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IDENTITIES RELATED TO DERIVATIONS AND CENTRALIZERS ON STANDARD OPERATOR ALGEBRAS

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Abstract. In this paper identities related to derivations and centralizers on operator algebras are considered. We prove the following result which is related to a classical result of Chernoff. Let X be a real or complex Banach space, let $L(X)$ and $F(X)$ be the algebra of all bounded linear operators and the ideal of all finite rank operators on X, respectively. Suppose there exist linear mappings $D, G : F(X) \to L(X)$ such that $D(A^2) = D(A)A + AG(A)$ and $G(A^2) = G(A)A + AD(A)$ is fulfilled for all $A \in F(X)$. In this case there exists $B \in L(X)$ such that $D(A) = G(A) = [A, B]$ holds for all $A \in F(X)$.

1. Introduction

This research has been motivated by the work of Chernoff [8], Molnár [17] and Jing and Lu [13]. Throughout, R will represent an associative ring with center $Z(R)$. Given an integer $n \geq 2$, a ring R is said to be $n-$torsion free, if for $x \in R$, $nx = 0$ implies $x = 0$. As usual the commutator$xy - yx$ will be denoted by $[x, y]$. Recall that a ring R is prime if for $a, b \in R$, $aRb = (0)$implies that either $a = 0$ or $b = 0$, and is semiprimein case $aRa = (0)$ implies $a = 0$. Let A be an algebra over the real or complex field and let B be a subalgebra of A. A linear mapping $D : B \to A$ is called a linear derivation in case $D(xy) = D(x)y + xD(y)$ holds for all pairs $x, y \in R$. In case we have a ring R an additive mapping $D : R \to R$ is called a derivation if $D(xy) = D(x)y + xD(y)$ holds for all pairs $x, y \in R$ and is called a Jordan derivation in case $D(x^2) = D(x)x + xD(x)$ is fulfilled for all $x \in R$. A derivation D is inner in case there exists $a \in R$, such that $D(x) = [a, x]$ holds

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for all $x \in R$. Every derivation is a Jordan derivation. The converse is in general not true. A classical result of Herstein $[11]$ asserts that any Jordan derivation on a $2-$torsion free prime ring is a derivation. A brief proof of Herstein's result can be found in $[4]$. Cusack $[9]$ generalized Herstein's result to $2-$torsion free semiprime rings (see also $[5]$ for an alternative proof). An additive mapping $T : R \to R$ is called a left centralizer in case $T(xy) = T(x)y$ holds for all pairs $x, y \in R$. The concept appears naturally in C^*- algebras. In ring theory it is more common to work with module homomorphisms. Ring theorists would write that $T : R_R \to R_R$ is a homomorphism of a ring module R into itself. For a semiprime ring R all such homomorphisms are of the form $T(x) = qx$ for all $x \in R$, where q is an element of Martindale right ring of quotients Q_r (see Chapter 2 in $[3]$). In case R has the identity element $T : R \to R$ is a left centralizer iff T is of the form $T(x) = ax$ for all $x \in R$ and some fixed element $a \in R$. An additive mapping $T : R \to R$ is called a left Jordan centralizer in case $T(x^2) = T(x)x$ holds for all $x \in R$.The definition of right centralizer and right Jordan centralizer should be self-explanatory. Following ideas from $[5]$ Zalar $[30]$ has proved that any left (right) Jordan centralizer on a $2-$torsion free semiprime ring is a left (right) centralizer. Molnár $[17]$ has proved that in case we have an additive mapping $T : A \to A$, where A is a semisimple H^*- algebra, satisfying the relation $T(x^3) = T(x)x^2$ ($T(x^3) = x^2T(x)$) for all $x \in A$, then T is a left (right) centralizer. Let us recall that a semisimple H^*-algebra is a semisimple Banach $^*-$algebra whose norm is a Hilbert space norm such that $(x, yz^*) = (xz, y) = (z, x^*y)$ is fulfilled for all $x, y, z \in A$ (see $[2]$). Vukman $[20]$ has proved that in case we have an additive mapping T, which maps a $2-$torsion free semiprime ring into itself, satisfying the relation $2T(x^2) = T(x)x + xT(x)$, for all $x \in R$ then T is a left and a right centralizer. Some results concerning centralizers on semiprime rings can be found in $[17, 19 - 28]$. Let X be a real or complex Banach space and let $L(X)$ and $F(X)$ denote the algebra of all bounded linear operators on X and the ideal of all finite rank operators in $L(X)$, respectively. An algebra $A(X) \subset L(X)$ is said to be standard in case $F(X) \subset A(X)$. Let us point out that any standard algebra is prime, which is a consequence of Hahn-Banach theorem. We denote by X^* the dual space of a real or complex Banach space X and by I the identity operator on X.

