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Review Article

Modeling Dynamics of Prey-Predator Fishery Model with Harvesting: A Bioeconomic Model

Charles Raymond, Alfred Hugo, and Monica Kung'aro

Department of Mathematics, University of Dodoma, P.O. Box 338, Dodoma, Tanzania

Correspondence should be addressed to Alfred Hugo; alfredkhugo@gmail.com

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A mathematical model is proposed and analysed to study the dynamics of two-prey one predator system of fishery model with Holling type II function response. The effect of harvesting was incorporated to both populations and thoroughly analysed. We study the ecological dynamics of the Nile perch, cichlid, and tilapia fishes as prey-predator system of lake Victoria fishery in Tanzania. In both cases, by nondimensionalization of the system, the equilibrium points are computed and conditions for local and global stability of the system are obtained. Condition for local stability was obtained by eigenvalue approach and Routh-Hurwitz Criterion. Moreover, the global stability of the coexistence equilibrium point is proved by defining appropriate Lyapunov function. Bioeconomic equilibrium is analysed and numerical simulations are also carried out to verify the analytical results. The numerical results indicate that the three species would coexist if cichlid and tilapia fishes will not be overharvested as these populations contribute to the growth rates of Nile perch population. The fishery control management should be exercised to avoid overharvesting of cichlid and tilapia fishes.

1. Introduction

In today's life, the relationship between predator and prey became an important aspect to discuss in ecology. The prey-predator system has attracted many researchers to study the interaction between the species [1]. Thus, we use mathematical ecology aspect to study the interacting species. The current study considers lake Victoria fishery found in Tanzania as a case study. However we have not gone to the field, but the current trends on the dynamics of species in the lake have been obtained from reading different literature such as NPFMP [2], FAO [3], LVFO [4], Barack [5], and Barilwa [6]. This literature explains the current trend of the lake and the fishery management of the lake Victoria. The lake is comprised of a lot of species such as stocked Nile perch, Lates niloticus, tilapia fish, Oreochromis niloticus, the cyprinid, Rastrineobola argentea, catfishes, insects, cichlids, crocodiles, and many zooplanktons and phytoplanktons NPFMP [2].

The particular study focuses on Nile perch as the predator while cichlid fishes and tilapia fishes are considered as prey populations. All three species are encountering the harvesting aspect. However harvesting without limitations may have detrimental effects on fish population because it decreases the population and sometimes leads a certain species to extinction, Ganguli [7]. The lake Victoria fishery today is either overexploited or in a state of full exploitation because of greater fishing effort and increased competition between fishers, vessels, or nations over the resource. The particular study intends to apply mathematical techniques to ensure the sustainability of the species in lake Victoria without compromising the biological, economic, and social objectives for the benefit of present and future generations. Prey-predator model in fishery was also studied by Kar [8], Chakraborty [9], and Yunfei and Yongzhen [10], while studies by Tapas et al. [11], Ganguli at el [7], Kar [8], Gian [12], Chaudhuri and Kar [13], and Kar [8] analysed the bioeconomic aspect of prey-predator system and observed that increasing harvesting efforts result in population decreases. The particular study intends to analyse the bioeconomic impact for the lake Victoria fishery activities in Tanzania.

2. Materials and Methods

2.1. Model Description, Formulation, and Analysis. It will be assumed that the Nile perch depends completely on cichlid and tilapia fishes as their favorite food (because of easy to capture and their taste) where cichlid and tilapia fishes have unlimited sources of food. The dynamics therefore follow the Holling type II function response. In this case, $x_1(t)$, $x_2(t)$, and $x_3(t)$ represent the population of cichlid fish, tilapia fish, and Nile perch, respectively, at any time t, where all species involved in modeling are also encountering harvesting aspect. The growth rate of cichlid fishes and tilapia fishes follows the logistic law and the birth rate should always be positive. Terms representing interspecific competition among the prey species are included in the model and the model is then divided into three nonlinear autonomous ordinary differential equations describing how the population densities of the three species would vary with time. The following assumptions are made in order to construct the model:

(i) Cichlid and tilapia fishes have an unlimited food supply in the lake Victoria.

(ii) The Nile perch is completely dependent on the cichlid and tilapia fishes as the only favorite food source.

(iii) Interspecific competition among tilapia and cichlid fishes is exploitative.

(iv) In absence of the predator, prey species grow logistically. That is, the population of the cichlid and tilapia fishes would increase exponentially until it reaches the maximum density of the Lake, which is its environmental carrying capacity K_i.

(v) The predation functional response of the Nile perch towards both cichlid fishes x_1 and tilapia fishes x_2 is assumed to follow Michaelis-Menten kinetics and is modeled using a Holling type II functional form with predation coefficients α_{13} and α_{23} and the half saturation constants β and γ.

By considering the underlying assumptions of the incorporated populations, we formulate the system of model equations as:

$$\begin{aligned}
\frac{dx_1}{dt} &= \lambda_1 x_1\left(1-\frac{x_1}{K_1}\right) - \alpha_{12}x_1x_2 - \frac{\alpha_{13}x_1x_3}{1+\beta x_1} - q_1E_1x_1\\
\frac{dx_2}{dt} &= \lambda_2 x_2\left(1-\frac{x_2}{K_2}\right) - \alpha_{23}x_1x_2 - \frac{\alpha_{23}x_2x_3}{1+\gamma x_2} - q_2E_2x_2\\
\frac{dx_3}{dt} &= -wx_3 + \alpha_{31}\frac{\alpha_{13}x_1x_3}{1+\beta x_1} + \alpha_{32}\frac{\alpha_{23}x_2x_3}{1+\gamma x_2} - q_3E_3x_3
\end{aligned} \tag{1}$$

with initial data values $x_1(0) \geqslant 0$, $x_2(0) \geqslant 0$, $x_3(0) \geqslant 0$.

All parameters in the model are assumed to be positive and λ_1 and λ_2 are per capita intrinsic growth rates of cichlid and tilapia fishes, respectively, while K_1 and K_2 are environmental carrying capacities of cichlid and tilapia fishes, respectively, α_{12} and α_{21} are coefficients for interspecific competition, α_{13} and α_{23} are predation coefficients for cichlid fishes and tilapia fishes, respectively, E_1, E_2, and E_3 are effort harvesting rates, w is natural mortality rate of Nile perch, and α_{31} and α_{32} are conversion parameters for cichlid fishes and tilapia fishes by Nile perch, while q_1, q_2, and q_3 are catchability coefficients.

For ease of computation, we are rescaling model (1) to reduce the number of parameters as follows: take $x = x_1/K_1$, $y = x_2/K_2$, $z = wx_3$, then the system of model (1) becomes

$$\begin{aligned}
\frac{dx}{dt} &= \lambda_1 x(1-x) - \sigma_1 xy - \frac{P_1xz}{1+Q_1x} - r_1x\\
\frac{dy}{dt} &= \lambda_2 y(1-y) - \sigma_2 xy - \frac{P_2yz}{1+Q_2y} - r_2y\\
\frac{dz}{dt} &= -wz + \frac{e_1xz}{1+Q_1x} + \frac{e_2yz}{1+Q_2y} - r_3z
\end{aligned} \tag{2}$$

with initial values $x(0) \geqslant 0$, $y(0) \geqslant 0$, and $z(0) \geqslant 0$.

2.2. Equilibrium Points of System (2). The equilibrium states of the model are obtained by setting $dx/dt = dy/dt = dz/dt = 0$ and we assume that the predator has positive mortality rate w. The following are the possible equilibrium points of the system $P_1(x^*, 0, 0)$, $P_2(0, y^*, 0)$, $P_3(x^*, y^*, 0)$, $P_4(x^*, 0, z^*)$, $P_5(0, y^*, z^*)$, and $P_6(x^*, y^*, z^*)$. Therefore,

(i) *The equilibrium point* $P_1(x^*, 0, 0)$ *with* $x^* > 0$

From system (2), in the absence of tilapia fish and Nile perch we have $P_1(x^*, 0, 0) = P_1((\lambda_1 - r_1)/\lambda_1, 0, 0)$ and this exists when $r_1 < \lambda_1$

(ii) *The equilibrium point* $P_2(0, y^*, 0)$ *with* $y^* > 0$

From system (2), in the absence of cichlid fishes and Nile perch ($x = 0$ and $z = 0$) we have $P_2(0, y^*, 0) = P_2((\lambda_2 - r_2)/\lambda_2, 0, 0)$ and this exists if $r_2 < \lambda_2$

(iii) *The equilibrium point* $P_3(x^*, y^*, 0)$ *with* $x^* > 0$ *and* $y^* > 0$

From system (2), in the absence of Nile perch ($z = 0$) we have

$$P_3(x^*, y^*, 0) = P_3\left(\frac{\lambda_2(\lambda_1 - r_1) - \sigma_1(\lambda_2 - r_2)}{\lambda_1\lambda_2 - \sigma_1\sigma_2}, \frac{\lambda_1(\lambda_2 - r_2) + \sigma_2(r_1 - \lambda_1)}{\lambda_1\lambda_2 - \sigma_1\sigma_2}, 0\right) \tag{3}$$

