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HYPERSURFACES WITH POINTWISE 1-TYPE GAUSS MAP

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Abstract. In this paper we prove that an oriented hypersurface M of a Euclidean space E^{n+1} has pointwise 1-type Gauss map of the first kind if and only if M has constant mean curvature. Then we conclude that all oriented isoparametric hypersurfaces of E^{n+1} has 1-type Gauss map. We also show that a rational hypersurface of revolution in a Euclidean space E^{n+1} has pointwise 1-type Gauss map of the second kind if and only if it is a right n-cone.

1. INTRODUCTION

A submanifold M of a Euclidean space E^m is said to be of finite type if its position vector x can be expressed as a finite sum of eigenvectors of the Laplacian Δ of M, that is, $x = x_0 + x_1 + \cdots + x_k$, where x_0 is a constant map, $x_1, \ldots, x_k$ are non-constant maps such that $\Delta x_i = \lambda_i x_i,\ \lambda_i \in \mathbb{R}$, $i = 1, 2, \ldots, k$. If $\lambda_1, \lambda_2, \ldots, \lambda_k$ are all different, then M is said to be of k-type (cf. [5, 6]). In [7], this definition was similarly extended to differentiable maps, in particular, to Gauss map of submanifolds. The notion of finite type Gauss map is especially a useful tool in the study of submanifolds (cf. [1-4, 7, 12]). In [7], Chen and Piccinni made a general study on compact submanifolds of Euclidean spaces with finite type Gauss map, and for hypersurfaces they prove that a compact hypersurface M of E^{n+1} has 1-type Gauss map G if and only if M is a hypersphere in E^{n+1}. In this work we show that all oriented isoparametric hypersurfaces of E^{n+1} have 1-type Gauss map.

If a submanifold M of a Euclidean space has 1-type Gauss map G, then $\Delta G = \lambda(G + C)$ for some $\lambda \in \mathbb{R}$ and some constant vector C. However, the Laplacian of the Gauss map of several surfaces such as helicoid, catenoid and right cones, and also some hypersurfaces that we study here take the form

$$\Delta G = f(G + C) \tag{1.1}$$

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for some non-constant function f on M and some constant vector C. A submanifold of a Euclidean space is said to have pointwise 1-type Gauss map if its Gauss map satisfies (1.1) for some smooth function f on M and some constant vector C. A pointwise 1-type Gauss map is called *proper* if the function f is non-constant. A submanifold with pointwise 1-type Gauss map is said to be of *the first kind* if the vector C in (1.1) is the zero vector. Otherwise, a submanifold with pointwise 1-type Gauss map is said to be of *the second kind*.

Surfaces in Euclidean spaces and in pseudo-Euclidean spaces with pointwise 1-type Gauss map were recently studied in [8-11] and [13].

In this work our main aim is to obtain a characterization of hypersurfaces of a Euclidean space E^{n+1} with pointwise 1-type Gauss map. We firstly prove that an oriented hypersurface M of a Euclidean space E^{n+1} has pointwise 1-type Gauss map of the first kind if and only if M has constant mean curvature. We also conclude that all oriented isoparametric hypersurfaces of E^{n+1} has 1-type Gauss map. Then we extend the results given by B.Y. Chen, M. Choi and Y.H. Kim for surfaces of revolution with pointwise 1-type Gauss map in E^3, [8].

2. Preliminaries

Let M be an n-dimensional hypersurface of a Euclidean space E^{n+1}. We denote by h, A and ∇, the second fundamental form, the Weingarten map and the induced Riemannian connection of M in E^{n+1}, respectively. Let $\{e_1, \ldots, e_n\}$ be an orthonormal local frame on M. For any real function g on M, the Laplacian Δg of the function g is defined by

$$\Delta g = \sum_{i=1}^{n} ((\nabla_{e_i} e_i)g - e_i e_i g).$$

The map $G : M^n \to S^n \subset E^{n+1}$ which sends each point of M to the unit normal vector to M at the point is called the Gauss map of the hypersurface M, where S^n is the unit sphere in E^{n+1} centered at the origin.