Let R be a ring and let $T : R \to R$ be a left (right) centralizer. In this case T satisfies the relation

$$T(x^3) = T(x^2)x \ (T(x^3) = xT(x^2)) \tag{1}$$

for all $x \in R$. The question arises under what additional assumptions the converse is true. More precisely, under what additional assumptions an additive mapping T satisfying the relation (1) is a left (right) centralizer. A routine calculation shows that the answer to this question is affirmative in arbitrary ring with the identity element.

In this paper we consider the relation (1) on standard operator algebras. The result below will be used in the proof of Theorem 3.

Theorem 1. *Let X be a real or complex Banach space and let $A(X)$ be a standard operator algebra on X. Suppose there exists an additive mapping $T : A(X) \to L(X)$ satisfying the relation*

$$T(A^3) = T(A^2)A \ (T(A^3) = AT(A^2))$$

for all $A \in A(X)$. In this case there exists $B \in L(X)$ such that the following statements are fulfilled:

(i) $T(A) = BA$ $(T(A) = AB)$, *for all* $A \in F(X)$.

(ii) $T(A^2) = BA^2(T(A^2) = A^2B)$, *for all* $A \in A(X)$.

Proof. We shall restrict our attention to the relation

$$T(A^3) = T(A^2)A, A \in A(X). \tag{2}$$

The proof in case we have the relation $T(A^3) = AT(A^2)$ will be omitted because of left-right symmetry.

Let A be from $F(X)$ and let $P \in F(X)$, be a projection with $AP = PA = A$. From the above relation one obtains $T(P) = T(P)P$. Putting $A + P$ for A in the above relation and applying the relation (2), we obtain

$$3T(A^2) + 3T(A) = 2T(A)A + T(P)A + T(A^2)P + 2T(A)P. \tag{3}$$

Putting in the above relation $-A$ for A and comparing the relation so obtained with the above relation we obtain

$$3T(A) = T(P)A + 2T(A)P, \tag{4}$$

Right multiplication of the relation (4) by P gives $T(A)P = T(P)A$, which reduces the relation (4) to

$$T(A) = T(P)A. \tag{5}$$

From the above relation one can conclude that T maps $F(X)$ into itself and that T is linear. Now using the relation (5) we obtain $T(A^2) = T(P)A^2 = (T(P)A)A = T(A)A$. We have therefore proved that for any $A \in F(X)$ the relation $T(A^2) = T(A)A$ is fulfilled. In other words, T is a left Jordan centralizer on $F(X)$. Since $F(X)$ is prime one can conclude according to Proposition 1.4 in

[30] that T is a left centralizer. We intend to prove that there exists an operator $B \in L(X)$, such that

$$T(A) = BA \tag{6}$$

holds for all $A \in F(X)$. In case X is finite-dimensional he relation (6) would follow from the relation (5) since in this case P can be choosen to be the identity operator on X. Suppose that X is infinite-dimensional. For any fixed $x \in X$ and $f \in X^*$ we denote by $x \otimes f$ an operator from $F(X)$ defined by $(x \otimes f)y = f(y)x$, for all $y \in X$. For any $A \in L(X)$ we have $A(x \otimes f) = ((Ax) \otimes f)$. Let us choose f and y such that $f(y) = 1$ and define $Bx = T(x \otimes f)y$. Obviously, B is linear. Using the fact that T is left centralizer on $F(X)$ we obtain

$$(BA)x = B(Ax) = T((Ax) \otimes f)y = T(A(x \otimes f))y = T(A)(x \otimes f)y = T(A)x,$$

for all $x \in X$. We have therefore $T(A) = BA$ for any $A \in F(X)$. Using closed graph theorem one can easily prove that B is continuous. The statement (i) of the theorem is therefore proved. It remains to prove the statement (ii). Let us introduce $T_1 : A(X) \to L(X)$ by $T_1(A) = BA$ and consider $T_0 = T - T_1$. The mapping T_0 is, obviously, additive and satisfies the relation (2). Besides, T_0 vanishes on $F(X)$. Let $A \in A(X)$, let P be an one-dimensional projection, and $S = A + PAP - (AP + PA)$. Since, obviously, $S - A \in F(X)$, we have $T_0(S) = T_0(A)$. Besides, $SP = PS = 0$. We have therefore the relation

$$T_0(A^3) = T_0(A^2)A, \tag{7}$$

for all $A \in A(X)$. Applying the above relation we obtain

$$T_0(S^2)S = T_0(S^3) = T_0(S^3 + P) = T_0((S + P)^3)$$
$$= T_0((S + P)^2)(S + P) = T_0(S^2)(S + P) = T_0(S^2)S + T_0(S^2)P.$$

