TAIWANESE JOURNAL OF MATHEMATICS
Vol. 6, No. 3, pp. 433-440, September 2002
This paper is available online at http://www.math.nthu.edu.tw/tjm/

A GENERAL ORLICZ-PETTIS THEOREM

Wu Junde and Lu Shijie

Abstract. In this paper, we show that the λ - multiplier convergent of series depends completely upon the AK - property of sequence space λ. From this conclusion we obtain a lot of new important theorems.

Let $[X,Y]$ be a dual pair and $\sigma(X,Y)$, $\tau(X,Y)$, $\beta(X,Y)$ the weak topology, Mackey topology and the strong topology of X, respectively. λ a scalar-valued sequence space. A series $\sum_i x_i$ in X is said to be λ - multiplier - $\sigma(X,Y)$ convergent if for each $(t_i) \in \lambda$, there exists a $x \in X$ such that for each $y \in Y$,

$$[x,y] = \sum_{i=1}^{\infty}[t_i x_i, y].$$

A series $\sum_i x_i$ in X is said to be subseries - $\sigma(X,Y)$ convergent, if for each strictly increasing sequence $\{i_n\}$, the series $\sum_n x_{i_n}$ is $\sigma(X,Y)$ convergent. If m_0 is the scalar-valued sequence space which satisfies that for each $(t_i) \in m_0$, $\{t_i : i \in \mathbf{N}\}$ is a finite set. Then the series $\sum_i x_i$ is subseries - $\sigma(X,Y)$ convergent if and only if $\sum_i x_i$ is m_0 - multiplier $\sigma(X,Y)$ convergent.

As is known, the famous Orlicz-Pettis theorem stated that ([1]): A series $\sum_i x_i$ in X is subseries - $\sigma(X,Y)$ convergent if and only if the series $\sum_i x_i$ in X is subseries - $\tau(X,Y)$ convergent.

Equivalently, the theorem may also be stated as follows: A series $\sum_i x_i$ in X is m_0 - multiplier - $\sigma(X,Y)$ convergent if and only if the series $\sum_i x_i$ in X is m_0 - multiplier - $\tau(X,Y)$ convergent.

With the restriction that X is a weakly sequentially complete normed space, the conclusion was proved by Orlicz in 1929 ([2]). However the theorem is also true without this restriction (see [3]). The first proof of the result for general normed

Received February 5, 2001; revised June 8, 2001.
Communicated by B. L. Lin.
2000 *Mathematics Subject Classification*: 46A03, 46E40.
Key words and phrases: Locally convex space, sequence space, λ - multiplier convergent, AK - property, Orlicz-Pettis theorem.

spaces was given by Pettis in 1938 ([4]), and it was Pettis who pointed out the applications of the result to vector measures. During the course of its evolution this theorem has evolved almost beyond recognition, and the techniques developed helped to illuminate a number of ideas in functional analysis.

The Orlicz-Pettis theorem has been an important catalyst in the development of the theory of (non-locally convex) F-spaces ([1]). A result of Orlicz-Pettis type is a theorem that asserts that a series which is subseries convergent in some topology is actually convergent in some stronger topology. The literature abounds with such Orlicz-Pettis type results ([5]). For historical remarks and extensive references to Orlicz-Pettis results, see ([6], [7]). In ([6, Th.2.2]), Dierolf improved the Orlicz-Pettis theorem as follows:

Let $[X, Y]$ be a dual pair, $\mu = \{M \subseteq Y, M$ is $\sigma(Y, X)$-bounded and for every continuous linear operator $S : (Y, \sigma(Y, X)) \to (l^1, \sigma(l^1, m_o))$ with the image $S(M)$ is relatively compact in $(l^1, \|.\|_1)\}$. Let $\mathcal{F}(\mu)$ denote the topology of uniform convergent on all sets in μ . Then we have:

(a) $\mathcal{F}(\mu)$ and $\sigma(X, Y)$ have the same subseries convergent series.