This exists if, for $r_1 < \lambda_1$ and $r_2 < \lambda_2$, $\lambda_2(\lambda_1 - r_1) > \sigma_1(\lambda_2 - r_2) \Longrightarrow \lambda_2(\lambda_1 - q_1E_1) > \alpha_{12}K_2(\lambda_2 - q_2E_2)$

Also $P_3(x^*, y^*, 0)$ exists if, for $\lambda_1 < r_1$ and $r_2 < \lambda_2$, $\sigma_2(r_1 - \lambda_1) > -\lambda_1(\lambda_2 - r_2)$

(iv) *The equilibrium point* $P_4(x^*, 0, z^*)$ *with* $x^* > 0$ *and* $z^* > 0$

From system (2), in the absence of tilapia fishes ($y = 0$) we have

$$P_4(x^*, 0, z^*) = P_4\left(\frac{w + r_3}{e_1 - Q_1(w - r_3)}, 0, \frac{-\lambda_1 Q_1 (w + r_3)^2}{P_1 (e_1 - Q_1 (w - r_3))^2} + \frac{(\lambda_1 Q_1 - r_1 Q_1 - \lambda_1)(w + r_3)}{P_1 (e_1 - Q_1 (w - r_3))} + \frac{\lambda_1 - r_1}{P_1}\right) \quad (4)$$

This exists if

$$(w + r_3)[-\lambda_1 Q_1 (w + r_3) - (\lambda_1 Q_1 - r_1 Q_1 - \lambda_1)(e_1 - wQ_1 - r_3 Q_1)] + (e_1 - wQ_1 - r_3 Q_1)^2 (\lambda_1 - r_1) > 0 \quad (5)$$

which is possible when $r_3 > -w$ and $\lambda_1 > r_1$

In terms of original parameter it implies $E_3 q_3 > -w$ and $\lambda_1 > E_1 q_1$

(v) *The equilibrium point* $P_5(0, y^*, z^*)$ *with* $y^* > 0$ *and* $z^* > 0$. From system (2), in the absence of cichlid fishes ($x = 0$) we have

$$P_5(0, y^*, z^*) = P_5\left(0, \frac{w + r_3}{e_2 - Q_2 (w - r_3)}, \frac{-\lambda_2 Q_2 (w + r_3)^2}{P_2 (e_2 - Q_2 (w - r_3))^2} + \frac{(\lambda_2 Q_2 - r_2 Q_2 - \lambda_2)(w + r_3)}{P_2 (e_2 - Q_2 (w - r_3))} + \frac{\lambda_2 - r_2}{P_2}\right) \quad (6)$$

This exists if $(w + r_3)[-\lambda_2 Q_2 (w + r_3) - (\lambda_2 Q_2 - r_2 Q_2 - \lambda_2)(e_2 - wQ_2 - r_3 Q_2)] + (e_2 - wQ_2 - r_3 Q_2)^2 (\lambda_2 - r_2) > 0$ and this is possible when $r_3 > -w$ and $\lambda_2 > r_2$ In terms of original parameter it means $E_3 q_3 > -w$ and $\lambda_2 > E_2 q_2$. The condition implies that $E_3 > -w$ and $\lambda_2 > E_2$

(vi) *Coexistence equilibrium point* $P_6(x^*, y^*, z^*)$

Following the procedure by Dubey [14], the endemic equilibrium point is obtained as follows:

$$\lambda_1 x (1 - x) - \sigma_1 xy - \frac{P_1 xz}{1 + Q_1 x} - r_1 x = 0 \quad (7)$$

$$\lambda_2 y (1 - x) - \sigma_2 xy - \frac{P_2 xz}{1 + Q_1 x} - r_2 y = 0 \quad (8)$$

$$-wz + \frac{e_1 xz}{1 + Q_1 x} + \frac{e_2 yz}{1 + Q_2 y} - r_3 z = 0 \quad (9)$$

From (7) we have

$$z = \frac{[\lambda_1 (1 - x) - \sigma_1 y - r_1](1 + Q_1 x)}{P_1} \quad (10)$$

From (8) we have

$$z = \frac{[\lambda_2 (1 - y) - \sigma_2 x - r_2](1 + Q_2 y)}{P_2} \quad (11)$$

From (8) and (9) we have

$$z = \frac{[\lambda_2 (1 - y) - \sigma_2 x - r_2 + w + r_3](1 + Q_2 y) - e_2 y}{P_2} - \frac{e_1 x (1 + Q_1 y)}{P_2 (1 + Q_1 x} \quad (12)$$

From (10) and (11) we have

$$f(x, y) = \frac{[(w + r_3)(1 + Q_2 y) - e_2 y](1 + Q_1 x) - e_1 x (1 + Q_2 y)}{P_2 (1 + Q_1 x)} \quad (13)$$

and from (10) and (12) we get

$$g(x, y) = \frac{[\lambda_2 (1 - y) - \sigma_2 x - r_2 + w + r_3](1 + Q_2 y) - e_2 y}{P_2} - \frac{e_1 x (1 + Q_1 y)}{P_2 (1 + Q_1 x} - \frac{[\lambda_1 (1 - x) - \sigma_1 y - r_1](1 + Q_1 x)}{P_1} \quad (14)$$

Equations (13) and (14) are two functions of x and y. To prove the existence of $P_6(x^*, y^*, z^*)$, the conditions under which $f(x, y)$ and $g(x, y)$ meet in the interior of the positive (x, y) plane at the point (x^*, y^*) are found. Now the values of x^*, y^* and z^* can be obtained from (7), then from (9) we observe that, as $x \longrightarrow 0$, y tends to y_f. y_f is the value of y at which the function $f(x, y)$ would cut the y axis in the (x, y) plane. So y_f is given by

$$y_f = \frac{w + r_3}{e_2 - Q_2 w - Q_2 r_3} \quad (15)$$

We notice that y_f is the same as y of $P_5(0, y^*, z^*)$. From (8), as $x \longrightarrow 0$, y tends to y_g given by

$$y_g = \frac{-D_2 + \sqrt{D_2^2 - 4 D_1 D_3}}{2 D_1} \quad (16)$$

where

$$D_1 = \frac{-\lambda_2 Q_2}{P_2}$$

$$D_2 = \frac{-\lambda_2 + \lambda_2 Q_2 - \sigma_2 r_2 + wQ_2 + r_3 Q_2 - e_2}{P_2} + \frac{\sigma_1}{P_1} \quad (17)$$

$$D_3 = \frac{P_1 (w + r_3 - r_2 + \lambda_2 + P_2 (r_1 - \lambda_2}{P_1 P_2}$$

y_f and y_g are the points at which the functions $f(x, y)$ and $g(x, y)$ would cut the y-axis in the (x, y) plane, respectively. Also from (13), $dy/dx = -(\partial f/\partial x)/(\partial f/\partial y)$ where

$$\frac{\partial f}{\partial x} = \frac{P_2(1+Q_1)\left[\left((w+r_3)(1+Q_2y)-e_2y\right)Q_1\right]}{P_2^2(1+Q_1x)^2} - \frac{P_2Q_1\left[\left((w+r_3)(1+Q_2y)-e_2y\right)(1+Q_1x-e_1x(1+Q_2y)\right]}{P_2^2(1+Q_1x)^2} \tag{18}$$

and

$$\frac{\partial f}{\partial y} = \frac{\left(wQ_2+r_3Q_2-e_2\right)(1+Q_1x)-e_2Q_2x}{P_2(1+Q_1x)} \tag{19}$$

We note that $dy/dx > 0$ if $\partial f/\partial x > 0$ and $\partial f/\partial y < 0$ and this requires $r_3 > -w$ and $\lambda_1 > r_1$. Similarly $dy/dx = -(\partial g/\partial x)/(\partial g/\partial y)$ from (14) where

$$\frac{\partial g}{\partial x} = \frac{-\sigma_2(1+Q_2y)}{P_2} + \frac{e_1(1+Q_2y}{P_2(1+Q_1x)^2} - \frac{Q_1(\lambda_1-\sigma_1y-r_1)-\lambda_1-2\lambda_1Q_1x}{P_1} \tag{20}$$

and

$$\frac{\partial g}{\partial y} = \frac{\left(w+r_3-r_2-\sigma_2x\right)Q_2}{P_2} - \frac{e_1Q_2x}{P_2(1+Q_1x)} \tag{21}$$

We also note that $dy/dx < 0$ if $\partial g/\partial x < 0$ and $\partial g/\partial y < 0$ and this requires $r_3 > -w$ and $r_3 > r_2$. Since for $f(x, y)$, we have $dy/dx > 0$ and for $g(x.y)$, we have $dy/dx < 0$, then $f(x, y)$ and $g(x, y)$ will meet if $y_f < y_g$. We therefore state the existence of the positive equilibrium point $P_6(x^*, y^*, z^*)$ in the following theorem.