We have therefore $T_0(A^2)P = 0$. Since this relation holds for all one-dimensional projections P on X, one can conclude that $T_0(A^2) = 0$, for all $A \in A(X)$ which means that we have $T(A^2) = BA^2$, for all $A \in A(X)$. The proof of the theorem is complete.

Corollary 1. *Let X be a real or complex Banach space. Suppose there exists an additive mapping $T : F(X) \to L(X)$ satisfying the relation*

$$T(A^3) = T(A^2)A \ (T(A^3) = AT(A^2))$$

for all $A \in F(X)$. In this case T is of the form $T(A) = BA$ $(T(A) = AB)$, for all $A \in F(X)$ and some $B \in L(X)$.

We proceed with the following well-known result first proved by Chernoff $[8]$ (see also $[18]$ and $[19]$).

Theorem A. *Let X be a real or complex Banach space and let $A(X)$ be a standard operator algebra on X. Suppose there exists a linear derivation D : $A(X) \to L(X)$. In this case D is of the form $D(A) = [A, B]$, for all $A \in A(X)$ and some $B \in L(X)$.*

Our next result generalizes Theorem A.

Theorem 2. *Let X be a real or complex Banach space and let $A(X)$ be a standard operator algebra on X. Suppose there exists a linear mapping D : $A(X) \to L(X)$ satisfying the relation*

$$D(A^2) = D(A)A + AD(A)$$

for all $A \in A(X)$. In this case D is of the form $D(A) = [A, B]$, for all $A \in A(X)$ and some $B \in L(X)$.

Proof. We have therefore the relation

$$D(A^2) = D(A)A + AD(A), \tag{8}$$

for all $A \in A(X)$. Similarly, as in the proof of Theorem 1, we will consider the restriction of D on $F(X)$. Let A be from $F(X)$ and let $P \in F(X)$ be a projection such that $AP = PA = A$.

From the relation (8) one obtains

$$D(P) = D(P)P + PD(P).$$

Right multiplication of the above relation by P gives $PD(P)P = 0$. Putting $A + P$ for A in the relation (8) we obtain after some calculation

$$2D(A) = D(P)A + D(A)P + PD(A) + AD(P).$$

From the above relation it follows that D maps $F(X)$ into itself. According to the relation (8) one can conclude that we have a Jordan derivation on $F(X)$ whence it follows that D is a derivation by primeness of $F(X)$. By Theorem A there exists $B \in L(X)$ such that

$$D(A) = [A, B], \tag{9}$$

holds for all $A \in F(X)$. It remains to prove that the relation (9) holds for all $A \in A(X)$ as well. Let us introduce $D_1 : A(X) \to L(X)$ by $D_1(A) = [A, B]$

and consider $D_0 = D - D_1$. The mapping D_0 is, obviously, linear and satisfies the relation (8). Besides, D_0 vanishes on $F(X)$. We intend to prove that it vanishes on $A(X)$ as well. Let $A \in A(X)$, let P be an one-dimensional projection and $S = A + PAP - (AP + PA)$. We have $D_0(S) = D_0(A)$ and $SP = PS = 0$. Now we have

$$D_0(S)S + SD_0(S) = D_0(S^2) = D_0(S^2 + P) = D_0((S + P)^2)$$
$$= D_0(S)(S + P) + (S + P)D_0(S) = D_0(S)S + D_0(S)P + SD_0(S) + PD_0(S).$$

We have therefore proved that

$$D_0(A)P + PD_0(A) = 0.(10)$$

Multiplying the relation (10) from both sides by P we obtain $PD_0(A)P = 0$. Now right multiplication of the relation (10) by P gives $D_0(A)P = 0$. Since P is arbitrary one-dimensional projection we have $D_0(A) = 0$, for all $A \in A(X)$ which completes the proof of the theorem.