(b) Let $\mathcal{N}$ be any system of bounded sets of Y which covers Y and denote by $\mathcal{F}(\mathcal{N})$ the topology of uniform convergent on all sets in $\mathcal{N}$. Then $\mathcal{F}(\mathcal{N})$ and $\sigma(X, Y)$ have the same subseries convergent series if and only if $\mathcal{N} \subseteq \mu$.

Thus $\mathcal{F}(\mu)$ is the finest $[X, Y]$ - polar topology on X which has the same subseries convergent series as $\sigma(X, Y)$.

Let c_{00} be the scalar valued sequence space which are 0 eventually, and the β - dual space of λ to be defined by: $\lambda^\beta = \{(u_i) : \sum_i u_i t_i$ is convergent for every $(t_i) \in \lambda\}$. It is obvious that if $c_{00} \subseteq \lambda$, then $[\lambda, \lambda^\beta]$ is a dual pair with respect to the bilinear pairing $[t, u] = \sum_i u_i t_i$, $t = (t_i) \in \lambda$, $u = (u_i) \in \lambda^\beta$.

Let (λ, τ_0) be a locally convex space, $c_{00} \subseteq \lambda$ and $t = (t_i) \in \lambda$, denote $t^{[0]} = t$, $t^{[n]} = (t_1, t_2, t_3, \cdots, t_n, 0, \cdots)$. If for every $t \in \lambda, \{t^{[n]}\}$ converges to t with respect to the topology τ_0, then (λ, τ_0) is said to be an AK - space.

Let $\mathcal{A} = \{\{t^{[n]}\}_{n=0}^\infty : t \in \lambda\}$ and $\tau_A(\lambda^\beta, \lambda)$ the topology of uniform convergent on all sets in $\mathcal{A}$. It is obviously that $\sigma(\lambda^\beta, \lambda) \subseteq \tau_A(\lambda^\beta, \lambda) \subseteq \beta(\lambda^\beta, \lambda)$. In ([8, Th.4-5]), Wen Songlong et al. further showed that:

Let $[X, Y]$ be a dual pair, $\mu_\lambda = \{M \subseteq Y, M$ is $\sigma(Y, X)$ - bounded and for every continuous linear operator $T : (Y, \sigma(Y, X)) \to (\lambda^\beta, \sigma(\lambda^\beta, \lambda))$ defined by $T(y) = ([x_i, y])_{i=1}^\infty$, where the series $\sum_i x_i$ is λ - multiplier - $\sigma(X, Y)$ - convergent, the image $T(M)$ is $\tau_A(\lambda^\beta, \lambda)$ - relatively sequentially compact$\}$. Let $\mathcal{F}(\mu_\lambda)$ denote the topology of uniform convergent on all sets in μ_λ. Then we have:

(c) $\mathcal{F}(\mu_\lambda)$ and $\sigma(X, Y)$ have the same λ - multiplier convergent series.

(d) Let $\mathcal{D}$ be any system of bounded sets of Y which covers Y and denote by $\mathcal{F}(\mathcal{D})$ the topology of uniform convergent on all sets in $\mathcal{D}$. Then $\mathcal{F}(\mathcal{D})$ and $\sigma(X, Y)$ have the same λ - multiplier convergent series if and only if $\mathcal{D} \subseteq \mu_\lambda$.

Thus $\mathcal{F}(\mu_\lambda)$ is the finest $[X,Y]$ - polar topology on X which has the same λ - multiplier convergent series as $\sigma(X,Y)$.

Note that $m_0^\beta = l^1$ and the Schur lemma ([9 , Th.1.3.2 and Remark 15.2.3]) that $(l^1, \sigma(l^1, m_0))$ and $(l^1, \|.\|_1)$ have the same relatively sequentially compact sets. Since $\sigma(l^1, m_0) \subseteq \tau_A(l^1, m_0) \subseteq \|.\|_1$, so $(l^1, \tau_A(l^1, m_0))$ and $(l^1, \|.\|_1)$ have the same relatively sequentially compact sets. Thus, Wen Songlong's result actually generalized the Dierolf Theorem.