Theorem 1. *The positive equilibrium point $P_6(x^*, y^*, z^*)$ will exist if the following conditions are satisfied:*

$$\begin{aligned} r_3 &> w, \\ \lambda_1 &> r_1, \\ r_3 &> r_2, \\ y_f &< y_g \end{aligned} \tag{22}$$

where y_f and y_g are as defined in (15) and (16), respectively.

In terms of original parameter, $\lambda_1 > r_1$ implies that $\lambda_1 > E_1q_1$; i.e., the growth rate of cichlid fishes must be greater than the harvesting effort imparted. Condition $r_3 > r_2$ gives $q_3E_3 > q_2E_2$. That is, the harvesting rate of Nile perch should be greater than the harvesting rate of tilapia fishes. For economic purpose the condition $r_3 > w$ needs to be satisfied which implies that the rate at which the Nile perch is harvested must be greater than its death rate.

2.3. Local Stability of Equilibrium Points. To analyse the local stability of the equilibrium point we consider the Jacobian matrix;

$$J(P_i) = \begin{pmatrix} A^{**} & \sigma_1x & \dfrac{-P_1x}{1+Q_1x} \\ -\sigma_1y & B^{**} & \dfrac{-P_2y}{1+Q_2y} \\ \dfrac{e_1z}{(1+Q_1x)^2} & \dfrac{e_2z}{(1+Q_2y)^2} & C^{**} \end{pmatrix} \tag{23}$$

where

$$\begin{aligned} A^{**} &= \lambda_1 - 2\lambda_1x - \sigma_1y - \frac{P_1z}{(1+Q_1x)^2} - r_1, \\ B^{**} &= \lambda_2 - 2\lambda_2y - \sigma_2x - \frac{P_2z}{(1+Q_2y)^2} - r_2 \\ C^{**} &= -w + \frac{e_1x}{1+Q_1x} + \frac{e_2y}{1+Q_2y} - r_3 \end{aligned} \tag{24}$$

(i) $P_0(0, 0, 0)$, the Jacobian matrix evaluated at $J(P_0)$ gives the eigenvalues, $\lambda_1 - r_1$, $\lambda_2 - r_2$, and $-(w + r_3)$. We see that $(\lambda_2 - r_2) > 0$ and $(\lambda_1 - r_1) > 0$ are always positive and so $P_0(0, 0, 0)$ is unstable.

(ii) $P_1(x^*, 0, 0) = ((\lambda_1 - r_1)/\lambda_1, 0, 0)$.

The Jacobian matrix (23) is evaluated at P_1 with the following eigenvalues:

$$\begin{aligned} L_1 &= -\lambda_1 - r_1, \\ L_2 &= \lambda_2 - \frac{\sigma_2(\lambda_1-r_1)}{\lambda_1} - \frac{P_2(\lambda_1-r_1)}{\lambda_1} - r_2, \\ L_3 &= -w - \frac{P_1(\lambda_1-r_1)}{\lambda_1+Q_1(\lambda_1-r_1)} - r_3. \end{aligned} \tag{25}$$

The eigenvalues are negative if $\lambda_1 > r_1$ and $\lambda_2 > (\sigma_2 - P_2)(\lambda_1 - r_1)/\lambda_1$.

Hence, the equilibrium point $P_1(x, 0, 0)$ is locally asymptotically stable if the following conditions hold: $\lambda_1 > r_1$, $Q_2 > P_2$, and $\lambda_2 > (Q_2 - P_2)(\lambda_1 - r_1)/\lambda_1$. The condition $\lambda_1 > r_1$ implies $E_1 < \lambda_1$. For the local stability of $P_1(x, 0, 0)$, E_1, the harvesting rate of cichlid fishes must be less than their intrinsic growth rate. Other inequalities show parameters that are vital for the local stability of $P_1(x, 0, 0)$.

(iii) $P_2(0, y^*, 0) = (0, (\lambda_2 - r_2)/\lambda_2, 0)$.

The Jacobian matrix (23) is evaluated at P_2 and the following eigenvalues obtained:

$$L_1 = \lambda_1 - \frac{\sigma_1(\lambda_2 - r_2}{\lambda_2} - r_1,$$
$$L_2 = r_2 - \lambda_2, \quad (26)$$
$$L_3 = -w - \frac{P_2(\lambda_2 - r_2)}{\lambda_2 + Q_2(\lambda_2 - r_2)} - r_3.$$

The eigenvalues above are negative if $\lambda_2 > r_2$. Hence, the equilibrium point $P_1(x, 0, 0)$ is locally asymptotically stable if conditions $\lambda_2 > r_2$ hold. The condition $\lambda_2 > r_2$ implies $E_2 < \lambda_2$. For the local stability of $P_1(x, 0, 0)$, E_2, the harvesting rate of tilapia fishes must be less than their intrinsic growth rate.

(iv) $P_3(x^*, y^*, 0) = ((\lambda_2(\lambda_1 - r_1) - \sigma_1(\lambda_2 - r_2))/(\lambda_1\lambda_2 - \sigma_1\sigma_2), (\lambda_1(\lambda_2 - r_2) + \sigma_2(r_1 - \lambda_1))/(\lambda_1\lambda_2 - \sigma_1\sigma_2), 0)$.

The eigenvalues of $J(P_3)$ are obtained by solving the characteristic equation;

$$\lambda^3 - (A^* + E^* + G^*)\lambda^2 + (A^*G^* + E^*G^* + A^*E^* - D^*B^*)\lambda + G^*D^*B^* - G^*A^*E^* = 0 \quad (27)$$

where $A^* = \lambda_1 - 2\lambda_1 x^* - \sigma_1 y^* - r_1$, $B^* = -\sigma_1 y^*$, $C^* = -P_1 x^*/(1 + Q_1 x^*)$, $D^* = -\sigma_1 y^*$, $E^* = \lambda_2 - 2\lambda_2 y^* - \sigma_2 x^* - r_2$, $F^* = -P_2 y^*/(1 + Q_2 y^*)$, and $G^* = -w + e_1 x^*/(1 + Q_1 x^*) + e_2 y^*/(1 + Q_2 y^*) - r_3$.

This can be expressed in form of

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0 \quad (28)$$

By Routh-Hurwitz criteria (Murray, 1989), the $\lambda's$ are negative if $a_1 > 0, a_3 > 0, a_1a_2 - a_3 > 0$.

(v) $P_4(x^*, 0, z^*) = P_4((w + r_3)/(e_1 - wQ_1 - r_3Q_1), 0, -\lambda_1Q_1(w+r_3)^2/P_1(e_1 - wQ_1 - r_3Q_1)^2 + (\lambda_1Q_1 - r_1Q_1 - \lambda_1)(w + r_3)/P_1(e_1 - wQ_1 - r_3Q_1) + (\lambda_1 - r_1)/P_1)$.

The eigenvalues $J(P_4)$ are obtained by solving the characteristics equation;

$$L^3 - (A_2^* + B_2^* + C_2^*)L^2 + \left[A_2^*(B_2^* + C_2^*) + B_2^*C_2^* + \frac{P_1e_1x^*z^*}{(1 + Q_1x^*)^3}\right]L - A_2^*B_2^*C_2^* - \frac{B_2^*P_1e_1x^*z^*}{(1 + Q_1x^*)^*} = 0 \quad (29)$$

where

$$A_2^* = \lambda_1 - r_1 - 2\lambda_1x^* - \frac{P_1z^*}{(1 + Q_1x^*)^2},$$
$$B_2^* = \lambda_2 - \sigma_2x^* - P_2z^* - r_2, \quad (30)$$
$$C_2^* = -w + \frac{e_1x^*}{1 + Q_1x^*} - r_3.$$

The characteristic equation is in the form

$$L^3 + a_1L^2 + a_2L + a_3 = 0 \quad (31)$$

where $a_1 = -(A_2^* + B_2^* + C_2^*)$, $a_2 = A_2^*(B_2^* + C_2^*) + B_2^*C_2^* + P_1e_1x^*z^*/(1 + Q_1x^*)^3$, and $a_3 = -A_2^*B_2^*C_2^* - B_2^*P_1e_1x^*z^*/(1 + Q_1x^*)^*$. By Rouths stability criterion, the equilibrium point P_4 is stable if (i) $a_1 > 0, a_2 > 0$, and $a_3 > 0$; (ii) $a_1a_2 > a_3$. Otherwise it is unstable.

(vi) $P_5(0, y^*, z^*) = P_5((w + r_3)/(e_2 - wQ_2 - r_3Q_2), 0, -\lambda_2Q_2(w+r_3)^2/P_2(e_2 - wQ_2 - r_3Q_2)^2 + (\lambda_2Q_2 - r_2Q_2 - \lambda_2)(w + r_3)/P_2(e_2 - wQ_2 - r_3Q_2) + (\lambda_2 - r_2)/P_2)$.