Let $x_1 = \varphi(v),\ \ x_{n+1} = \psi(v)$ be a curve in the $x_1 x_{n+1}$-half plane lying in halfspace $x_1 = \varphi(v) > 0$. Rotating this curve around the x_{n+1}-axis we obtain a rotational hypersurface M in E^{n+1}, (cf. [14]). Let $\{\eta_1, \ldots, \eta_{n+1}\}$ be the standard orthonormal basis of E^{n+1} and $S^{n-1}(1)$ be the unit sphere in E^n spanned by $\{\eta_1, \ldots, \eta_n\}$. We can have an orthogonal parametrization of $S^{n-1}(1) \subset E^n$ as

$$\begin{aligned} &Y_1 = \cos u_1,\ Y_2 = \sin u_1 \cos u_2,\ \ldots, \\ &Y_{n-1} = \sin u_1 \cdots \sin u_{n-2} \cos u_{n-1}, \\ &Y_n = \sin u_1 \cdots \sin u_{n-2} \sin u_{n-1}. \end{aligned} \tag{2.1}$$

It follows that

$$x(u_1,\ldots,u_{n-1},v)=(\varphi(v)Y_1,\ \varphi(v)Y_2,\ldots,\varphi(v)Y_n,\ \psi(v)),\ \ Y_i=Y_i(u_1,\ldots,u_{n-1}),$$

is a parametrization of the rotational hypersurface M. Let us put

$$Y(u_1,\ldots u_{n-1}) = (Y_1(u_1,\ldots,u_{n-1}),\ldots,Y_n(u_1,\ldots,u_{n-1}),0), \tag{2.2}$$

which is the position vector of the sphere $S^{n-1}(1) \subset E^n$ in E^{n+1}. Then we can write

$$x(u_1,\ldots,u_{n-1},v) = \varphi(v)Y(u_1,\ldots,u_{n-1}) + \psi(v)\eta_{n+1}, \tag{2.3}$$

where $\eta_{n+1} = (0,0,\ldots,0,1)$ is the axis of the rotation. Taking derivative we have the orthogonal coordinate vector fields on M as

$$x_{u_i} = \varphi(v)Y_{u_i},\ \ i = 1,\ldots,n-1,\quad x_v = \varphi'(v)Y + \psi'(v)\eta_{n+1}. \tag{2.4}$$

Hence the Gauss map of the hypersurface of revolution is given by

$$G = \frac{1}{\sqrt{p}}(\psi' Y - \varphi'\eta_{n+1}),\quad p = {\varphi'}^2 + {\psi'}^2. \tag{2.5}$$

3. Hypersurfaces with Pointwise 1-Type Gauss Map

In this section we give a characterization theorem for hypersurfaces of Euclidean spaces with pointwise 1-type Gauss map of the first kind. To do this we need the following lemma.

Lemma 3.1. *Let M be an oriented hypersurface of a Euclidean space E^{n+1}. Then the Laplacian of the Gauss map G is given by*

$$\Delta G = \|A_G\|^2 G + n\nabla\alpha, \tag{3.1}$$

where $\nabla\alpha$ is the gradient of the mean curvature and $\|A_G\|^2 = tr(A_G A_G)$.

Proof. For a fixed vector $C_0 \in E^{n+1}$, we put $G_0 =< G, C_0 >$. Then, for vector fields X, Y tangent to M using the formulas of Gauss and Weingarten we have

$$YXG_0 = - < \nabla_Y(A_G(X)) + h(A_G(X),Y), C_0 > . \tag{3.2}$$

Let $\{e_1,\ldots,e_{n+1}\}$ be an adapted local orthonormal frame in E^{n+1} such that $e_1,\ldots,e_n$ are tangent to M and $e_{n+1} = G$. Moreover we assume that $e_1,\ldots,e_n$

