\sum_{j=m_1+1}^{m_2} A_{ij}x_j^{(2)}\right) > 2.$$

Inductively, we can obtain a sequence $\{m_n\}$ of N such that

$$q\left(\sum_{j=1}^{m_1} A_{ij}x_j^{(1)}\right) > 1,$$

$$q\left(\sum_{j=m_n+1}^{m_{n+1}} A_{ij}x_j^{(n+1)}\right) > n+1, \; n = 1, 2, ...$$

Let

$$z_j = x_j^{(1)}; \; 1 \quad j \quad m_1,$$

$$z_j = \frac{x_j^{(n+1)}}{n+1}, \; m_n + 1 \quad j \quad m_{n+1}, n = 1, 2, ...$$

From that M is a bounded subset of $(X,\ \tau_X)$ it follows that $(z_j) \in c_0(X)$.

On the other hand, from $q(\sum_{j=m_n}^{m_{n+1}} A_{ij}z_j) > 1$, the series $\sum_j A_{ij}z_j$ is not convergent. This is a contradiction and so the conclusion holds.

Now, we show that the inequality (1) is not true. In fact, let $K = 1$, there exist $i_1, m_1 \in N$ and $x_j^{(1)} \in M, j = 1, 2, ..., m_1$, such that

$$q\left(\sum_{j=1}^{m_1} A_{i_1 j}x_j^{(1)}\right) > 1.$$

There exist $i_2,\ m_2 \in N$ and $x_j^{(2)} \in M, j = 1, 2, ..., m_2$, such that

$$q\left(\sum_{j=1}^{m_2} A_{i_2 j}x_j^{(2)}\right) > \sum_{i=1}^{i_1} K_i + \sum_{j=1}^{m_1} \sup\{q(A_{ij}x) : i \in N, x \in M\} + 2. \tag{3}$$

It is obvious that $m_2 > m_1$ and $i_2 > i_1$. From (3) it follows that

$$q\left(\sum_{j=m_1+1}^{m_2} A_{i_2 j}x_j^{(2)}\right) > 2.$$

Inductively, we can obtain two strictly increasing sequences $\{m_n\}$ and $\{i_n\}$ such that

$$q\left(\sum_{j=1}^{m_1} A_{i_1 j}x_j^{(1)}\right) > 1,$$

$$q\left(\sum_{j=m_n+1}^{m_{n+1}} A_{i_{n+1} j}x_j^{(n+1)}\right) > n+1, n = 1, 2, ...$$

Let

$$z_j = x_j^{(1)}, 1 \quad j \quad m_1,$$

$$z_j = \frac{x_j^{(n+1)}}{n+1},\ m_n + 1 \quad j \quad m_{n+1},\ n = 1, 2, ...$$

Then $q(\sum_{j=m_n+1}^{m_{n+1}} A_{i_{n+1} j}z_j) > 1,\ n = 1, 2, ...$. This shows that the series $\sum_j A_{ij}z_j$ does not converge uniformly with respect to $i \in N$. This contradicts Lemma 1 and hence Lemma 2 holds.

Theorem 1. *Let (X, τ_X) be a barrelled space and $(X^*, \beta(X^*, X))$ has property (B), (Y, τ_Y) a sequentially complete locally convex space, then $A = (A_{ij})$ is a conservative matrix if and only if*

(1) *For every bounded subset* M *of* (X, τ_X) *and* $q \in D_Y$, *there exists* $K_M > 0$ *such that for any* $i, m \in N$ *and* $x_j \in M$,

$$q\left(\sum_{j=1}^{m} A_{ij} x_j\right) \qquad K_M.$$

(2) *For every* $x \in X$, $\sum_j A_{ij} x$ *converges and* $\lim_{i\to\infty} \sum_{j=1}^{\infty} A_{ij} x$ *exists.*

(3) *For every* $x \in X$ *and* $j \in N$, $\lim_{i\to\infty} A_{ij} x$ *exists.*

About the (B) *property of* $(X^*,\ \beta(X^*,\ X))$ *see* ([4, P_{30}]).

Proof. The necessity follows from Lemma 2 and the fact that for every $x \in X$ and $j \in N, (x, x, ...)$ and $e_j \quad x = (0, 0, ..., x, 0, ...) \in c(X)$, where the ith place of $e_j \quad x$ is x.

Sufficiency. Since (X, τ_X) is a barrelled space and $(X^*,\ \beta(X^*,\ X))$ has the (B) property, it follows from ([5]) that $c_0(X)$ is a barrelled space.