The eigenvalues evaluated at $J(P_5)$ are obtained by solving the characteristic equation

$$L^3 - (A_3^* + B_3^* + C_3^*)L^2 + \left[B_3^*C_3^* + A_3^*(B_3^* + C_3^*) + \frac{P_2e_2z^*y^*}{(1 + Q_2y^*)^3}\right]L - A_3^*B_3^*C_3^* - \frac{P_2e_2z^*y^*}{(1 + Q_2y^*)^3} = 0 \quad (32)$$

where $A_3^* = \lambda_1 - \sigma_1y^* - P_1z^* - r_1$, $B_3^* = \lambda_2 - r_2 - 2\lambda_2y^* - P_2z^*/(1 + Q_2x^*)^2$, and $C_3^* = -w + e_2y^*/(1 + Q_2y^*) - r_3$. The characteristic equation is in the form $L^3 + a_1L^2 + a_2L + a_3 = 0$;

where $a_1 = -(A_3^* + B_3^* + C_3^*)$, $a_2 = B_3^*C_3^* + A_3^*(B_3^* + C_3^*) + P_2e_2z^*y^*/(1 + Q_2y^*)^3$, and $a_3 = -A_3^*B_3^*C_3^* - P_2e_2z^*y^*/(1 + Q_2y^*)^3$. By Rouths stability criterion, the equilibrium point P_5 is stable if (i) $a_1 > 0, a_2 > 0$, and $a_3 > 0$ and (ii) $a_1a_2 > a_3$. Otherwise it is unstable.

2.4. Global Stability of the Coexistence Equilibrium Point $P_6(x^*, y^*, z^*)$. To analyse global stability of coexistence equilibrium point, a suitable Lyapunov function is chosen, from which conditions for the global asymptotic stability of the coexistence point $P_6(x^*, y^*, z^*)$ are derived. The approach is based on work by Chaudhuri [13] and Dubey [14].

Theorem 2. *The coexistence equilibrium point* $P_6(x^*, y^*, z^*)$ *is globally asymptotically stable if*

(i) $\lambda_1 > 0, \lambda_2 > 0$, *(ii)* $P_1 > e_1, P_2 > e_2$, *and (ii)* $\lambda_1\lambda_2 > (\sigma_1 + \sigma_2)^2$.

Proof. Consider the following Lyapunov function:

$$V(x, y, z) = (x - x^*) - x^* \log\left(\frac{x}{x^*}\right) + (y - y^*) - y^* \log\left(\frac{y}{y^*}\right) + (z - z^*) - z^* \log\left(\frac{z}{z^*}\right) \quad (33)$$

Simplifying the above Lyapunov function,

$$\begin{aligned} V(x,y,z) &= x - x^* - (x^* \log x - x^* \log x^*) + y \\ &\quad - y^* - (y^* \log y - y^* \log y^*) + z \\ &\quad - z^* - (z^* \log z - z^* \log z^*) \\ V(x,y,z) &= x - x^* - x^* \log x + x^* \log x^* + y - y^* \\ &\quad - y^* \log y + y^* \log y^* + z - z^* \\ &\quad - z^* \log z + z^* \log z^* \end{aligned} \tag{34}$$

where $\partial V/\partial x = 1 - x^*/x = (x - x^*)/x$, $\partial V/\partial y = 1 - y^*/y = (y - y^*)/y$,and $\partial V/\partial z = 1 - z^*/z = (z - z^*)/z$.

Differentiating V with respect to time, i.e.,

$$\frac{dV}{dt} = \frac{\partial V}{\partial x}\frac{dx}{dt} + \frac{\partial V}{\partial y}\frac{dy}{dt} + \frac{\partial V}{\partial z}\frac{dz}{dt}$$

$$\begin{aligned} \frac{dV}{dt} &= \frac{x - x^*}{x}\left[\lambda_1 x(1-x) - \sigma_1 xy - \frac{P_1 xz}{1+Q_1 x} - r_1 x\right] + \frac{y - y^*}{y}\left[\lambda_2 y(1-x) - \sigma_2 xy - \frac{P_2 xz}{1+Q_1 x} - r_2 y\right] + \frac{z - z^*}{z}\left[-wz + \frac{e_1 xz}{1+Q_1 x} + \frac{e_2 yz}{1+Q_2 y} - r_3 z\right] \\ \frac{dV}{dt} &= (x - x^*)\left[\lambda_1(1-x) - \sigma_1 y - \frac{P_1 z}{1+Q_1 x} - r_1\right] + (y - y^*)\left[\lambda_2(1-x) - \sigma_2 x - \frac{P_2 z}{1+Q_1 x} - r_2\right] + (z - z^*)\left[-w + \frac{e_1 x}{1+Q_1 x} + \frac{e_2 y}{1+Q_2 y} - r_3\right] \\ \frac{dV}{dt} &= (x - x^*)\left[\lambda_1 - \lambda_1 x - \sigma_1 y - \frac{P_1 z}{1+Q_1 x} - r_1 - \lambda_1 - \lambda_1 x^* + \sigma_1 y^* + \frac{P_1 z^*}{1+Q_1 x^*} + r_1\right] + (y - y^*)\left[\lambda_2 - \lambda_2 y - \sigma_2 x - \frac{P_2 z}{1+Q_2 y} - r_2 - \lambda_2 + \lambda_2 y^* + \sigma_2 x^* + \frac{P_2 z^*}{1+Q_2 y^*} + r_2\right] + (z - z^*)\left[-w + \frac{e_1 x}{1+Q_1 x} + \frac{e_2 y}{1+Q_2 y} - r_3 + w - \frac{e_1 x^*}{1+Q_1 x^*} - \frac{e_2 y^*}{1+Q_2 y^*} + r_3\right] \end{aligned}$$

$$\begin{aligned} \frac{dV}{dt} &= (x - x^*)\left[\lambda_1(x^* - x) + \sigma_1(Z^* - z)\right] + (y - y^*)\left[\lambda_2(y^2 - y) + \sigma_2(x^* - x) + \frac{P_2(z^* - z)}{1+Q_2 y}\right] + (z^* - z)\left[\frac{e_1(x - x^*}{1+Q_1 x} + \frac{e_2(y - y^*)}{1+Q_2 y}\right] \end{aligned} \tag{35}$$

which simplifies to

$$\begin{aligned} \frac{dV}{dt} &= -(x - x^*)^2 \lambda_1 - (x - x^*)(y - y^*)(\sigma_1 - \sigma_2) - (x - x^*)(z - z^*)\left(\frac{P_1 - e_1}{1+Q_1 x}\right) - (y - y^*)^2 \lambda_2 - (y - y^*)(z - z^*)\left(\frac{P_2 - e_2}{1+Q_2 y}\right) \end{aligned} \tag{36}$$

Thus, dV/dt is a quadratic form which can be expressed as $dV/dt = -X^T AX$, where $X^T = (x - x^*, y - y^*, z - z^*)$ and A is symmetric matrix given by

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{pmatrix} \tag{37}$$

with $a_{11} = \lambda_1, a_{12} = \sigma_1 - \sigma_2, a_{13} = (P_1 - e_1)/(1+Q_1 x), a_{22} = \lambda_2$, $a_{23} = (P_2 - e_2)/(1+Q_2 y)$, and $a_{33} = 0$. We note that the point $P_6(x^*, y^*, z^*)$ is globally asymptotically stable if $dV/dt < 0$; that is, the matrix A is positive definite Chaudhuri [13]. Now the matrix A is positive if $a_{11} > 0$, $a_{13} = 0$, $a_{12} > 0$, $a_{22} > 0$, $a_{23} = 0$, and $a_{11}a_{22} - a_{12}^2 > 0$. $a_{11} > 0$ gives $\lambda_1 > 0$, $a_{13} = 0$ gives $P_1 = e_1$, $a_{12} > 0$ gives $\sigma_1 > \sigma_2$, $a_{22} > 0$ gives $\lambda_2 > 0$, and $a_{11}a_{22} - a_{12}^2 > 0$ gives $\lambda_1\lambda_2 > (\sigma_1 + \sigma_2)^2$. This completes the proof. □

2.5. Bioeconomic Equilibrium. The term bionomic equilibrium is an amalgamation of the concepts of biological equilibrium as well as economic equilibrium Kar [8]. From system (2), a biological equilibrium is given by $dx/dt = dy/dt = dz/dt = 0$. The economic equilibrium is said to be achieved when the total revenue obtained by selling the harvested biomass (TR) equals the total cost for the effort devoted to harvesting (TC).