Theorem 2 and the result below will be used in the proof of Theorem 4.

Theorem 3. *Let X be a Banach space over the real or complex field F and let $A(X)$ be a standard operator algebra on X. Suppose there exists an additive mapping $T : A(X) \to L(X)$, such that*

$$T(A^2) = [T(A), A]$$

holds for all $A \in A(X)$. In this case the following statements are fulfilled.

(*i*) $T(A) = 0$, *for all* $A \in F(X)$
(*ii*) $T(A^2) = 0$, *for all* $A \in A(X)$.
(*iii*) $T(A) = \alpha(A)I$, *for all* $A \in A(X)$ *where* $\alpha : A(X) \to F$ *is an additive mapping.*

Proof. We have therefore the relation

$$T(A^2) = [T(A), A]\,, \tag{11}$$

for all $A \in A(X)$. Putting in the above relation $A + B$ for A, we obtain $T(AB + BA) = [T(A), B] + [T(B), A]$, for all pairs $A, B \in A(X)$. In particular putting $B = A^2$ and applying the relation (11) we obtain

$$2T(A^3) = \left[T(A), A^2\right] + \left[T(A^2), A\right] = [T(A), A]\, A +$$

$$A\,[T(A),A] + [[T(A),A]\,,A] = 2\,[T(A),A]\,A = 2T(A^2)A.$$

We have therefore

$$T(A^3) = T(A^2)A,$$

for all $A \in A(X)$. Since, obviously, all the assumptions of Theorem 1 are fulfilled one can conclude that there exists $B \in L(X)$, such that

$$T(A) = BA \tag{12}$$

is fulfilled for all $A \in F(X)$ and

$$T(A^2) = BA^2 \tag{13}$$

holds for all $A \in A(X)$.It is our aim to prove that $B = 0$. Combining the relation (11) with the relation (12) we obtain $BA^2 = [BA, A] = BA^2 - ABA$. We have therefore

$$ABA = 0, \tag{14}$$

for all $A \in F(X)$. Putting $A+P$ for A, where P is an one-dimensional projection, in the above relation we obtain

$$ABP + PBA = 0$$

Putting in the above relation PA for A, and applying the relation (14) we obtain $PABP = 0$ which means that we have $(BP)A(BP) = 0$, for all $A \in F(X)$, whence it follows $BP = 0$ by primeness of $F(X)$. Since P is arbitrary one-dimensional projection it follows that $B = 0,$ which gives according to (12) and (13) first two statements of the theorem. It remains to prove the statement (iii). According to the statement (ii) of the theorem the relation (11) reduces to

$$[T(A), A] = 0, \tag{15}$$

for all $A \in A(X)$. Putting in the above relation $A + P$ for A, where P is an one-dimensional projection, and applying the fact that T vanishes on $F(X)$, we obtain $[T(A), P] = 0$. Therefore, $T(A)$ commutes with all one-dimensional projections. which means that for any $A \in A(X)$ there exists $\alpha(A) \in F$, such that $T(A) = \alpha(A)I$ holds. Obviously, $A \to \alpha(A)$ is an additive mapping on $A(X)$. The proof of the theorem is complete.

Corollary 2. *Let X be a real or complex Banach space. Suppose there exists an additive mapping $T : F(X) \to L(X)$, such that*

$$T(A^2) = [T(A), A]$$

holds for all $A \in F(X)$. In this case we have $T(A) = 0$, for all $A \in F(X)$.

We proceed with the concept of so-called generalized derivations. In the theory of operator algebras one usually means by a generalized derivation of an algebra A a mapping of the form $x \to ax + xb$ where a and b are fixed elements of A. We prefer, however, to call such mappings generalized inner (or alternatively inner generalized) derivations for they present a generalization of the concept of inner derivations. In the theory of operator algebras, they are considered as an important class of so-called elementary operators (i.e., mappings of the form $x \to \sum_{i=1}^{n} a_i x b_i$). We refer the reader to $[10]$ for a good account of this theory. Now let R be a ring and let F be a generalized inner derivation of R given by $F(x) = ax + xb$. In this case we have $F(xy) = F(x)y + x\,[y, b]$ for all pairs $x, y \in R$. In view of this observations we now give the following definition. An additive mapping F of a ring R into itself is called a generalized derivation if there exists a derivation $D : R \to R$ such that $F(xy) = F(x)y + xD(y)$ holds for all pairs $x, y \in R$. By our knowledge the concept of generalized derivations was introduced by Brešar $[6]$. The concept of generalized derivations covers both concepts the concept of derivations and the concept of generalized inner derivations. Moreover, the concept of generalized derivations covers the concept of left centralizers as well. Namely, it is easy to see that an additive mapping F, which maps a ring R into itself, is a generalized derivation iff F is of the form $F = D + T$ where $D : R \to R$ is a derivation and $T : R \to R$ is a left centralizer. For results concerning generalized derivations we refer to $[1, 6, 7, 12, 13, 15, 16]$. Recently, Jing and Lu $[13]$ introduced the concept of generalized Jordan derivation. An additive mapping $F : R \to R$ is a generalized Jordan derivation if $F(x^2) = F(x)x + xD(x)$ holds for all $x \in R$, where $D : R \to R$ is a Jordan derivation. The concept of generalized Jordan derivation was the motivation for the following problem. Let R be a ring and let $D, G : R \to R$ be additive mappings satisfying relations