In this paper, we show that the λ - multiplier convergent of series depends completely upon the AK-property of λ. From this conclusion we can obtain some new important facts. In particularly, we will give Wen Songlong's result a direct proof.

Let ω be the space of all scalar valued sequences. A non-zero vector sequence $\{z^{(n)}\}$ in ω is said to be a block sequence if there exists $k_o = 0$ and a strictly increasing sequence of positive integers sequence $\{k_n\}$ such that

$$z^{(n)} = (0, 0, \cdots, z^{(n)}_{k_{n-1}+1}, \cdots, z^{(n)}_{k_n}, 0, 0, \cdots).$$

The sequence space λ is said to have the signed-weak gliding hump property (S-WGHP) if given any $t = (t_i) \in \lambda$ and any block sequence $\{t^{(n)}\}$ with $t = \sum_{n=1}^\infty t^{(n)}$ (pointwise sum), then for each strictly increasing positive integers sequence $\{m_k\}$ has a further subsequence $\{n_k\}$ and a signed sequence $\{\theta_k\}$ with $\theta_k = 1$ or $\theta_k = -1 (k \in \mathbf{N})$, such that $\bar{t} = \sum_{k=1}^\infty \theta_k t^{(n_k)} \in \lambda$ (pointwise sum) (see [10]).

Lemma 1 ([11]). *Let $c_{00} \subseteq \lambda$ and τ_0 be a locally convex topology of λ^β such that $\sigma(\lambda^\beta, \lambda) \subseteq \tau_0$. The following states are equivalent* :

(1) *B is τ_0 - compact*;

(2) *B is τ_0 - countable compact*;

(3) *B is τ_0 - sequentially compact*;

(4) *B is τ_0 - bounded, and each sequence $\{u^{(n)}\} \subseteq B$ which is coordinate convergent to $u^{(0)} \in \omega$, then $\{u^{(n)}\}$ must be τ_0 - convergent to $u^{(0)}$ and $u^{(0)} \in B$.*

Lemma 2 ([12]). *Let $c_{00} \subseteq \lambda$ and λ have the S-WGHP, then $(\lambda, \tau(\lambda, \lambda^\beta))$ is an AK - space.*

Lemma 3 ([13]). *If (X, τ_1) is a sequentially complete locally convex space and $\{x_i\} \subseteq X$ is a τ_1 - convergent sequence, then the absolutely convex closure of $\{x_i\}$ is a τ_1 - compact set and is also a τ_1 - sequentially compact set.*

A property (P) is said to be continuous linear invariant, if the property (P) is conserved with respect to all continuous linear mappings. Hence compact sets, countable compact sets, sequentially compact sets, convex compact sets, bounded sets, convergent sequences and finitely sets are continuous linear invariants.

Let (X, τ_1) be a locally convex space. A sequence $\{x_k\}$ in (X, τ_1) is said to be τ_1 - $\mathcal{K}$ - convergent if every subsequence of $\{x_k\}$ has a subsequence $\{x_{n_k}\}$ such that the series $\sum_k x_{n_k}$ is τ_1 - convergent to an element of X ([5, §3]). While a τ_1 - $\mathcal{K}$ - convergent sequence is τ_1 - convergent to 0, the converse does not hold, except in complete metric linear spaces ([5, §3]). A subset B of (X, τ_1) is said to be τ_1 - $\mathcal{K}$ - bounded if whenever $\{x_k\} \subseteq B$ and $\{t_k\}$ is a scalar sequence converging to 0, the sequence $\{t_k x_k\}$ is τ_1 - $\mathcal{K}$ - convergent ([5, §3]).

It is obvious that τ_1 - $\mathcal{K}$ - convergent sequence and τ_1 - $\mathcal{K}$ - bounded sets are also continuous linear invariants.

