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# ON GEOMETRIC AND TOPOLOGICAL PROPERTIES OF THE CLASSES OF HEREDITARILY $\ell_p$ BANACH SPACES

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**Abstract.** A class of hereditarily $\ell_p$ $(1 \le p < \infty)$ Banach sequence spaces is constructed and denoted by $X_{\alpha,p}$. Any constructed space is a dual space. We show that (i) the predual of any member $X$ of the class of $X_{\alpha,1}$ contains asymptotically isometric copies of $c_0$.(ii) Every infinite dimensional subspace of $X$ contains asymptotically isometric complemented copies of $\ell_1$, and consequently, the dual X$^*$ of X contains subspaces isometrically isomorphic to $C[0,1]^*$. (iii) Every member of the class of $X_{\alpha,p}$ $(1 \le p < \infty)$ fails the Dunford-Pettis property. (iv) We observe that all $X_{\alpha,p}$ spaces are Banach spaces without unconditional basis but all constructed spaces contain a subspace which is weakly sequentially complete with an unconditional basis which is weakly null sequence but not in norm. (v) All spaces have asymptotic-norming and Kadec-Klee property. The predual of any $X_{\alpha,p}$ is an Asplund space.

## 1. Introduction

S. Chen and B.-L. Lin [4] proved that a Banach space contains an asymptotically isometric copy of $\ell_1$ if its dual space contains an isometric copy of $\ell_\infty$, and if a Banach space contains an asymptotically isometric copy of $c_0$, then its dual space contains an asymptotically isometric copy of $\ell_1$.

J. Dilworth, M. Girardi and J. Hagler [7] have shown that a Banach space contains asymptotically isometric copies of $\ell_1$ if and only if its dual space contains an isometric copy of $L_1$. In [3] a class of hereditarily $\ell_1$ Banach space failing the Schur property was studied. Hagler in an unpublished result showed that all of the spaces contain $\ell_1$ hereditarily complemented, and their predual contains many subspaces

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isomorphic to $c_0$ [8]. In this paper we study further properties of the spaces. In particular, we prove that the predual of any member $X$ of this class contains asymptotically isometric copies of $c_0$ and consequently $X$ contains asymptotically isometric copies of $\ell_1$.

The Banach spaces of this class was extended to the $X_{\alpha,p}$ spaces. Let $X$ denote a specific $X_{\alpha,p}$ space, then $X$ contains $\ell_p$ hereditarily complemented $(1 \leq p < \infty)$ [2]. Every member $X$ fails the Dunford-Pettis property. We also observe that all constructed spaces have asymptotic-norming and Kadec-Klee property. Since any $X_{\alpha,p}$ is a dual space, it follows that the Banach space $Y$ the predual of any $X_{\alpha,p}$ is an Asplund space. The $X_{\alpha,p}$ spaces for $p > 1$ contain reflexive subspaces which are weakly sequentially complete with unconditional basis. Excellent sources of information on the asymptotic-norming property of Banach spaces are [9, 10, 12].

A Banach space X is said to be an Asplund space if every convex subset of X is Frechet differentiable at all points of a dense $G_\delta$ subset of its domain.

It is known that a Banach space $X$ is an Asplund space if and only if $X^*$ has the Radon-Nikodym property if and only if every separable subspace of $X$ has a separable dual [1, 14, 15]. We observe that the predual of any $X_{\alpha,p}$ is an Asplund space.

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Now we go through the construction of the spaces.

A block $F$ is an interval (finite or infinite) of integers. For any block $F$, and $x = (t_1, t_2, \ldots)$ a finitely non-zero sequence of scalars, we let $\langle x, F\rangle = \sum_{j\in F} t_j$. A sequence of blocks $F_1, F_2, \ldots$ is admissible if $\max F_i < \min F_{i+1}$ for each $i$. Finally, let $1 = \alpha_1 \geq a_2 \geq \alpha_3 \geq \ldots$ be a sequence of real numbers with $\lim_{i\to\infty} \alpha_i = 0$ and $\sum_{i=1}^{\infty} \alpha_i = \infty$.

We now define a norm which uses the $\alpha_i$'s and admissible sequence of blocks in its definition. Let $1 \leq p < \infty$ and $x = (t_1, t_2, \ldots)$ be finitely non-zero sequence of reals. Define

$$\|x\| = \max \left[ \sum_{i=1}^{n} \alpha_i |\langle x, F_i\rangle|^p \right]^{\frac{1}{p}}$$

where the max is taken over all $n$, and admissible sequences $F_1, F_2, \ldots$. The Banach space $X_{\alpha,p}$ is the completion of the finitely non-zero sequences of scalars in this norm.