For every $i,\ m \in N$, let $T_{im} : c_0(X) \to (Y, \tau_Y)$ be defined by $T_{im}(\overline{x}) = \sum_{j=1}^{m} A_{ij}(x_j)$. Then T_{im} is a continuous linear operator. For any $(x_j) \in c_0(X)$, note that $\{x_j\}$ is a bounded subset of (X, τ_X), it follows from Lemma 2 that $\{T_{im} : i, m \in N\}$ is pointwise bounded on $c_0(X)$ and, therefore, is equicontinuous.

Now, we show that for every $i \in N$ and $(x_j) \in c_0(X)$, the series $\sum_{j=1}^{\infty} A_{ij}(x_j)$ convergent and converges uniformly with respect to $i \in N$. If not, note that $(Y,\ \tau_Y)$ is sequentially complete, so there exist $(x_j) \in c_0(X)$, $\varepsilon_0 > 0$, $q \in D_Y$, $i_k, n_k, m_k \in N, n_k \quad m_k \quad n_{k+1}, \{n_k\}$ is strictly increasing and satisfies that

$$q\left(\sum_{j=n_k}^{m_k} A_{i_k j} x_j\right) \geq \varepsilon_0, k \in N. \tag{4}$$

Note that $z^{(k)} = (0, 0, ..., x_{n_k}, x_{n_k+1}, ..., x_{m_k}, 0, ...) \in c_0(X), \{z^{(k)}\}$ converges to 0 in $c_0(X)$ and $\{T_{im} : i, m \in N\}$ is equicontinuous, we have $\{q(T_{im} z^{(k)})\}$ converges to 0 uniformly with respect to $i, m \in N$. In particularly,

$$q(T_{i_k m_k} z^{(k)}) = q\left(\sum_{j=n_k}^{m_k} A_{i_k j} x_j\right) \to 0.$$

This contradicts (4) and so for every $(x_j) \in c_0(X)$, the series $\sum_j A_{ij} x_j$ is convergent and converges uniformly with respect to $i \in N$.

Thus, for any $(x_j) \in c(X)$, if $\lim_{j\to\infty} x_j = l$, then $(x_j) = (x_j - l) + (l, l, ...)$, so $\sum_j A_{ij}x_j = \sum_j A_{ij}(x_j - l) + \sum_j A_{ij}l$ converges. Moreover, it follows from the series $\sum_j A_{ij}(x_j - l)$ converges uniformly with respect to $i \in N$ and conditions (2), (3) that $\{\sum_j A_{ij}x_j\}_{i=1}^{\infty}$ is a Cauchy sequence and so is a convergent sequence by the sequentially completeness of (Y, τ_Y). That is, $A = (A_{ij})$ transforms $c(X)$ into $c(Y)$, A is a conservative matrix. We complete the proof of Theorem 1.

Let (X, τ_X) be a barrelled space and $A = (A_{ij})$ a conservative matrix. Then the columns of A are pointwise convergent on X. Let $\lim_{i\to\infty} A_{ij}(x) = A_j(x)$, it follows from the barrelledness of (X, τ_X) that $A_j \in L(X, Y)$.

In order to show the chi function χ is a continuous linear operator, we need the following Theorem:

Theorem 2. *Let (X, τ_X), (Y, τ_Y) be barrelled spaces and $(Y, \sigma(Y, Y^*))$ sequentially complete, $A = (A_{ij})$ a conservative matrix. Then for any $x \in X, \sum_j A_j(x)$ converges in $\sigma(Y, Y^*)$. Moreover, if $\sum_j A_j : X \to Y$ is defined by $x \to$ weak $\sum_j A_j(x)$, then $\sum_j A_j$ is $\tau_X - \tau_Y$ continuous.*

Proof. Fix x in X and $f \in Y^*$. Consider the matrix

$$B = \begin{pmatrix} f(A_{11}(x)) & f(A_{12}(x)) & \dots \\ f(A_{21}(x)) & f(A_{22}(x)) & \dots \\ \dots\dots & \dots\dots & \dots \end{pmatrix}.$$

It is easily to show that B is a conservative scalar matrix. It follows from ([6, Theorem 1.3.7]) that $\sum_j f(A_j(x))$ is convergent. By the sequentially completeness of $(Y, \sigma(Y, Y^*))$, $\sum_j A_j(x)$ is convergent in $\sigma(Y, Y^*)$.