Let c_1 be the fishing cost per unit effort for cichlid fishes, c_2 the fishing cost per unit effort for tilapia fishes, c_3 the fishing cost per unit effort for Nile perch, p_1 the price per unit biomass of cichlid fishes, p_2 the price per unit biomass of tilapia fishes, and p_3 the price per unit biomass of Nile perch; q_1, q_2, and q_3 are catchability coefficients of cichlid fishes, tilapia fishes, and Nile perch, respectively.

Then we have $\pi_1 = (p_1 q_1 x - c_1)E_1$, $\pi_2 = (p_2 q_2 y - c_2)E_2$, and $\pi_3 = (p_3 q_3 x - c_3)E_3$.

π_1, π_2, and π_3 are the economic rent (net revenue) of cichlid fishes, tilapia fishes, and Nile perch, respectively.

Therefore, the economic rent (net revenue) at any time is given by $\pi = \pi_1 + \pi_2 + \pi_3$, which is

$$\pi = (p_1 q_1 x - c_1) E_1 + (p_2 q_2 y - c_2) E_2 + (p_3 q_3 z - c_3) E_3 \tag{38}$$

Then $P(x_\infty, y_\infty, z_\infty, E_{1\infty}, E_{2\infty}, E_{3\infty})$ is the bioeconomic equilibrium where x_∞, y_∞, z_∞, $E_{1\infty}$, $E_{2\infty}$, and $E_{3\infty}$ are the bioeconomic values of cichlid fishes, tilapia fishes, Nile perch, harvesting effort of cichlid fishes, harvesting effort of tilapia fishes, and harvesting effort of Nile perch, respectively, and it is given by the simultaneous equation:

$$\lambda_1 (1 - x) - \sigma_1 y - \frac{P_1 z}{1 + Q_1 x} - q_1 E_1 = 0 \tag{39}$$

$$\lambda_2 (1 - y) - \sigma_2 x - \frac{P_2 z}{1 + Q_2 y} - q_2 E_2 = 0 \tag{40}$$

$$-w + \frac{e_1 x}{1 + Q_1 x} + \frac{e_2 y}{1 + Q_2 y} - q_3 E_3 = 0 \tag{41}$$

$$\pi = (p_1 q_1 x - c_1) E_1 + (p_2 q_2 y - c_2) E_2 + (p_3 q_3 z - c_3) E_3 = 0 \tag{42}$$

In order to determine the bioeconomic equilibrium, we now consider the following cases.

Case I. If $c_1 > p_1 q_1$, that is, the fishing cost per unit effort for cichlid fishes is greater than the revenue in the cichlid fish fishery, then fishermen will be in loss and naturally they would withdraw their participation from cichlid fish and the fishery will be closed ($E_1 = 0$). Only tilapia fish and Nile perch fishery remain operational (i.e., $c_2 < p_2 q_2 y$ and $c_3 < p_3 q_3 z$). Now we have $y_\infty = c_2/p_2 q_2$ and $z_\infty = c_3/p_3 q_3$. From (39) when $E_1 = 0$, we have

$$\begin{aligned} &\lambda_1 - \lambda_1 x_\infty - \sigma_1 y_\infty - \frac{P_1 z_\infty}{1 + Q_1 x_\infty} = 0 \\ &\lambda_1 (1 + Q_1 x_\infty) - \lambda_1 x_\infty (1 + Q_1 x_\infty) \\ &\quad - \sigma_1 y_\infty (1 + Q_1 x_\infty - P_1 z_\infty = 0 \\ &\lambda_1 + \lambda_1 Q_1 x_\infty) - \lambda_1 x_\infty + \lambda_1 Q_1 x_\infty^2 - \sigma_1 y_\infty \\ &\quad + \sigma_1 Q_1 x_\infty y_\infty - P_1 z_\infty = 0 \end{aligned} \tag{43}$$

$$\begin{aligned} &-\lambda_1 Q_1 x_\infty^2 + (\lambda_1 Q_1 - \lambda_1 - \sigma_1 Q_1 y_\infty) x_\infty + \lambda_1 \\ &\quad - \sigma_1 y_\infty - P_1 z_\infty = 0 \end{aligned} \tag{44}$$

Since $y_\infty = c_2/p_2 q_2$ and $z_\infty = c_3/p_3 q_3$, then (44) can be written in quadratic form as

$$\lambda_1 Q_1 x_\infty^2 - \left(\lambda_1 Q_1 - \lambda_1 - \frac{\sigma_1 Q_1 c_2}{p_2 q_2}\right) x_\infty + \lambda_1 - \frac{\sigma_1 c_2}{p_2 q_2} - \frac{P_1 c_3}{p_3 q_3} = 0 \tag{45}$$

Therefore

$$x_{\infty 1,2} = \frac{-R_2 \pm \sqrt{R_2^2 - 4 R_1 R_2}}{2 R_3} \tag{46}$$

where $R_1 = \lambda_1 Q_1$, $R_2 = (\lambda_1 Q_1 p_2 q_2 - \lambda_1 p_2 q_2 - \sigma_1 Q_1 c_2)/p_2 q_2$, $R_3 = \lambda_1 - \sigma_1 c_2/p_2 q_2 - P_1 c_3/p_3 q_3$.

From (46),

$$E_{1\infty} = \frac{\lambda_2}{q_2} (1 - y_\infty) - \frac{\sigma_2}{q_2} x_\infty - \frac{P_2 z_\infty}{q_2 (1 + Q_2 y_\infty)} \tag{47}$$

Therefore, $E_{1\infty} > 0$ if

$$\frac{\lambda_2 q_2 p_2 - \lambda_2 c_2 - \sigma_2 p_2 q_2 x^\infty}{p_2 q_2^2} > \frac{P_2 p_2 q_2 c_3}{p_3 q_3 p_2 q_2 - Q_2 p_3 q_3 c_2} \tag{48}$$

Also,

$$\begin{aligned} E_{2\infty} &= \frac{-wz}{q_3} + \frac{e_1 xz}{q_3 + Q_1 q_3 x} + \frac{e_2 yz}{q_3 + Q_2 q_3 y} \\ &= -\frac{w c_3}{p_3 q_3^2} + \frac{e_1 c_3 x^\infty}{p_3 q_3^2 + Q_1 p_3 q_3^2 x^\infty} \\ &\quad + \frac{e_2 c_2 c_3 p_2 q_2}{p_2^2 q_2^2 p_3 q_3^2 + Q_2 p_2 q_2 p_3 q_3^2 c_2} \end{aligned} \tag{49}$$

Thus, $E_{2\infty} > 0$ if

$$\frac{e_1 c_3 x^\infty \left(p_2^2 q_2^2 p_3 q_3^2 + Q_2 p_2 q_2 p_3 q_3^2 c_2\right) + e_2 c_2 c_3 p_2 q_2 \left(p_3 q_3^2 + Q_1 p_3 q_3^2 x^\infty\right)}{\left(p_3 q_3^2 - Q_1 p_3 q_3^2 x^\infty\right)\left(p_2^2 q_2^2 q_3^2 p_3 + Q_2 p_2 q_2 p_3 q_3^2 c_2\right)} > \frac{w c_3}{p_3 q_3^2} \tag{50}$$

Therefore, the bioeconomic equilibrium exists if conditions (48) and (50) hold.

Case II. If $c_2 > p_2 q_2$, that is, the fishing cost per unit effort for tilapia fishes is greater than the revenue in the tilapia

fish fishery, then fishermen will be in loss and naturally, they would withdraw their participation from tilapia fish fishery and the fishery will be closed ($E_2 = 0$). Only cichlid fishes and Nile perch fishery remain operational (i.e., $c_1 < p_1q_1x$ and $c_3 < p_3q_3z$). Now we have $x_\infty = c_1/p_1q_1$ and $z_\infty = c_3/p_3q_3$. From (46) when $E_2 = 0$, we have

$$\lambda_2 - \lambda_1 y_\infty - \sigma_2 x_\infty - \frac{P_2 z_\infty}{1 + Q_1 y_\infty} = 0$$

$$\begin{aligned}&\lambda_2 (1 + Q_2 y_\infty) - \lambda_2 y_\infty (1 + Q_2 y_\infty)\\ &\quad - \sigma_2 x_\infty (1 + Q_2 y_\infty - P_2 z_\infty = 0\end{aligned} \tag{51}$$

$$\begin{aligned}&\lambda_2 + \lambda_2 Q_2 y_\infty) - \lambda_2 y_\infty + \lambda_2 Q_2 y_\infty^2 - \sigma_2 x_\infty\\ &\quad + \sigma_2 Q_2 x_\infty y_\infty - P_2 z_\infty = 0\\ &- \lambda_2 Q_2 y_\infty^2 + (\lambda_2 Q_2 - \lambda_2 - \sigma_2 Q_2 x_\infty) y_\infty + \lambda_2\\ &\quad - \sigma_2 x_\infty - P_2 z_\infty = 0\end{aligned} \tag{52}$$