Let $[X, Y]$ be a dual pair and $\mathcal{P} = \{D \subseteq Y : D$ is finite set or D is $\sigma(X, Y)$ - bounded and has the property $(P)\}$. Now, we denote $\tau_P(X, Y)$ the topology of uniform convergent on all sets in $\mathcal{P}$. That $\tau_P(X, Y)$ is a $[X, Y]$ - polar topology is clear. Similarly, we can also define the topology $\tau_P(\lambda, \lambda^\beta)$.

Theorem 1. *Let $c_{00} \subseteq \lambda$ and the property (P) be a continuous linear invariant. Then for every dual pair $[X, Y]$, every λ - multiplier - $\sigma(X, Y)$ convergent series $\sum_i x_i$ in X must be λ - multiplier - $\tau_P(X, Y)$ convergent if and only if $(\lambda, \tau_P(\lambda, \lambda^\beta))$ is an AK - space.*

Proof. $(\Longleftarrow)$. Let $[X, Y]$ be a dual pair and $\sum_i x_i$ be a λ - multiplier - $\sigma(X, Y)$ convergent series in X. If $\sum_i x_i$ is not λ - multiplier - $\tau_P(X, Y)$ convergent, then there exist $(t_i^{(0)}) \in \lambda$, $x_0 \in X$ and $D \in \mathcal{P}$ such that $\sum_i t_i^{(0)} x_i$ is $\sigma(X, Y)$ - convergent to x_0, but $\sum_i t_i^{(0)} x_i$ does not converge to x_0 uniformly on D. That is, there exists $\varepsilon_0 > 0$ such that

$$\underline{\lim}_{n\to\infty} \sup{}_{y \in D} \Big\{ \Big| \Big[\sum_{i=n}^{\infty} t_i^{(0)} x_i, y \Big] \Big| \Big\} \geq \varepsilon_0. \tag{1}$$

Note that the series $\sum_i x_i$ is λ - multiplier - $\sigma(X, Y)$ - convergent, so for every $(t_i) \in \lambda$, there exists $x \in X$ such that for every $y \in Y$,

$$\Big[\sum_i t_i x_i, y \Big] = [x, y]. \tag{2}$$

(2) can be written as follows :

$$\sum_i t_i [x_i, y] = [x, y]. \tag{3}$$

This shows that for every $y \in Y$, $([x_i, y])_{i=1}^{\infty} \in \lambda^\beta$. If we define $T : y \to ([x_i, y])_{i=1}^{\infty}$, it follows from (3) that $T : (Y, \sigma(Y, X)) \to (\lambda^\beta, \sigma(\lambda^\beta, \lambda))$ is a continuous linear mapping. Since the property (P) is a continuous linear invariant, therefore $\{([x_i, y])_{i=1}^{\infty} : y \in D\} \subseteq \lambda^\beta$ has also property (P). Thus, it follows from $(\lambda, \tau_P(\lambda, \lambda^\beta))$ being an AK - space that there exists $n_0 \in \mathbf{N}$ such that whenever $n \geq n_0$, we have

$$\sup_{y\in D}\Big\{\Big|\sum_{i=n}^{\infty} t_i^{(0)}[x_i, y]\Big|\Big\} < \frac{\varepsilon_0}{2}.$$

This contradicts (1) and the sufficiency is proved.

($\Longrightarrow$). If $(\lambda, \tau_P(\lambda, \lambda^\beta))$ is not AK - space, there exist $(t_i^{(0)}) \in \lambda$, $D \subseteq \lambda^\beta$ and D has the property (P) such that

$$\underline{\lim}_{n} \sup\Big\{\Big|\sum_{i=n}^{\infty} t_i^{(1)} u_i\Big| : (u_i) \in D\Big\} > 0. \tag{4}$$

Let $X = \lambda$ and $Y = \lambda^\beta$, e^i be the sequence with a 1 in the ith coordinate and 0 elsewhere. Then $\sum_i e_i$ is λ - multiplier - $\sigma(\lambda, \lambda^\beta)$ convergent. On the other hand, it follows from (4) that $\sum_i e_i$ is not λ - multiplier - $\tau_P(\lambda, \lambda^\beta)$ convergent. This is a contradiction. ■

From Theorem 1 and Lemma 3 we have :

Corollary 1. *Let $c_{00} \subseteq \lambda$ and λ have the S-WGHP. Then for every dual pair $[X, Y]$, every λ - multiplier - $\sigma(X, Y)$ convergent series must be also λ - multiplier - $\tau(X, Y)$ convergent.*

Since m_0 has the S-WGHP, so Corollary 1 substantially improves the classical Orlicz-Pettis theorem.