## 2. Definitions and Notation

Definitions and notation are standard, but we give some of these here.

The dual space of $X$ is denoted by $X^*$. A subspace $Y$ of $X$ is complemented in $X$ if there is a projection $P : X \to X$ such that $P(X) = Y$ and $\|P\| < \infty$.

Let $\ell_1$ be the space of absolutely summable sequences and $L_1$ the space of Lebesgue-integrable functions on $[0,1]$. $c_0$ is the space of all null sequences $x = (t_1, t_2, \ldots)$ with $\|x\| = \max_n |t_n|$.

A Banach space $X$ is called hereditarily $\ell_1$ if every infinite dimensional subspace of $X$ contains a subspace isomorphic to $\ell_1$.

**Definition 2.1.** Let $X$ be a Banach space. We say that $X$ contains asymptotically isometric copies of $\ell_1$ if for some sequence $\lambda_0 < \lambda_1 < \ldots$ with $\lim_n \lambda_n = 1$, there is sequence $(x_n)$ in $X$ such that for all $m$ and scalars $(t_n : 0 \leq n \leq m)$

$$\sum_{n=0}^{m} \lambda_n |t_n| \leq \| \sum_{n=0}^{m} t_n x_n \| \leq \sum_{n=0}^{m} |t_n|$$

$X$ contains asymptotically isometric copy of $c_0$ if

$$\max_i \lambda_i |t_i| \leq \| \sum_{n=0}^{m} t_n x_n \| \leq \max_i |t_i|$$

**Definition 2.2.** A Banach space $X$ is said to have the Dunford-Pettis property (DPP) if for every weakly null sequences $(x_n)$ in $X$ and $(x_n^*)$ in $X^*$, then $\lim_n x_n^* (x_n) = 0$.

**Definition 2.3.** An infinite-dimensional Banach space $X$ is said to be prime if every infinite-dimensional complemented subspace of $X$ is isomorphic to $X$.

It is known that $c_0$, $\ell_p$, $1 \leq p < \infty$ and $\ell_\infty$ are prime.

## 3. The Results

The key to the analysis of the space $X$ is via the following result(lemma 4 of [3]).

**Lemma 3.1.** *Let the sequence $(\alpha_i)$ be as above, let $N > 0$ be an integer and let $\varepsilon > 0$. Then there exist a $\delta > 0$ such that, if $b_1, b_2, \ldots, b_n$ are $\geq 0$, $b_i < \delta$ for all $i$, and $\sum_{i=1}^{n} \alpha_i b_i = 1$, then $\sum_{i=1}^{n} \alpha_{i+N} b_i \geq 1 - \varepsilon$.*

The following summarize the main result of [2]. Let $(e_i)$ denote the sequence of usual unit vectors in $X_{\alpha,p}$, $e_i(j) = \delta_{ij}$.

**Theorem 3.2.** *Let $X_{\alpha,p}$ denote a specific space of the class, we have the following:*

(1) *$X_{\alpha,p}$ is hereditarily complementably $\ell_p$.*

(2) *The sequence* $(e_i)$ *is a normalized boundedly complete bases for* $X_{\alpha,p}$. *Thus,* $X_{\alpha,p}$ *is a dual space.*

(3) *The predual of* $X_{\alpha,p}$ *contains complemented subspaces isomorphic to* $\ell_q$ *where* $\frac{1}{p}+\frac{1}{q}=1$.

(4) $X_{\alpha,p}$ *spaces have some other properties similar to [3], which we state some of them here.*

   (a) *Each complemented non weakly sequentially complete subspace of* $X_{\alpha,p}$ *contains a complemented isomorph of* $X_{\alpha,p}$.
   (b) $X_{\alpha,p}$ *and* $X_{\beta,p}$ *are isomorphic if and only if they are equal as sets.*
   (c) *The sequence* $(x_n)$ *with* $x_n = e_{2n-1} - e_{2n}$ *is weakly null sequence in* $X_{\alpha,p}$ *but not in norm.*
   *Since* $X_{\alpha,p}$ *contains* $\ell_p$ *hereditarily complementably, thus,*
   (d) $X_{\alpha,p}$ *spaces are not prime.*
   *Since for* $p > 1$, $X_{\alpha,p}$ *does not contain* $\ell_1$ *and is not reflexive,*
   (e) $X_{\alpha,p}$ *is a Banach space without unconditional basis.*

**Remark 3.3.** Let $(f_i)$ in $X^*$ be the biorthogonal sequence to the usual basis $(e_i)$ in $X$, and let $Y$ be the subspace of $X^*$ generated by the sequence $(f_i)$. Theorem 3.2(2) and well known result [13](proposition 1.b.4 page 9) show that $X = Y^*$. For $p = 1$, Hagler proved that $Y$ contains many subspaces isomorphic to $c_0$. For $p > 1$, Theorem 3.2(3) shows that $Y$ contains complemented subspaces isomorphic to $\ell_q$ where $\frac{1}{p}+\frac{1}{q}=1$.