Since $x_\infty = c_1/p_1q_1$ and $z_\infty = c_3/p_3q_3$, then (52) can be written in quadratic form as

$$\begin{aligned}&\lambda_2 Q_2 y_\infty^2 - \left(\lambda_2 Q_2 - \lambda_2 - \frac{\sigma_2 Q_2 c_1}{p_1 q_1}\right) y_\infty + \lambda_1 - \frac{\sigma_2 c_1}{p_1 q_1}\\ &\quad - \frac{P_2 c_3}{p_3 q_3} = 0\end{aligned} \tag{53}$$

Therefore

$$y_\infty = \frac{-S_2 \pm \sqrt{S_2^2 - 4S_1S_2}}{2S_3} \tag{54}$$

where

$$\begin{aligned}S_1 &= \lambda_2 Q_2\\ S_2 &= \frac{\lambda_2 Q_2 p_1 q_1 - \lambda_2 p_1 q_1 - \sigma_2 Q_2 c_1}{p_1 q_1}\\ S_3 &= \lambda_2 - \frac{\sigma_2 c_1}{p_1 q_1} - \frac{P_2 c_3}{p_3 q_3}\end{aligned} \tag{55}$$

From (39),

$$\lambda_1 (1 - x_\infty) - \sigma_1 y_\infty - \frac{P_1 z_\infty}{1 + Q_1 x_\infty} - q_1 E_{1\infty} = 0$$

$$E_{1\infty} = \frac{\lambda_1}{q_1} - \frac{\lambda_1 c_1}{p_1 q_1^2} - \frac{\sigma_1 y^\infty}{q_1} - \frac{P_1 p_1 q_1 c_3}{p_3 q_3 p_1 q_1 - Q_1 p_3 q_3 c_1} \tag{56}$$

$E_{1\infty} > 0$ if

$$\frac{\lambda_1 q_1 p_1 - \lambda_1 c_1 - \sigma_1 p_1 q_1 y^\infty}{p_1 q_1^2} > \frac{P_1 p_1 q_1 c_3}{p_3 q_3 p_1 q_1 - Q_1 p_3 q_3 c_1} \tag{57}$$

Also,

$$\begin{aligned}E_{3\infty} &= \frac{-wz}{q_3} + \frac{e_1 xz}{q_3 + Q_1 q_3 x} + \frac{e_2 yz}{q_3 + Q_2 q_3 y}\\ &= -\frac{wc_3}{p_3 q_3^2} + \frac{e_1 c_3 y^\infty}{p_3 q_3^2 + Q_2 p_3 q_3^2 y^\infty}\\ &\quad + \frac{e_2 c_1 c_3 p_1 q_1}{p_1^2 q_1^2 p_3 q_3^2 + Q_1 p_1 q_1 p_3 q_3^2 c_1}\end{aligned} \tag{58}$$

Thus, $E_{2\infty} > 0$ if

$$\frac{e_2 c_3 y^\infty \left(p_1^2 q_1^2 p_3 q_3^2 + Q_1 p_1 q_1 p_3 q_3^2 c_1\right) + e_2 c_1 c_3 p_1 q_1 \left(p_3 q_3^2 + Q_2 p_3 q_3^2 y^\infty\right)}{\left(p_3 q_3^2 - Q_2 p_3 q_3^2 y^\infty\right)\left(p_1^2 q_1^2 q_3^2 p_3 + Q_1 p_1 q_1 p_3 q_3^2 c_1\right)} > \frac{wc_3}{p_3 q_3^2} \tag{59}$$

Therefore, the bioeconomic equilibrium exists if conditions (57) and (59) hold.

Case III. If $c_3 > p_3q_3$, that is, the fishing cost per unit effort for Nile perch is greater than the revenue in the Nile perch fishery, then fishermen will be in loss and naturally, they would withdraw their participation from Nile perch fishery and the fishery will be closed. Only cichlid fish and tilapia fish fishery remain operational (i.e., $c_1 < p_1q_1x$ and $c_2 < p_2q_2z$). Now we have $x_\infty = c_1/p_1q_1$ and $y_\infty = c_2/p_2q_2$. Substituting x_∞ and y_∞ in (44), we get

$$\begin{aligned}E_{1\infty} &= \lambda_1 \left(\frac{1}{q_1} - \frac{c_1}{p_1 q_1^2}\right)\\ &\quad - \frac{\sigma_1 c_2 \left(p_1 q_1^2 - Q_1 q_1 c_1\right) + P1 p_1 q_1^2 p_2 q_2 z^\infty}{p_2 q_2 q_1 \left(p_1 q_1^2 - Q_1 q_1 c_1\right.}\end{aligned} \tag{60}$$

Thus, $E_{1\infty} > 0$ if

$$\begin{aligned}&\lambda_1 \left(\frac{1}{q_1} - \frac{c_1}{p_1 q_1^2}\right)\\ &\quad > \frac{\sigma_1 c_2 \left(p_1 q_1^2 - Q_1 q_1 c_1\right) + P1 p_1 q_1^2 p_2 q_2 z^\infty}{p_2 q_2 q_1 \left(p_1 q_1^2 - Q_1 q_1 c_1\right.}\end{aligned} \tag{61}$$

Also,

$$\begin{aligned}E_{2\infty} &= \lambda_1 \left(\frac{1}{q_1} - \frac{c_2}{p_2 q_2^2}\right)\\ &\quad - \frac{\sigma_2 c_1 \left(p_2 q_2^2 - Q_2 q_2 c_2\right) + P2 p_2 q_2^2 p_1 q_1 z^\infty}{p_1 q_1 q_2 \left(p_2 q_2^2 - Q_2 q_2 c_2\right.}\end{aligned} \tag{62}$$

Thus, $E_{2\infty} > 0$ if

$$\lambda_2\left(\frac{1}{q_2}-\frac{c_2}{p_2q_2^2}\right) > \frac{\sigma_2c_1\left(p_2q_2^2-Q_2q_2c_2\right)+P2p_2q_2^2p_1q_1z^{\infty}}{p_1q_1q_2\left(p_2q_2^2-Q_2q_2c_2\right.} \tag{63}$$

Therefore, the bioeconomic equilibrium exists if conditions (61) and (63) hold.

Case IV. If $c_1 > p_1q_1x, c_2 > p_2q_2y$ and $c_3 > p_3q_3z$, then the fishing cost is greater than revenues for all three species and the whole fishery will be closed.

Case V. If $c_1 < p_1q_1x, c_2 < p_2q_2y$ and $c_3 < p_3q_3z$, then the fishing cost is less than revenues for all three species; that is, the fishery is more profitable and hence it would attract more fishermen and the whole fishery will be in operation. Now we have $x_\infty = c_1/p_1q_1$, $y_\infty = c_2/p_2q_2$, and $z_\infty = c_3/p_3q_3$, then

$$E_{1\infty} = \lambda_1\left(\frac{1}{q_1}-\frac{c_1}{p_1q_1^2}\right) - \frac{\sigma_1c_2\left(p_1q_1^2p_3q_3-Q_1p_3q_3q_1c_1\right)+P_1p_1q_1c_3}{p_2q_2q_1\left(p_3q_3P_1q_1^2-Q_1p_3q_3q_1c_1\right)} \tag{64}$$

Thus, $E_{1\infty} > 0$, if

$$\lambda_1\left(\frac{1}{q_1}-\frac{c_1}{p_1q_1^2}\right) > \frac{\sigma_1c_2\left(p_1q_1^2p_3q_3-Q_1p_3q_3q_1c_1\right)+P_1p_1q_1c_3}{p_2q_2q_1\left(P_3q_3P_1q_1^2-Q_1p_3q_3q_1c_1\right)} \tag{65}$$

And,

$$E_{2\infty} = \lambda_2\left(\frac{1}{q_2}-\frac{c_2}{p_2q_2^2}\right) - \frac{\sigma_2c_1\left(p_2q_2^2p_3q_3-Q_2p_3q_3q_2c_2\right)+P_2p_2q_2c_3}{p_1q_1q_2\left(p_3q_3p_2q_2^2-Q_2p_3q_3q_2c_2\right)} \tag{66}$$

Thus, $E_{2\infty} > 0$,

$$\lambda_2\left(\frac{1}{q_2}-\frac{c_2}{p_2q_2^2}\right) > \frac{\sigma_2c_1\left(p_2q_2^2p_3q_3-Q_2p_3q_3q_2c_2\right)+P_2p_2q_2c_3}{p_1q_1q_2\left(P_3q_3p_2q_2^2-Q_2p_3q_3q_2c_2\right)} \tag{67}$$

Also,

$$E_{3\infty} = -\frac{wc_3}{p_3q_3}+\frac{e_1c_1c_3p_1q_1}{p_1^2q_1^2p_3q_3+p_3q_3p_1q_1c_1} + \frac{e_2c_2c_3p_2q_2}{p_3q_3p_2^2q_2^2-p_3q_3p_2q_2c_2} \tag{68}$$

Thus, $E_{3\infty} > 0$ if

$$\frac{e_1c_1c_3p_1q_1\left(p_3q_3p_2^2q_2^2-p_3q_3p_2q_2c_2\right)+e_2c_2c_3p_2q_2\left(p_1^2q_1^2p_3q_3+p_3q_3p_1q_1c_1\right)}{\left(p_1^2q_1^2p_3q_3+p_3q_3p_1q_1c_1\right)\left(p_3q_3p_2^2q_2^2-p_3q_3p_2q_2c_2\right)} > 0 \tag{69}$$

Thus, the nontrivial bioeconomic equilibrium point $P_\infty(x_\infty, y_\infty, z_\infty, E_{1\infty}, E_{2\infty}, E_{3\infty})$ exists if and only if conditions (65), (67), and (69) hold together.