are eigenvectors of the Weingarten map A_G corresponding to the eigenvalues λ_i, $i = 1, 2, \ldots, n$, that is, $A_G(e_i) = \lambda_i e_i$. Denote by $\{\omega_1, \ldots, \omega_{n+1}\}$ and $\{\omega_{ij}\}$, $i, j = 1, 2, \ldots, n$, the dual frame and the connection forms associated to $\{e_1, \ldots, e_{n+1}\}$, respectively. Then, by using the connection equations $\nabla_{e_i} e_i = \sum_{k=1}^{n} \omega_{ik}(e_i) e_k$ and the equation of Codazzi $(\nabla_{e_i} A_G) e_j = (\nabla_{e_j} A_G) e_i$ we have

$$e_j(\lambda_i) = (\lambda_i - \lambda_j)\omega_{ij}(e_i), \quad i \neq j. \tag{3.3}$$

Hence, considering (3.3) we obtain

$$\begin{aligned}
\sum_{i=1}^{n} (\nabla_{e_i} A_G) e_i &= \sum_{i=1}^{n} \{\nabla_{e_i}(A_G(e_i)) - A_G(\nabla_{e_i} e_i)\} \\
&= \sum_{i=1}^{n} \{e_i(\lambda_i) e_i + \sum_{j=1}^{n} (\lambda_i - \lambda_j)\omega_{ij}(e_i) e_j\} \\
&= \sum_{i=1}^{n} \{e_i(\lambda_i) e_i + \sum_{i \neq j, j=1}^{n} e_j(\lambda_i) e_j\} \\
&= \sum_{i,j=1}^{n} e_j(\lambda_i) e_j = n\nabla\alpha,
\end{aligned} \tag{3.4}$$

and also we have $\sum_{i=1}^{n} h(A_G(e_i), e_i) = \mathrm{tr}(A_G A_G) G = \|A_G\|^2 G$.

By using (3.2) and (3.4) we calculate the Laplacian of $< G, C_0 >$ as follows:

$$\begin{aligned}
\Delta < G, C_0 > &= \sum_{i=1}^{n} (\nabla_{e_i} e_i - e_i e_i) < G, C_0 > \\
&= - \sum_{i=1}^{n} < A_G(\nabla_{e_i} e_i)), C_0 > \\
&\quad + \textstyle\sum_{i=1}^{n} < \nabla_{e_i}(A_G(e_i)) + h(A_G(e_i), e_i), C_0 > \\
&= < \sum_{i=1}^{n} \{\nabla_{e_i}(A_G(e_i)) - A_G(\nabla_{e_i} e_i))\}, C_0 > \\
&\quad + < \sum_{i=1}^{n} h(A_G(e_i), e_i), C_0 > \\
&= < n\nabla\alpha, C_0 > + < \|A_G\|^2 G, C_0 > .
\end{aligned} \tag{3.5}$$

Since (3.5) holds for any $C_0 \in E^{n+1}$, then the proof is completed. ■

Now, from the definition (1.1) and the equation (3.1) we state the following theorem which characterizes the hypersurfaces of Euclidean spaces with pointwise 1-type Gauss map of the first kind.

Theorem 3.2. *Let M be an oriented hypersurface of a Euclidean space E^{n+1}. Then M has proper pointwise 1-type Gauss map of the first kind if and only if M has constant mean curvature and $\|A_G\|^2$ is non-constant.*

We can have the following corollary on hypersurfaces with 1-type Gauss map.

Corollary 3.3. *All oriented isoparametric hypersurfaces of a Euclidean space E^{n+1} has 1-type Gauss map.*

For example, hyperplanes, hyperspheres and the generalized cylinder $S^{n-k} \times E^k$ of E^{n+1} have 1-type Gauss map.

We can also state

Theorem 3.4. *If an oriented hypersurfaces M of a Euclidean space E^{n+1} has proper pointwise 1-type Gauss map of the second kind, then the mean curvature of M is non-constant.*

4. Hypersurface of Revolution with Pointwise 1-Type Gauss Map of the First and Second Kind

The aim of this section is to study the hypersurfaces of revolution of a Euclidean space E^{n+1} in terms of pointwise 1-type Gauss map of the first and second kind. We mainly extend the results given by B.Y. Chen, M. Choi and Y.H. Kim for surfaces of revolution with pointwise 1-type Gauss map in E^3, [8].