There are a number of possible future directions that one might take in studying further the structure of the space $Y$. We list two of them:

(1) For $p = 1$, is $Y$ hereditarily $c_0$ ?

(2) For $p > 1$, is $Y$ hereditarily complementably $\ell_q$ ?

**Theorem 3.4.** *The predual of* $X_{\alpha,1}$ *contains asymptotically isometric copies of* $c_0$.

*Proof.* Let $V$ be an infinite dimensional subspace of $X_{\alpha,1}$. The proof of Theorem 1.(1) in [3] shows that we may assume the following:

There exist sequences $(v_i)$ in $V$, $(n_i)$ of integers, and $\delta_i > 0$ satisfying

1. $\|v_i\| = 1$ for all $i$.
2. Put $N_i = n_1 + n_2 + \ldots + n_{i-1}$ for $i > 1$ and $N_1 = 0$. Then $\delta_i$ satisfies Lemma 3.1 for $\varepsilon_i < \varepsilon_{i-1} < \ldots < 1$ and $N = N_i$.
3. For each block $F$ and $i$, $|\langle v_i, F\rangle| \le \delta_i$.
4. For each $i$, there is a sequence of admissible blocks $F_1^i, F_2^i, \ldots, F_{n_i}^i$ with

(a) $\max F^i_{n_i} < \min F^{i+1}_1$ for each $i$
(b) $\sum_{j=1}^{n_i} \alpha_j |\langle v_i, F^i_j \rangle| = 1.$
(c) $\langle v_k, F^i_j \rangle = 0$ if $k \neq i$.

A trivial modification $(1 - \varepsilon_i, i = 1, 2, ...$ instead of 1/2) in proof of theorem 1.(1) in [3] shows that

$$\begin{aligned} \| \sum_{i=1}^{n} t_i v_i \| &\geq \sum_{i=1}^{n} \sum_{j=1}^{n_i} \alpha_{j+N_i} |\langle \sum_{k=1}^{n} t_k v_k, F^i_j \rangle| \\ &= \sum_{i=1}^{n} |t_i| \sum_{j=1}^{n_i} \alpha_{j+N_i} |\langle v_i, F^i_j \rangle| \\ &\geq \sum_{i=1}^{n} (1 - \varepsilon_i) |t_i| \end{aligned}$$

for any $n$, and scalars $t_1, t_2, \ldots t_n$.

Let $\phi_i \in X^*_{\alpha,1}$ be defined by

$$\phi_i(x) = \sum_{j=1}^{n_i} \varepsilon^i_j \alpha_{j+N_i} \langle x, F^i_j \rangle$$

where $\varepsilon^i_j = sgn\left(\langle v_i, F^i_j \rangle\right)$ for each $j$ and $i$.

Properties (1-4) for the $v_i$s imply that

$$\begin{aligned} \phi_i(v_i) &= \sum_{j=1}^{n_i} \varepsilon^i_j \alpha_{j+N_i} \langle v_i, F^i_j \rangle \\ &= \sum_{j=1}^{n_i} \alpha_{j+N_i} |\langle v_i, F^i_j \rangle| \\ &\geq (1 - \varepsilon_i) \sum_{j=1}^{n_i} \alpha_j |\langle v_i, F^i_j \rangle| \\ &= 1 - \varepsilon_i \end{aligned}$$

for each $i$ and $\phi_i(v_j) = 0$ for $i \neq j$.