3. Results and Discussion

3.1. Numerical Results. The model system is simulated using the inbuilt ODE solvers coded in Matlab programming language and figures are plotted using parameter values presented in Table 1.

Figure 1 presents the dynamics of the population interaction with respect to time. The figure indicates that cichlid fishes population grows faster compared to other species. The dynamics was also visualized in $3D$ as presented in Figure 2. Figure 3 shows the impact in harvesting cichlid fish population; it is observed that increasing harvesting effort leads to decrease Nile perch population as well. In Figure 4 the same scenario happens when tilapia fish population harvested tends also to decrease Nile perch population while increasing harvesting effort in Nile perch population tends to increase cichlid and tilapia fishes as presented in Figure 5. Figure 6 presents the exponential growth rate of cichlid and tilapia fishes in absence of Nile perch population.

3.2. Discussion. A mathematical model was proposed and analysed to study the dynamics of a two-prey-one predator system with harvesting aspects. The model was used to study the ecological dynamics of the Nile perch-cichlid-tilapia fishes prey-predator system of the lake Victoria fishery. The harvesting rate was found to play a crucial role in stabilizing the system. Figures 3 and 4 show that cichlid and tilapia fishes tend to extinction when the harvesting rates E_1 and E_2 exceed their intrinsic growth rates λ_1 and λ_2. For sustainability harvesting of cichlid and tilapia fishes their intrinsic growth rates should be kept smaller. However, harvesting of the three species at a rate much lower than their intrinsic growth rate

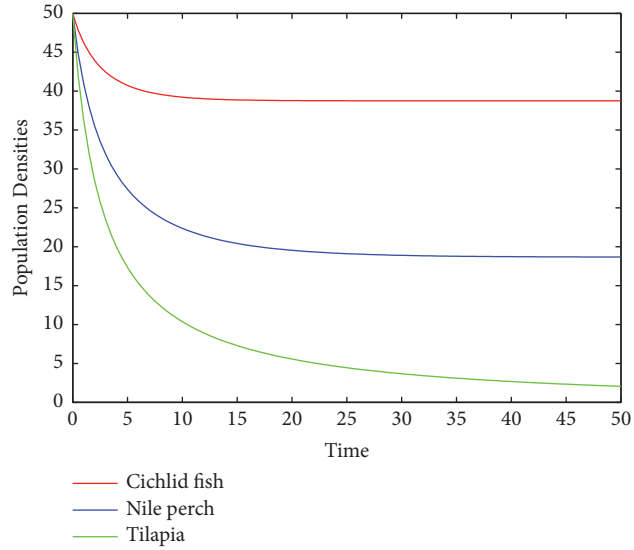

Figure 1: Graph of prey x_1 and x_2 and predator x_3 against time (in years). It shows the variation of population density with time.

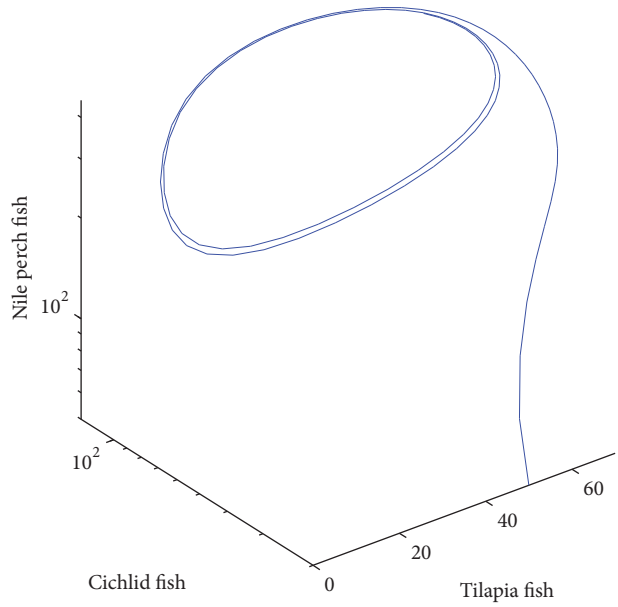

Figure 2: Graph of prey x_1 and x_2 and predator x_3 against time (in years). The limit cycle shows that population density will slightly change but not oscillate the boundary.

would not lead to collapse of the system. This result is similar to one obtained by Chaudhuri and Kar [13] on the existence conditions for the system they studied. In absence of the predator, the two prey species can coexist and are stable if the interspecific competition among them is maintained at minimum level or negligible also if both prey species are not harvested beyond its intrinsic growth rate. Numerical analysis results indeed confirmed this as shown in Figure 6.

Theorem 1 showed that the three species would coexist if cichlid and tilapia fishes were not harvested beyond their intrinsic growth rates; the Nile perch converted the biomass of cichlid and tilapia fishes into fertility at a rate greater than the Nile perch's natural mortality rate and the time it took to handle the cichlid or tilapia fish. The findings of Dubey and Upadhyay [14] indicated that the predator's mortality rate and food conversion coefficients played a crucial role in determining the stability behaviour of the equilibrium points.

In order for the bioeconomic equilibrium to exist, the fishing cost per unity effort for all species, price per unity biomass, catchability coefficient, harvesting effort, intrinsic growth rate of prey species, and the mortality rate of predator play a vital role. Hence for the bioeconomic equilibrium to exist, the fishing cost per unity effort for all three species should be less than the revenue in their fishery. Similar results were also obtained by Ganguli and Kar [7] when they studied

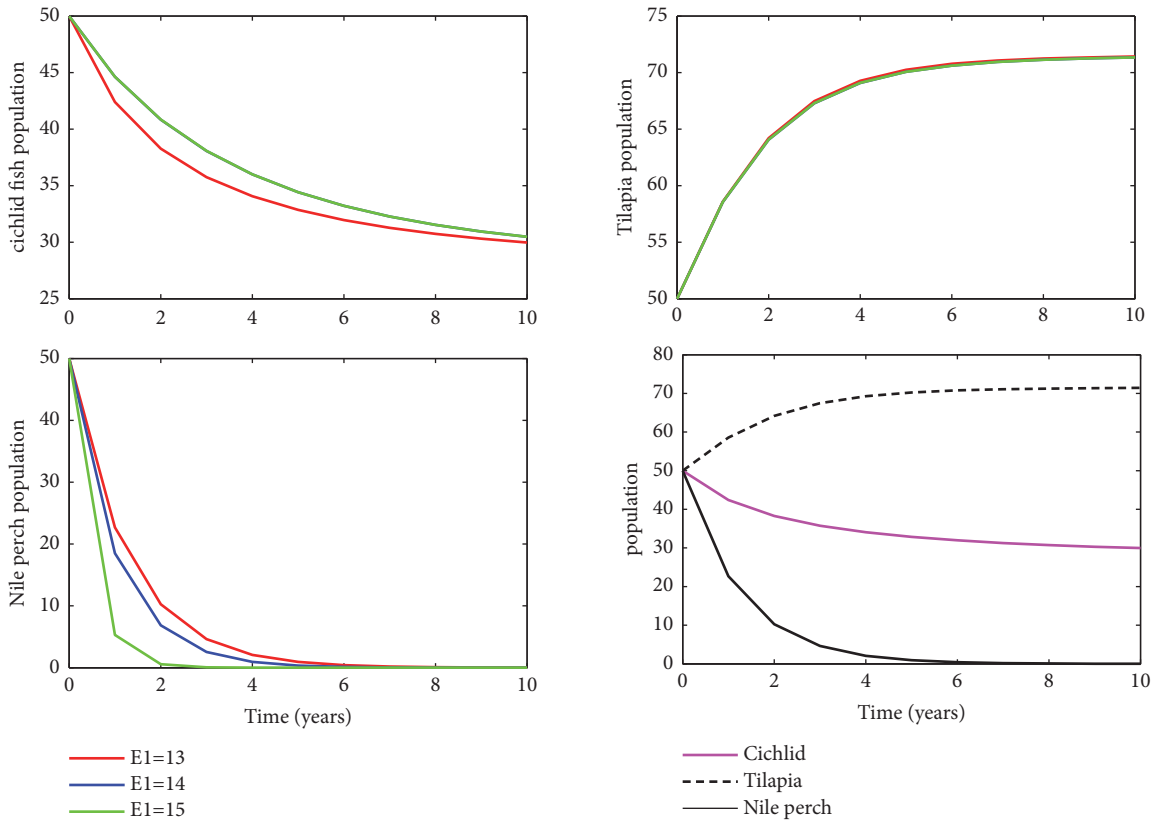

Figure 3: Variation of the fishes population with different harvesting efforts in cichlid $E1$ with fixed values of $E2$ and $E3$.