Now, we prove that $x \to$ weak $\sum_j A_j(x)$ is $\tau_X - \tau_Y$ continuous.

Let $T_n = \sum_{j=1}^{n} A_j$, that $\{T_n\}$ are pointwise bounded continuous linear operators on X is clear. It follows from the barrelledness of (X, τ_X) that $\{T_n\}$ is equicontinuous. Then $\{T_n\}$ is also equicontinuous into $(Y, \sigma(Y, Y^*))$. Note that $\{T_n\}$ is pointwise weak convergent to $\sum_j A_j$, so $\sum_j A_j$ is $\tau_X - \sigma(Y, Y^*)$ continuous, it follows from ([7, Corollary 11.1.3]) that $\sum_j A_j$ is also $\sigma(X, X^*) - \sigma(Y, Y^*)$ continuous. By the Hellinger-Toeplitz theorem ([7, Corollary 11.2.6]) that $\sum_j A_j$ is also $\beta(X, X^*) - \beta(Y, Y^*)$ continuous. Since (X, τ_X) and (Y, τ_Y) are barrelled spaces, $\beta(X, X^*) = \tau_X, \beta(Y, Y^*) = \tau_Y$. Thus, $\sum_j A_j$ is $\tau_X - \tau_Y$ continuous. We complete the proof of Theorem 2.

Now, we introduce the chi function χ as following:

Let (X, τ_X) and (Y, τ_Y) be barrelled spaces and (Y, τ_Y) weak sequentially complete, $A = (A_{ij})$ a conservative matrix, denote

$$\chi(A)(x) = \lim_i \sum_j A_{ij}x - weak \sum_j A_j x.$$

From the barrelledness of (X, τ_X), it is easily to see that $\lim_{i\to\infty}\sum_j A_{ij} : X \to Y$ is $\tau_X - \tau_Y$ continuous, thus, by Theorem 2 we have:

Corollary 1. *If (X, τ_X) and (Y, τ_Y) are barrelled spaces and (Y, τ_Y) weak sequentially complete, $A = (A_{ij})$ a conservative matrix, then the chi function $\chi(A) : X \to Y$ is a $\tau_X - \tau_Y$ continuous linear operator.*

3. The Conull Matrix

If (X, τ_X) and (Y, τ_Y) be barrelled spaces and (Y, τ_Y) weak sequentially complete, $A = (A_{ij})$ a conservative matrix, we may prove the following facts:

(I) If $F \in (\omega_A)^*$, then there exists a sequence $\{f_j\} \subseteq X^*$ such that for every $\overline{x} = (x_j) \in \omega_A$, $F(\overline{x}) = \sum_j f_j(x_j)$. If $G \in (c_0(X))^*$, then there exist a sequence $\{g_j\} \subseteq X^*$ such that for every $\overline{x} = (x_j) \in c_0(X)$, $G(\overline{x}) = \sum_j g_j(x_j)$.

If p is a continuous seminorm and $f \in X^*$, denote $\|f\|_p = \sup\{|f(x)| : x \in X,\ p(x) \quad 1\}$, $l^1(X^*) = \{(f_j) : f_j \in X^*$ and there exists a continuous seminorm p of (X, τ_X) such that $\sum_j \|f_j\|_p < \infty\}$.

(II) $(c_0(X))^* = l^1(X^*)$.

(III) For any $G \in (c(X))^*$, there exist $\{g_0, g_j\}_{j=1}^{\infty} \subseteq X^*$ such that $(g_j) \in l^1(X^*)$ and for every $\overline{x} = (x_j) \in c(X)$,

$$G(\overline{x}) = g_0\Big(\lim_j x_j\Big) + \sum_j g_j\Big(x_j - \lim_j x_j\Big).$$

(IV) For any $f \in (c_A)^*$, there exist $F \in (\omega_A)^*$ and $\{g_0, g_j\}_{j=1}^{\infty} \subseteq Y^*$ such that $(g_j) \in l^1(Y^*)$ and for every $\overline{x} = (x_j) \in c_A$,

$$f(\overline{x}) = F(\overline{x}) + \Big(g_0 - \sum_j g_j\Big)\Big(\lim_A x\Big) + \sum_j g_j(Ax)_j.$$

Where $\lim_A x = \lim_i \sum_j A_{ij}x_j$, $(Ax)_i = \sum_j A_{ij}x_j$.