Let $k(\lambda, \lambda^\beta), c(\lambda, \lambda^\beta)$, and $v(\lambda, \lambda^\beta)$ be the topologies of uniform convergent on all $\sigma(\lambda^\beta, \lambda)$ - compact sets, $\sigma(\lambda^\beta, \lambda)$ - countable compact sets and $\sigma(\lambda^\beta, \lambda)$ - sequentially compact sets, respectivelly.

Lemma 1 showed that $k(\lambda, \lambda^\beta) = c(\lambda, \lambda^\beta) = v(\lambda, \lambda^\beta)$. Furthermore, we have

Lemma 4. *Let $c_{00} \subseteq \lambda$ and $(\lambda^\beta, \sigma(\lambda^\beta, \lambda))$ be a sequentially complete space, if $(\lambda, \tau(\lambda, \lambda^\beta))$ is an AK - space, then $(\lambda, k(\lambda, \lambda^\beta))$ is also an AK - space.*

In fact, if not, there exist $t \in \lambda$ and $D \subseteq \lambda^\beta$, D is a $\sigma(\lambda^\beta, \lambda)$ - compact set, and a strictly increasing positive integers sequence $\{n_k\}$, and $\{u^{(k)}\} \subseteq D$ such that $|[t^{[n_k]} - t, u^{(k)}]| \geq \varepsilon_1 > 0, k \in \mathbf{N}$. By Lemma 1, we may assume that $\{u^{(k)}\}$ is $\sigma(\lambda^\beta, \lambda)$ - convergent. Since $(\lambda^\beta, \sigma(\lambda^\beta, \lambda))$ is sequentially complete, it follows from Lemma 3 that the absolutely convex closure of $\{u^{(k)}\}$ is also $(\lambda^\beta, \sigma(\lambda^\beta, \lambda))$ - compact set. Thus, by the definition of Mackey topology $\tau(\lambda, \lambda^\beta)$, and because $(\lambda, \tau(\lambda, \lambda^\beta))$ is an AK - space, we have $[t^{[n_k]} - t, u^{(k)}] \longrightarrow 0$. This is a contradiction.

Since if $c_{00} \subseteq \lambda$ and λ has the S - WGHP, then $\sigma(\lambda^\beta, \lambda)$ is a sequentially complete space [10]. Thus, from Corollary 1 and Lemma 4 we can further improve the classical Orlicz-Pettis theorem as follows :

Corollary 2. *Let $c_{00} \subseteq \lambda$ and λ has the S - WGHP, then for every dual pair $[X, Y]$ and every λ - multiplier - $\sigma(X, Y)$ - convergent series must be also λ - multiplier - $k(X, Y)$ convergent.*

Let $\mathcal{OP} = \{Q \subseteq \lambda^\beta : Q$ is $\sigma(\lambda^\beta, \lambda)$ - bounded set and for every $(t_i) \in \lambda$, the series $\sum_i u_i t_i$ converges uniformly with respect to $(u_i) \in Q\}$. It is clear that the topology τ_{OP} of uniform convergent on all sets in $\mathcal{OP}$ is a $[\lambda, \lambda^\beta]$ - polar topology, and $(\lambda, \tau_{OP}(\lambda, \lambda^\beta))$ is the following meaning finest AK - topology, that is, if τ' is a $[\lambda, \lambda^\beta]$ - polar topology, and (λ, τ') is an AK - space, then $\tau' \subseteq \tau_{OP}(\lambda, \lambda^\beta)$.