Let $n$, and scalars $t_1, \ldots, t_n$ be given. Since $\|v_i\| = 1$ for all $i$ and

$$|\sum_{i=1}^{n} t_i \phi_i(v_j)| \geq (1 - \varepsilon_j) |t_j|.$$

This implies that

$$\|\sum_{i=1}^{n} t_i\phi_i\| \geq \max_j (1-\varepsilon_j)\,|t_j|.$$

Now by definition of $\phi_i$, for each $x \in X$, $\sum_i |\phi_i(x)| \leq \|x\|$. So if $\|x\| = 1$,

$$\begin{aligned} |\sum_{i=1}^{n} t_i\phi_i(x)| &\leq \sum_{i=1}^{n} |t_i||\phi_i(x)| \\ &\leq (\max_i |t_i|)\left(\sum_{i=1}^{n} |\phi_i(x)|\right) \\ &\leq \max_i |t_i|. \end{aligned}$$

Taking sup over all $\|x\| = 1$ shows that

$$\|\sum_{i=1}^{n} t_i\phi_i\| \leq \max_i |t_i|.$$

Let $X = Y^*$ then clearly each $\phi_i \in Y$ (remark 3.3) and therefore $X$ contains asymptotically isometric copies of $c_0$. ∎

Theorem 3.4 and theorem 5 of [4] have the following consequence.

**Theorem 3.5.** *The Banach space $X_{\alpha,1}$ contains asymptotically isometric copies of $\ell_1$.*

The following Theorem is an immediate consequence of theorem 2 of [7] and corollary 3.5.

**Theorem 3.6.**

$(i)$ *The dual $X^*_{\alpha,1}$ of $X_{\alpha,1}$ contains subspaces isometrically isomorphic to $C[0,1]^*$,*

$(ii)$ *$C(\Delta)$ is isometric to a quotient space of $X_{\alpha,1}$ where $\Delta$ is the Cantor set and*

$(iii)$ *$L_1$ is linearly isometric to a subspace of $X^*_{\alpha,1}$.*

**Definition 3.7.** A norming set for a Banach space $X$ is defined to be a subset $\phi$ of the unit ball of $X^*$ such that, for each $x \in X$,

$$\|x\| = \sup\{\varphi(x) : \varphi \in \phi\}.$$

The next definition make use of a convergence criteria for a bounded sequence $(x_i)$ in a Banach space.

**Definition 3.8.** A Banach space $X$ have the asymptotic-norming property (ANP) if it has an equivalent norm for which there is a norming set $\phi$ which has the property that the sequence $(x_n)$ converges strongly if $\|x_n\| = 1$ for each $n$ and $(x_n)$ is asymptotically normed by $\phi$, meaning that, for each positive $\varepsilon$, there exist $\varphi \in \phi$ and $N$ such that

$$\varphi(x_n) > 1 - \varepsilon \text{ if } n > N$$

The following theorem which is from [12] is essential in this study.

**Theorem 3.9.**

$(i)$ *If $X^*$ is separable and also is a dual of a Banach space $X$, then $X^*$ has ANP.*

$(ii)$ *There is a separable Banach space that has ANP and is not isomorphic to any subspace of a separable dual.*

$(iii)$ *If a Banach space $X$ has ANP, then $X$ has RNP.*

It is not known whether RNP implies ANP, even for Banach spaces that are dual.

**Definition 3.10.** A Banach space is said to have Kadec-Klee property if $(x_n)$ converges strongly to $x$ whenever $(x_n)$ converges weakly to $x$ and $\|x\| = \|x_n\|$ for each $n$.

The following result of Stegall shows that $Y$ the predual of any $X_{\alpha,p}$ is an Asplund space [15].

**Theorem 3.11.** *If $X^*$ has the Radon-Nikodym property then $X$ is an Asplund space.*

Theorems 3.9, 3.11 and theorem 3.1 of [12] imply that,

**Theorem 3.12.** *Let $X$ be a member of the class of $X_{\alpha,p}$ spaces then $X$ has the following properties.*

1. *$X$ has asymptotic-norming property.*
2. *$X$ has Kadec-Klee property.*
3. *Banach space $Y$ the predual of $X$ is an Asplund space.*

**Remark 3.13.** A subspace $W$ of the dual of a Banach space has the $w^*-$Kadec-Klee property ($w^*$-KK property) if $(w_i)$ in $W$ converges strongly to $w$ whenever $w \in W$, $\|w\| = \|w_i\|$ for each i, and $w$ is the $w^*$-limit of $(w_i)$. Since $X$ is the separable dual of a Banach space $Y$ it follows from results of Davis and Johnson [5] that $Y$ can be given an equivalent norm for which $X$ then has $w^*$-KK property.

Before we go through the proof of theorem 3.15 we prove the following lemma.