Let $(X, \tau_X), (Y, \tau_Y)$ be two barrelled spaces and (Y, τ_Y) weak sequentially complete, for $x \in X$, denote $x^{(0)} = (x, x, \ldots)$. A conservative matrix (A_{ij}) is said to be conull if and only if for every $x \in X, U_n(x^{(0)})$ converges weakly to $x^{(0)}$ in c_A. Otherwise A is coregular.

Theorem 3. *A conservative matrix $A = (A_{ij})$ is conull if and only if $\chi(A) = 0$.*

Proof. Let A be a conull matrix and $x \in X$, note that

$$\lim_A (x^{(0)} - U_n(x^{(0)})) = \chi(A)(x) + \sum_{j=n+1}^{\infty} A_j(x).$$

Since $A : c_A \to c(Y)$ is continuous and $\lim : c(X) \to Y$ is also continuous, so $\lim_A = \lim \circ A : c_A \to Y$ is continuous. Using ([7, Corollary 11.1.3]) again, we obtain that $\lim_A$ is also $(c_A, \sigma(c_A, (c_A)^*)) - (Y, \sigma(Y, Y^*))$ continuous. Thus we have $\mathcal{X}(A)(x) = 0$.

If $\mathcal{X}(A) = 0$, now we prove that A is a conull matrix.

In fact, let $f \in (c_A)^*$, it follows from fact (IV) that there exist $F \in (\omega_A)^*$, $g_0 \in Y^*$ and $(g_j) \in l^1(Y^*)$ such that

$$\begin{aligned} f(x^{(0)} - U_n(x^{(0)})) =& F(x^{(0)} - U_n(x^{(0)})) + \left(g_0 - \sum_j g_j\right)\left(\lim_A(x^{(0)} - U_n(x^{(0)}))\right) \\ &+ \sum_i g_i(A(x^{(0)} - U_n(x^{(0)})))_i = F(x^{(0)} - U_n(x^{(0)})) \\ &+ \left(g_0 - \sum_j g_j\right)(\mathcal{X}(A)(x) + \sum_{j=n+1}^{\infty} A_j(x)) \\ &+ \sum_i g_i\left(\sum_{j=n+1}^{\infty} A_{ij}(x)\right). \end{aligned}$$

Since $\{x^{(0)} - U_n(x^{(0)})\}$ converges to 0 in ω_A and $\{\sum_{j=n+1}^{\infty} A_j(x)\}$ converges to 0 in $(Y, \sigma(Y, Y^*))$, so $\{F(x^{(0)} - U_n(x^{(0)}))\}$ converges to 0, $\{(g_0 - \sum_j g_j) \sum_{j=n+1}^{\infty} A_j(x)\}$ converges to 0. It follows from $(g_i) \in l^1(Y^*)$, there exist a continuous seminorm q on (Y, τ_Y) such that $\sum_j \|g_j\|_q < \infty$. Moreover, by Lemma 2 that there exist $K > 0$ such that for all n, $i \in N$, $q(\sum_{j=n+1}^{\infty} A_{ij}(x)) \quad K$. Pick $i_0 \in N$ such that $\sum_{i=i_0+1}^{\infty} \|g_i\|_q \quad \frac{\varepsilon}{K}$. Thus, we have

$$\begin{aligned} \left|\sum_i g_i\left(\sum_{j=n+1}^{\infty} A_{ij}(x)\right)\right| \quad & \left|\sum_{i=1}^{i_0} g_i\left(\sum_{j=n+1}^{\infty} A_{ij}(x)\right)\right| + \textstyle\sum_{i=i_0+1}^{\infty} \|g_i\|_q K \\ & \left|\sum_{i=1}^{i_0} g_i\left(\sum_{j=n+1}^{\infty} A_{ij}(x)\right)\right| + \varepsilon. \end{aligned}$$

So $\{\sum_{i=1}^{\infty} g_i(\sum_{j=n+1}^{\infty} A_{ij}(x))\}$ converges to 0. This shows that $\{f(x^{(0)} - U_n(x^{(0)}))\}$ converges to 0. We complete the proof of Theorem 3.

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Wu Junde
Department of Mathematics, Zhejiang University
Yuquan Campus, Hangzhou, 310027, China
E-mail: WJD@math.zju.edu.cn

Kim Dohan
Department of Mathematics, Seoul National University, Seoul, 151-747, Korea

Cho Minhyung
Department of Applied Mathematics, Kumoh National Institute of Technology
Kyungbuk, 730-701, Korea