Let M be a hypersurface of revolution in E^{n+1} defined by (2.3). By straightforward calculation we can have the Weingarten map as

$$A_G = \begin{pmatrix} -\frac{\psi'}{\varphi\sqrt{p}} I_{n-1} & 0 \\ 0 & \frac{\psi'\varphi'' - \varphi'\psi''}{p\sqrt{p}} \end{pmatrix}$$

where I_{n-1} is the $(n-1) \times (n-1)$ identity map and $p = \varphi'^2 + \psi'^2$. Thus the mean curvature of M is

$$\alpha = \frac{1}{n}\left(-\frac{(n-1)\psi'}{\varphi\sqrt{p}} + \frac{\psi'\varphi'' - \varphi'\psi''}{p\sqrt{p}} \right) \tag{4.1}$$

and

$$\|A_G\|^2 = \frac{(n-1)\psi'^2}{\varphi^2 p} + \frac{(\psi'\varphi'' - \varphi'\psi'')^2}{p^3}. \tag{4.2}$$

Since the mean curvature α is the function of v, using (2.4) we can have the gradient of α as

$$\nabla \alpha = \frac{\alpha'}{p}\left(\varphi' Y + \psi' \eta_{n+1}\right). \tag{4.3}$$

Lemma 4.1. *Let M be a hypersurface of revolution in Euclidean space E^{n+1} with pointwise 1-type Gauss map. Then either the Gauss map is harmonic, that is, $\Delta G = 0$ or the function f defined in (1.1) depends only on v and the vector C in (1.1) is parallel to the axis of the hypersurface of revolution.*

Proof. Using (3.1) and (4.3) the Laplacian of the Gauss map (2.5) becomes

$$\Delta G = \left(\frac{\|A_G\|^2\psi'}{\sqrt{p}} + \frac{n\alpha'\varphi'}{p}\right) Y + \left(\frac{n\alpha'\psi'}{p} - \frac{\|A_G\|^2\varphi'}{\sqrt{p}}\right)\eta_{n+1}. \tag{4.4}$$

Suppose that the generating curve of (2.3) is of unit speed, that is, $p = {\varphi'}^2 + {\psi'}^2 = 1$. By a direct calculation we can be have

$$\begin{aligned} \Delta G = & \left(\frac{(n-1)\psi'}{\varphi^2} - \frac{(n-1)\varphi'\psi''}{\varphi} - \psi'''\right) Y \\ & + \left(\frac{(n-1)\varphi'\varphi''}{\varphi} + \varphi'''\right)\eta_{n+1}. \end{aligned} \tag{4.5}$$

If M has pointwise 1-type Gauss map, then (1.1) holds for some function f and some vector C. When the Gauss map is not harmonic, (1.1), (2.1), (2.5) and (4.5) imply that the first n components of C must be zero and

$$\begin{aligned} & \frac{(n-1)\psi'}{\varphi^2} - \frac{(n-1)\varphi'\psi''}{\varphi} - \psi''' = f\psi'(v), \\ & \frac{(n-1)\varphi'\varphi''}{\varphi} + \varphi''' = f(c - \varphi'(v)) \end{aligned} \tag{4.6}$$

where $C = (0, \ldots, 0, c)$. Since $\varphi'(v)$ and $\psi'(v)$ are not both zero, the function f is independent of $u_1, \ldots, u_{n-1}$. ■

We can have the following examples of hypersurfaces of revolution with proper pointwise 1-type Gauss map of the first kind and the second kind, respectively.