Let $\mathcal{A} = \{\{t^{[n]}\}_{n=0}^\infty : t \in \lambda\}$ and $\tau_A(\lambda^\beta, \lambda)$ the topology of uniform convergent on all sets in $\mathcal{A}$. We have the following important results :

Theorem 2. *Every $Q \in \mathcal{OP}$ is a $\tau_A(\lambda^\beta, \lambda)$ - relatively sequentially compact set.*

Proof. Let $\{u^{(k)}\}$ be a sequence in Q; since Q is $\sigma(\lambda^\beta, \lambda)$ - bounded and $c_{00} \subseteq \lambda$, by using a diagonal argument we can abtain a subsequence $\{u^{(k_j)}\}$ of $\{u^{(k)}\}$ such that $\{u^{(k_j)}\}$ is coordinates convergent to $u^{(0)} \in \omega$. Now we prove that $\{u^{(k_j)}\}$ is $\tau_A(\lambda^\beta, \lambda)$ convergent to $u^{(0)}$ and $u^{(0)} \in \lambda^\beta$.

In fact, let $t = (t_i) \in \lambda$; since the series $\sum_i u_i t_i$ converges uniformly with respect to $(t_i) \in Q$, for every $\varepsilon > 0$, there exists $k_0 \in \mathbf{N}$, whenever $k \geq k_0$, for all $j \in \mathbf{N}$, we have

$$|\sum_{i=k}^{\infty} u_i^{(k_j)} t_i| < \frac{\varepsilon}{4}. \tag{5}$$

Let $m, n \in \mathbf{N}$ and $m \geq n \geq k_0$, note that $\{u^{(k_j)}\}$ is coordinates convergent to $u^{(0)}$, so $\lim_j \sum_{i=n}^m (u_i^{(k_j)} - u_i^{(0)}) t_i = 0$. Thus

$$\begin{aligned} |\sum_{i=n}^{m} u_i^{(0)} t_i| &\cdot \ |\sum_{i=n}^{m} (u_i^{(0)} - u_i^{(k_j)}) t_i| + |\sum_{i=n}^{m} u_i^{(k_j)} t_i| \\ &\cdot \ |\sum_{i=n}^{m} (u_i^{(0)} - u_i^{(k_j)}) t_i| + |\sum_{i=n}^{\infty} u_i^{(k_j)} t_i| + |\sum_{i=m+1}^{\infty} u_i^{(k_j)} t_i| \\ &\cdot \ |\sum_{i=n}^{m} (u_i^{(0)} - u_i^{(k_j)}) t_i| + \frac{\varepsilon}{2}. \end{aligned}$$

Therefore, the series $\sum_i u_i^{(0)} t_i$ is convergent, i.e., $u^{(0)} \in \lambda^\beta$. Similarly, we may prove that $\{u^{(k_j)}\}$ is $\tau_A(\lambda^\beta, \lambda)$ convergent to $u^{(0)}$. By the Lemma 1 it follows that Q is $\tau_A(\lambda^\beta, \lambda)$ - relatively sequentially compact set.

Reark 1. Let $[X, Y]$ be a dual pair, $\sum_i x_i$ a λ - multiplier - $\sigma(X, Y)$ - convergent series in X, $\mathcal{N}$ any system of $\sigma(X, Y)$ - bounded subsets of Y which cover Y and denote $\tau_N(\lambda^\beta, \lambda)$ the topology of uniform convergent on all sets in $\mathcal{N}$. It follows from Theorem 1 and its proof that $\sum_i x_i$ is also λ - multiplier - $\tau_N(X, Y)$ convergent if and only if for every $D \in \mathcal{N}$, $\{([x_i, y])_{i=1}^\infty : y \in D\} \in \mathcal{OP}$.

Theorem 3. *Let* $D \subseteq \lambda^\beta$ *and* D *be a* $\tau_A(\lambda^\beta, \lambda)$ *- relatively sequentially compact set, then* $D \in \mathcal{OP}$.