**Lemma 3.14.** *Let $(x_n)$ be a sequence of vectors in a Banach space $X$, such that for every increasing sequence, $(n_k)$ of integers,*

$$\lim_{k\to\infty} \frac{\|x_{n_1} + x_{n_2} + \ldots + x_k\|}{k} = 0$$

*then $x_n \to 0$ weakly.*

*Proof.* If this is not true, then there exit $f \in X^*$ with $\|f\| = 1, \delta > 0$ and a sequence $(n_i)$ of integers such that $f(x_{n_i}) \geq \delta$. This implies that $\sum_{i=1}^k f(x_{n_i}) \geq k\delta$. Therefore,

$$\frac{\|\sum_{i=1}^{k} x_{n_i}\|}{k} \geq \frac{\sum_{i=1}^{n} f(x_{n_i})}{k} \geq \delta$$

which is a contradiction. ∎

**Lemma 3.15.** *The Banach spaces $X_{\alpha,p}$ $(1 \leq p < \infty)$ fail the DPP.*

*Proof.* Let $u_i = e_{2i} - e_{2i-1}$ and $f_i : X_{\alpha,p} \to R$ such that for any $x = (t_1, t_2, \ldots) \in X_{\alpha,p}$, we have $f_i(x) = t_i$ for integers $i$. Then for $g_n = f_{2n} - f_{2n-1}$, we have $g_n(u_n) = 2$. To complete the proof we need to show that $u_n \to 0$ weakly, and $g_n \to 0$ weakly. The first one follows from Lemma 3.14. We claim that $g_n \to 0$ weakly. If not there are $F \in X_{\alpha,p}^{**}$ with $\|F\| = 1$, $\delta > 0$ and a subsequence $(g_{n_k})$ such that $F(g_{n_k}) > \delta$ for all integers $k$. So for integer $N$, we have $\sum_{k=1}^N F(g_{n_k}) > N\delta$ and hence

$$\frac{\|\sum_{k=1}^{N} g_{n_k}\|}{N} > \delta.$$

This implies that for any integer $N$, there exist $x = (t_1, t_2, \ldots) \in X_{\alpha,p}$ such that

$$\frac{1}{N}\sum_{k=1}^{N} g_{n_k}(x) > \delta.$$

Then $\lim_{n\to\infty} t_n = 0$ for integers $N$ and corresponding $x = (t_1, t_2, \ldots)$, since $\sum_{i=1}^{\infty} \alpha_i = \infty$. Therefore,

$$\begin{aligned} \left|\frac{1}{N}\sum_{k=1}^{N} g_{n_k}(x)\right| &= \frac{1}{N}\left|\sum_{k=1}^{N}(t_{2n_k} - t_{2n_k-1})\right| \\ &\leq \frac{1}{N}\sum_{k=1}^{N}|t_{2n_k}| + \frac{1}{N}\sum_{k=1}^{N}|t_{2n_k-1}| \to 0 \end{aligned}$$

as $N \to \infty$ which is a contradiction. ∎

**Remark 3.16.** It is known that if $X^*$ has the DPP, then so does $X$. This implies that $X^*_{\alpha,p}$ also fails the DPP.

It is known that if an infinite-dimensional Banach space has no normalized weakly null sequence then it contains infinite unconditional basic sequence, in fact it contains a subspace isomorphic to $\ell_1$. In [2], we proved that $X_{\alpha,p}$ is a class of hereditarily complementably $\ell_p$ Banach spaces. Here is some other properties of these spaces.

**Theorem 3.17.**

$(i)$ *Let* $u_i = e_{2i} - e_{2i-1}$ $(i \in N)$ *and* $Y$ *be the closed subspace of an specific* $X_{\alpha,p}$ *generated by* $u_i$, *i.e.,* $Y = [u_i]$. *Then the sequence* $(u_i)$ *is an unconditional basis of* $Y$.

$(ii)$ $Y$ *is weakly sequentially complete and* $u_i \to 0$ *weakly, but in norm.*

*Proof.* Part(i) is a consequence of the fact that for any sequence $(t_i)$, and any $j$, we have $\|\sum_{i\neq j} t_i u_i\| \leq \|\sum_i t_i u_i\|$, See [13] (Proposition 1.c.6 page 18).

For part(ii), since $(u_i)$ is unconditional basis for $[u_i]$ and since $[u_i]$ does not contain a copy of $c_0$, it follows from [6] (Theorem 2, page 74) that $[u_i]$ is weakly sequentially complete.

Theorem 3.2 shows that $u_i \to 0$ weakly but not in norm. In fact $\|u_i\| = (1+\alpha_2)^{\frac{1}{p}}$. ∎

**Remark 3.18.** A result of James [11] asserts that a Banach space with an unconditional basis is either reflexive or has a subspace isomorphic to $c_0$ or $\ell_1$. This implies that the Banach spaces $Y = [u_i]$ for $p > 1$ is reflexive.

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