$$D(x^2) = D(x)x + xG(x) \text{and} G(x^2) = G(x)x + xD(x), \tag{16}$$

for all $x \in R$. The question arises about the solution of the equations above. Let R be a ring, let $H : R \to R$ be a Jordan derivation and let $f : R \to Z(R)$ be an additive mapping with $f(x^2) = 0$, for all $x \in R$. In this case an easy calculation shows that mappings D and G of the form $D(x) = H(x) + f(x), G(x) = H(x) - f(x)$ satisfy the relations (16). The observations above lead to the following result which generalizes Theorem 2 as well as Theorem A.

Theorem 4. *Let X be a Banach space over the real or complex field F. Suppose there exist linear mappings $D, G : A(X) \to L(X)$, such that*

$$D(A^2) = D(A)A + AG(A) \text{ and } G(A^2) = G(A)A + AD(A)$$

is fulfilled for all $A \in A(X)$. *In this case there exists* $B \in L(X)$ *and a linear mapping* $\alpha : A(X) \to F$ *such that the following statements are fulfilled:*

(i) $D(A) = [A, B] + \alpha(A)I, G(A) = [A, B] - \alpha(A)I$, *for all* $A \in A(X)$.

(ii) $D(A^2) = G(A^2)$, *for all* $A \in A(X)$.

(iii) $D(A) = G(A) = [A, B]$, *for all* $A \in F(X)$.

Proof. We have the relations

$$D(A^2) = D(A)A + AG(A) \tag{17}$$

and

$$G(A^2) = G(A)A + AD(A), \tag{18}$$

for all $A \in A(X)$. From the above relations it follows that the mapping H, defined by $H(A) = D(A) + G(A)$, $A \in A(X)$ satisfies the relation $H(A^2) = H(A)A + AH(A)$, for all $A \in A(X)$. According to Theorem 2 there exists $C \in A(X)$ such that

$$H(A) = [A, C], \tag{19}$$

holds for all $A \in A(X)$. Subtracting the relation (18) from the relation (17) we obtain

$$T(A^2) = [T(A), A], \tag{20}$$

for all $A \in A(X)$, where T stands for $D - G$. We have therefore a linear mapping $T : A(X) \to L(X)$ satisfying the relation (20). By Theorem 3 T vanishes on $F(X)$, $T(A^2) = 0$, for all $A \in A(X)$, and there exists a linear mapping $\beta : A(X) \to F$ such that $T(A) = \beta(A)I$ is fulfilled for all $A \in A(X)$. Combining these facts with the relation (19) one can conclude that $D(A) = [A, B] + \alpha(A)I$ and $G(A) = [A, B] - \alpha(A)I$ is fulfilled for all $A \in A(X)$, where $B = \frac{1}{2}C$ and $\alpha = \frac{1}{2}\beta$. Besides, $D(A) = G(A) = [A, B]$, for all $A \in F(X)$ and $D(A^2) = G(A^2)$, for all $A \in A(X)$. The proof of the theorem is complete.

Corollary 3. *Let* X *be a real or complex Banach space. Suppose there exist linear mappings* $D, G : F(X) \to L(X)$ *such that*

$$D(A^2) = D(A)A + AG(A) \text{ and } G(A^2) = G(A)A + AD(A)$$

is fulfilled for all $A \in F(X)$. *In this case there exists* $B \in L(X)$ *such that* $D(A) = G(A) = [A, B]$ *holds for all* $A \in F(X)$.

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