Proof. If not, there exist $t = (t_i) \in \lambda$ and a strictly increasing positive integers sequence $\{i_k\}$, a sequence $\{u^{(k)}\} \in D$ and $\varepsilon_0 > 0$ such that

$$|\sum_{i=i_k}^\infty u_i^{(k)} t_i| \geq \varepsilon_0, k \in \mathbf{N}. \tag{6}$$

Since D is $\tau_A(\lambda^\beta, \lambda)$ - relatively sequentially compact, we may assume that $\{u^{(k)}\}$ is $\tau_A(\lambda^\beta, \lambda)$ - convergent to $u^{(0)} \in \lambda^\beta$. That is

$$\lim_k \sup\{|[t^{[n]}, u^{(k)} - u^{(0)}]| : n = 0, 1, 2, \cdots\} = 0. \tag{7}$$

It follows from (7) that there exists $k_1 \in \mathbf{N}$, such that whenever $k \geq k_1$, for any $n \in \mathbf{N}$,

$$|[t^{[0]} - t^{[n]}, u^{(k)} - u^{(0)}]| < \frac{\varepsilon_0}{4}.$$

Since $u^{(0)} \in \lambda^\beta$, so there exists $n_1 \in \mathbf{N}$ such that whenever $n \geq n_1$,

$$|[t^{[0]} - t^{[n]}, u^{(0)}]| < \frac{\varepsilon_0}{4}.$$

So whenever $n \geq n_1$ and $k \geq k_1$, we have

$$|[t^{[0]} - t^{[n]}, u^{(k)}]| \cdot \quad |[t^{[0]} - t^{[n]}, u^{(k)} - u^{(0)}]| + |[t^{[0]} - t^{[n]}, u^{(0)}]| < \frac{\varepsilon_0}{2}.$$

This contradicts (6) and the Theorem is proved.

From Theorem 2, Remark 1 and Theorem 3 we obtain immediately the Wen Songlong' conclusion.

References

1. N. J. Kalton, The Orlicz-Pettis Theorem. *Contemp. Math.* **2** (1980), 91-100.
2. W. Orlicz, Beitrage zur Theorie der Orthogonalent Wicklungen, *Studia Math.* **1** (1929), 241-255.
3. S. Banach, *Theorie des Operations Lineaires*, Warsaw, 1932.
4. B. J. Pettis, On Integration in Vector Spaces. *Trans. Amer. Math. Soc.* **44** (1938), 277-304.
5. P. Antosik and C. Swartz, *Matrix Methods in Analysis*. Lecture Notes in Math., 1113, Springer - Verlag, 1985.
6. P. Dierolf, Theorems of the Orlicz-Pettis Type for Locally Convex Spaces. *Manuscripta Math.* **20** (1977), 73-94.
7. J. Diestel, J. Uhl, *Vector Measure*. Amer. Math. Soc. Surveys. Providence, 1977.
8. Wen Songlong, Cui Chengri and Li Ronglu, s-Multiplier Convergent and Theorems of the Orlicz-Pettis-Type. *Acta Math. Sinica.* **43** (2000), 275-282.
9. A. Wilansky, *Modern Methods in Topological Vector Spaces*. McGraw-Hill, New York, 1978.
10. C. Swartz, *Infinite Matrices and the Gliding Hump*. World Sci. Publ., Singapore 1996.
11. Wu Junde and Qu Wenbo, The Compact Sets in the Infinite Matrix Topological Algebras. to appear
12. J. Boos and T. Leiger, The Signed Weak Gliding Hump Property. *Acta Comn. Univ. Tartuensis.* **970** (1994), 13-22.
13. Wu Junde and Wu Yajuan, Null Sequence in the Mapping System, *J. of Math. (PRC)*, **18** (1998), 264-266.

Wu Junde and Lu Shijie
Department of Mathematics, Zhejiang University, Yuquan Campus,
Hang Zhou, Zhejiang, 310027, People's Republic of China