Example 4.2. Consider the generalized catenoid, [14], which is the minimal hypersurface of revolution parameterized by

$$x(u_1, \ldots, u_{n-1}, v) = vY(u_1, \ldots, u_{n-1}) + \left(\int \frac{a dv}{\sqrt{v^{2(n-1)} - a^2}}\right)\eta_{n+1}, \quad v > 0,$$

where a is nonzero constant, $\eta_{n+1} = (0, 0, \ldots, 0, 1) \in E^{n+1}$ and $Y(u_1, \ldots, u_{n-1})$ is defined in (2.2). Then, the Gauss map G of the generalized catenoid is given by

$$G = \frac{1}{v^{n-1}}(aY - \sqrt{v^{2(n-1)} - a^2}\eta_{n+1}),$$

and hence, the Laplacian of the Gauss map satisfies

$$\Delta G = \frac{n(n-1)a^2}{v^{2n}}G,$$

which implies that the generalized catenoid has proper pointwise 1-type Gauss map of the first kind.

Example 4.3. Consider the right n-cone C_a based on the sphere $S^{n-1}(1)$ which is parameterized by

$$x(u_1, \ldots, u_{n-1}, v) = vY(u_1, \ldots, u_{n-1}) + av\eta_{n+1}, \quad a \geq 0,$$

where $\eta_{n+1} = (0, 0, \ldots, 0, 1) \in E^{n+1}$ and $Y(u_1, \ldots, u_{n-1})$ is defined in (2.2). Then, the Gauss map G of C_a is given by

$$G = \frac{1}{\sqrt{1+a^2}}(aY - \eta_{n+1}).$$

Hence, by using (3.1), (4.1), (4.2) and (4.3) for $\varphi(v) = v, \ v > 0$ and $\psi(v) = av$ we can have

$$\Delta G = \frac{n-1}{v^2}(G + \frac{1}{\sqrt{1+a^2}}\eta_{n+1}),$$

which means that the right n-cone has pointwise 1-type Gauss map of the second kind.

Let M be a hypersurface of revolution in E^{n+1} parameterized by taking $\varphi(t) = t, \ t > 0$ and $\psi(t) = g(t)$ in (2.3)

$$x(u_1, \ldots, u_{n-1}, t) = tY(u_1, \ldots, u_{n-1}) + g(t)\eta_{n+1}, \tag{4.7}$$

where Y is given by (2.2). The Gauss map G of M parameterized by (4.7) is given by

$$G = \frac{1}{\sqrt{1+g'^2}}(g'Y - \eta_{n+1}). \tag{4.8}$$

When we consider (4.3) for the parametrization (4.7) we obtain from the equation (3.1)

$$\Delta G = \left(\|A_G\|^2 + \frac{n\alpha'}{g'\sqrt{1+g'^2}}\right)G + \frac{n\alpha'}{g'}\eta_{n+1}, \tag{4.9}$$

where

$$\|A_G\|^2 = \frac{(n-1)g'^2}{t^2(1+g'^2)} + \frac{g''^2}{(1+g'^2)^3} \tag{4.10}$$

and

$$n\alpha' = -\frac{(n-1)g''}{t(1+g'^2)^{3/2}} - \frac{g'''}{(1+g'^2)^{3/2}} + \frac{(n-1)g'}{t^2\sqrt{1+g'^2}} + \frac{3g'g''^2}{(1+g'^2)^{5/2}}. \tag{4.11}$$

Suppose that M has pointwise 1-type Gauss map of the second kind. Then, by definition, the vector C in (1.1) is nonzero and by Lemma 4.1 $C = (0, \ldots, 0, c) = c\eta_{n+1}$. Therefore the equations (1.1) and (4.9) imply that

$$\|A_G\|^2 + \frac{n\alpha'}{g'\sqrt{1+g'^2}} = f \quad \text{and} \quad \frac{n\alpha'}{g'} = cf. \tag{4.12}$$

Eliminating f in (4.12) and, using (4.10) and (4.11) we obtain

$$\begin{aligned} &(n-1)g''(1+g'^2)^2 t + g'''(1+g'^2)^2 t^2 \\ &-(n-1)g'(1+g'^2)^3 - 3g'g''^2(1+g'^2)t^2 \\ = {}& c\sqrt{1+g'^2}\{g'''(1+g'^2)t^2 + (n-1)g''(1+g'^2)t \\ &-4g'g''^2t^2 - (n-1)g'(1+g'^2)^3\}. \end{aligned} \tag{4.13}$$