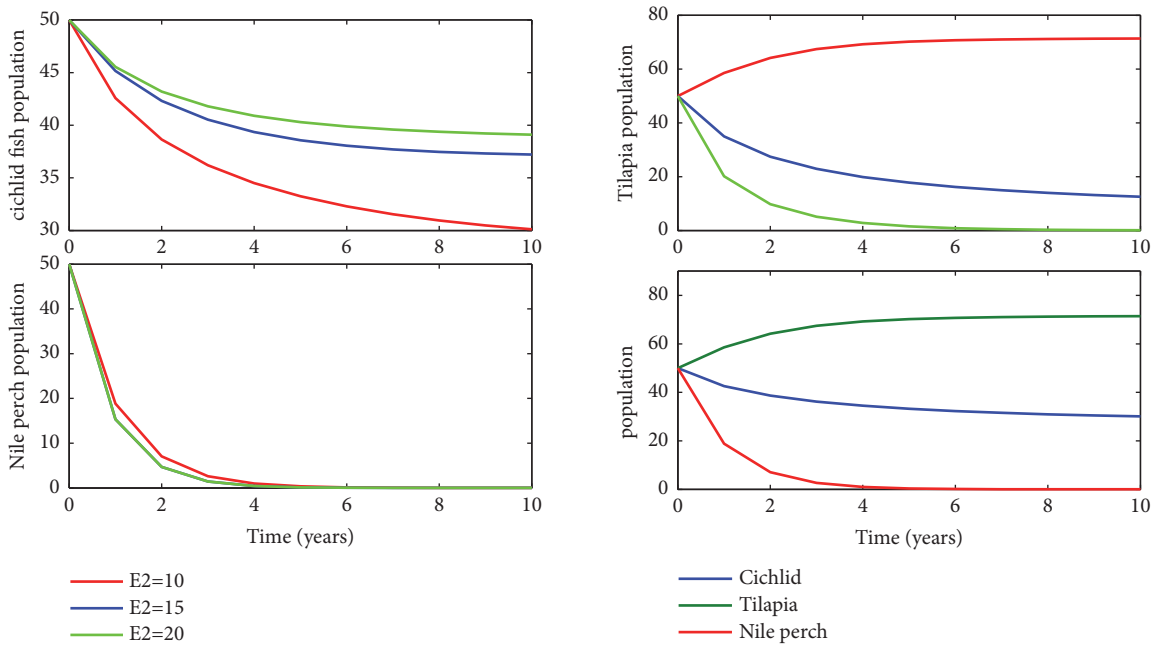

Figure 4: The effect of Tilapia harvest with different values of $E2$ with fixed values of $E1$ and $E3$.

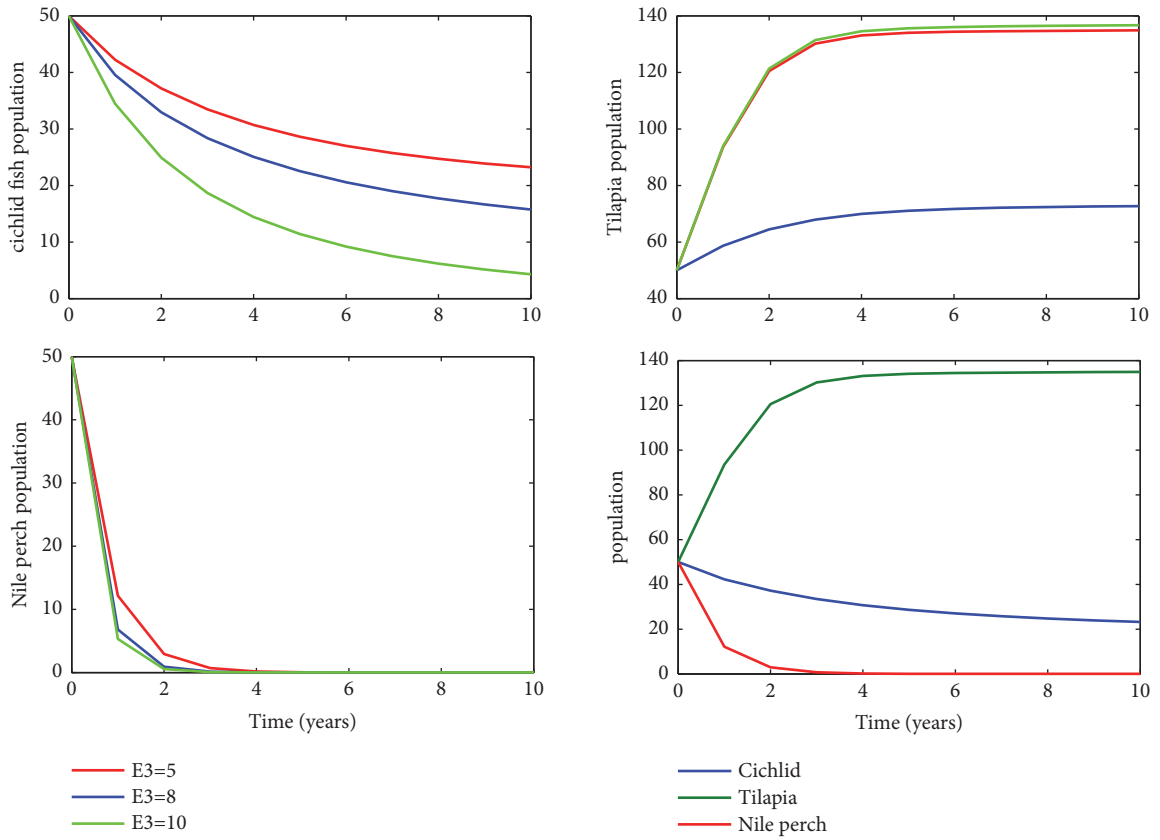

Figure 5: The effect of Nile perch harvest with different values of $E3$ with fixed values of $E1$ and $E2$.

Table 1: Parameter Values of the model.

Parameter	Value	source
λ_1	2.07	[13]
λ_2	2.09	[13]
K_1	200	[13]
K_2	100	[13]
α_{12}	0.001	[15]
α_{13}	0.02	[15]
α_{21}	0.002	[16]
α_{23}	0.03	[16]
α_{31}	1.5	[17]
β	0.1	[17]
γ	0.2	[17]
q_1	0.14	[17]
q_2	0.13	[18]
q_3	0.125	[18]
w	1	[17]

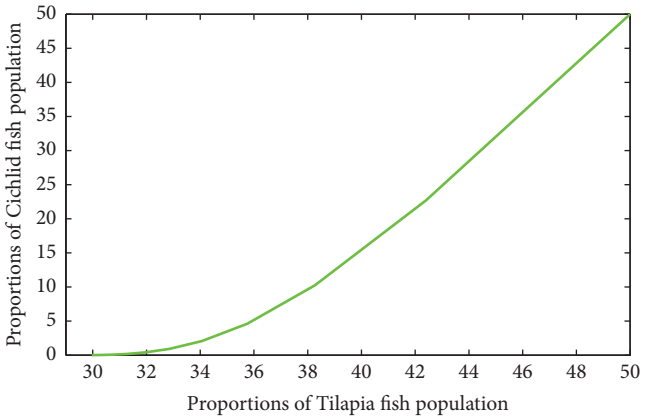

Figure 6: The interaction between cichlid and tilapia fishes in absence of Nile perch.

the optimal harvesting of a prey-predator model with variable carrying capacity.

One of the major observations from numerical simulation results is that the predator population density increased significantly when the harvesting rate of both prey species decreased. This implies that a gradual increase in the number of cichlid and tilapia fishes would result in significant increase in the number of Nile perch and vice versa, which would in the long term lead the population density of the cichlid and tilapia fishes to fall to a level lower than the original one. Kar and Chaudhuri [16] also discovered that predator population density can increase significantly when the harvesting rate of prey species is decreased.

4. Conclusion

We deduce from both analytical and numerical results that if the harvesting rate of the cichlid and tilapia fishes exceeds their intrinsic growth rate, the population of the cichlid, tilapia fishes, and Nile perch would become extinct with time. However, analytical and numerical results also show that harvesting of the three species at a rate much lower than their intrinsic growth rate would not lead to collapse of the system. Thus, in order to use fish as a resource and produce maximum economic benefit while maintaining sustainable fishery species, the harvesting rate of species should never be allowed to exceed their growth rate.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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