Suppose that M is a hypersurface of revolution of polynomial kind, that is, $g(t)$ is a polynomial in t. For $n = 2$, in [8], it was shown that the polynomial $g(t)$ that satisfies (4.13) has degree 1. Following the method used in [8], it is easily seen that $g(t) = at + b,\ \ a, b \in \mathbb{R},\ a \neq 0$ is the only solution of (4.13) for $n \geq 2$. Also, applying (4.13) we have $c = \frac{1}{\sqrt{1+a^2}}$. So the parametrization of M reduces to

$$x(u_1, \ldots, u_{n-1}, t) = tY(u_1, \ldots, u_{n-1}) + (at + b)\eta_{n+1}, \ \ a \neq 0, \tag{4.14}$$

which is the right n-cone.

As a result we have the following.

Theorem 4.4. *A hypersurface of revolution of polynomial kind in a Euclidean space E^{n+1} has pointwise 1-type Gauss map of the second kind if and only if it is a right n-cone.*

Let M be a hypersurface of revolution of rational kind, that is, $g(t)$ is a rational function in t. In [8], it was proven that there is no rational function $g(t)$, except polynomial, which satisfies the equation (4.13) for $n = 2$. Following [8], one can see that the equation (4.13) does not have any rational solution, except polynomial, for $n \geq 2$ because the factor $n-1$ appeared in some terms of the equation (4.13) does not change the method used in [8]. Therefore we can state the following.

Theorem 4.5. *There do not exist rational hypersurface of revolution, except polynomial kind, in a Euclidean space E^{n+1} with pointwise 1-type Gauss map of the second kind.*

We finally prove the following theorem:

Theorem 4.6. *A rational hypersurface of revolution of Euclidean space E^{n+1} has pointwise 1-type Gauss map if and only if it is an open portion of a hyperplane, a generalized cylinder, or a right n-cone.*

Proof. Let M be a hypersurface of revolution parameterized by (2.3). If $\varphi = \varphi_0$ is constant, then the hypersurface is an open portion of the generalized cylinder $S^{n-1}(\varphi_0) \times \mathbb{R}$. When φ is not constant, we can consider the parametrization given by (4.7) for the hypersurface of revolution. The hypersurface of revolution has constant mean curvature if and only if $g = g(t)$ is a solution of the differential equation

$$g'' + \frac{(n-1)(1+g'^2)g'}{t} + n\alpha(1+g'^2)^{3/2} = 0, \tag{4.15}$$

for some constant α. Following the solution of the differential equation (4.15) for $n = 2$ given in [8], we can obtain the solution of (4.15) as

$$g(t) = \int \frac{a - \alpha t^n}{\sqrt{t^{2(n-1)} - (a - \alpha t^n)^2}} dt + c_1, \tag{4.16}$$

where a and c_1 are constant. If $a = \alpha = 0$, g is constant. Then, the hypersurfaces is an open portion of a hyperplane. If $a \neq 0$ and $\alpha = 0$, that is, M is a minimal hypersurface of revolution which is called a generalized catenoid for $n > 2$, [14], then (4.16) implies that $g(t)$ can be expressed in terms of elliptic functions and it is not of rational kind. For example, if $n = 2$, then (4.16) gives $g(t) = a\cosh^{-1}(t/a) + c_1$, and the surface is a catenoid. If $a = 0$, $\alpha \neq 0$, then from (4.16) we have $g(t) = \sqrt{1 - \alpha^2 t^2}/\alpha$. In this case, the hypersurface M is an n-sphere which is not rational kind. If a, $\alpha \neq 0$, then (4.16) implies that $g(t)$ can be expressed in terms elliptic functions. Thus, $g(t)$ is not a rational function of t.

If M is a rational hypersurface of revolution with pointwise 1-type Gauss map of the second kind, then M is an open portion of a right n-cone according to Theorem 4.4 and 4.5.

The converse is followed by Corollary 3.3 and Example 4.3. ∎

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