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Research Article

Numerical Simulation of an Air Pollution Model on Industrial Areas by Considering the Influence of Multiple Point Sources

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A numerical simulation on a two-dimensional atmospheric diffusion equation of an air pollution measurement model is proposed. The considered area is separated into two parts that are an industrial zone and an urban zone. In this research, the air pollution measurement by releasing the pollutant from multiple point sources above an industrial zone to the other area is simulated. The governing partial differential equation of air pollutant concentration is approximated by using a finite difference technique. The approximate solutions of the air pollutant concentration on both areas are compared. The air pollutant concentration levels influenced by multiple point sources are also analyzed.

1. Introduction

A rapid growth of industrial sector can explain that air pollution affects the health of human being who lives around industrial areas. The air pollution has become a major problem of human life and environment. The purpose of this research was to study the air pollution assessment problem in two adjacent zones: industrial and urban zones by using the atmospheric diffusion model. In [1], the simulation of two-dimensional advection-diffusion model with a point source was presented. The numerical solutions were solved by using the finite difference techniques. In [2], the researchers used the mathematical model to simulate the dispersion of sulfur dioxide concentration with the wind and diffusion parameters regarding the reference atmospheric stability. In [3], the mass transport model consisted of the stream function, vorticity, and convection-diffusion equation. The smoke dispersion which released into the atmosphere from one and two-point sources was considered with obstacle domain. The approximated solutions were solved by using the finite element techniques. In [4], the researchers studied the smoke dispersion model in a two-dimensional space by considering two and three point sources with a two obstacles in the domain. In [5], the two-dimensional advection-diffusion equation with mesoscale wind, eddy diffusivity profiles, and removal mechanisms was introduced. Then, the primary pollutant released into the atmosphere from an area source, which was also considered. In [6], the researchers studied the two-dimensional advection-diffusion equation of primary and secondary pollutants. The area source with removal mechanisms and the point source considering on the boundary were proposed. The solutions of air pollution in [5, 6] were estimated by using the Crank-Nicolson implicit methods. In [7], the air-quality model in the three-dimensional with variations of the atmospheric stability classes and wind velocities from multiple sources was analyzed. The fractional step methods were used in order to predict the air pollutant concentration in [2, 4, 7]. In [8], the atmospheric diffusion model was used to describe the dispersion of air pollutant concentration near an industrial zone. The problem was considered by controlling the air pollution emission under a point source. From the numerical experiments, it was indicated that the air pollution control was necessary for air-quality management. In [9], the researchers studied the dispersion behavior of air pollution in the tunnel under a Bangkok sky train platform by using the simulation of a three-dimensional air-quality model. This model was considered varied cases on the wind inflow with obstacles. In [10], the three-dimensional advection-diffusion equation was considered to approximate the concentration of air pollutant in a heavy traffic area under the Bangkok sky train station. The numerical simulations were studied for three cases that were the average of source or sink emissions, the moving of source or sink emissions, and the mix of source and sink emissions. The explicit finite difference scheme was used to solve the air pollutant concentration in [8–10].

The source that is smokestack of industrial factory or power plant discharges the air pollution into the system. The genesis of air pollution is the cause of problems. In this research, the simple finite difference methods are used for solving the atmospheric diffusion equation.

2. Governing Equation

2.1. The Atmospheric Diffusion Equation. The diffusion model generally uses Gaussian plume idea, which is the well-known atmospheric diffusion equation. It represents the behavior of air pollution in industrial areas. The dispersion of pollutant concentration from multiple point sources is described by the following three-dimensional advection-diffusion equation:

$$\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}+w\frac{\partial c}{\partial z}=k_x\frac{\partial^2 c}{\partial x^2}+k_y\frac{\partial^2 c}{\partial y^2}+k_z\frac{\partial^2 c}{\partial z^2}+s, \quad (1)$$

where $c = c(x, y, z, t)$ is the concentration of air pollutant at (x, y, z) and time $t\,(kg/m^3)$, u, v, and w are the wind velocity component (m/sec) in x-, y-, z-directions, respectively, k_x, k_y, and k_z are the diffusion coefficient (m^2/sec) in x-, y-, z-directions respectively, and s is the sink rate of air pollutants (sec^{-1}).

The assumptions of (1) are defined that the concentrations of air pollutant are emitted from continued point sources. The advection and diffusion in y-direction are laterally averaged. By the assumption, we can also eliminate all terms in y-direction. Therefore, the governing equation can be written as

$$\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}+w\frac{\partial c}{\partial z}=k_x\frac{\partial^2 c}{\partial x^2}+k_z\frac{\partial^2 c}{\partial z^2}+s. \quad (2)$$

The initial condition is assumed under the cold start assumption. That is,

$$c(x, z, 0) = 0, \quad (3)$$

for all $x > 0$ and $z > 0$. The boundary conditions are assumed that

$$c(0, z, t) = 0, \quad (4)$$

$$\frac{\partial c}{\partial x}(L, z, t) = 0, \quad (5)$$

$$\frac{\partial c}{\partial z}(x, 0, t) = 0, \quad (6)$$

$$\frac{\partial c}{\partial z}(x, H, t) = 0, \quad (7)$$

for all $t > 0$ where L is the length of the domain in x-direction and H is the height of the inversion layer. The concentration at the point sources is assumed to be the constant variables as

$$c\left(x_p, 0, t\right) = c_{s_p}, \quad (8)$$

for $p = 1, 2$ where x_p is the position of the point source p in the x-direction and c_{s_p} is the concentration value at the point source of p.

2.2. The Nondimensional Form Equation. Now, we introduce the dimensionless form of equation (2). The nondimensional variables are denoted by letting $C = c/c_{\max}$, $X = x/l_x$, $Z = z/l_z$, $T = t/t_{\max}$, $D_x = k_x/l_x u_{\max}$, $D_z = k_z/l_z u_{\max}$, $U = u/u_{\max}$, and $W = \beta w_{\max}/u_{\max}$ when $\beta = w/w_{\max}$. We define $c_{\max} = \max\{c(x,z,t) : 0 \leqslant x \leqslant L,\ 0 \leqslant z \leqslant H,\ 0 \leqslant t \leqslant t_{\max}\}$, $u_{\max} = \max\{u(x,z,t) : 0 \leqslant x \leqslant L,\ 0 \leqslant z \leqslant H,\ 0 \leqslant t \leqslant t_{\max}\}$, $w_{\max} = \max\{w(x,z,t) : 0 \leqslant x \leqslant L,\ 0 \leqslant z \leqslant H,\ 0 \leqslant t \leqslant t_{\max}\}$, and $t_{\max}$ is a stationary time. Thus the nondimensional equation of air pollution is as follows:

$$\frac{1}{ST}\frac{\partial C}{\partial T}+U\frac{\partial C}{\partial X}+W\frac{\partial C}{\partial Z}=D_x\frac{\partial^2 C}{\partial X^2}+D_z\frac{\partial^2 C}{\partial Z^2}+S, \quad (9)$$

where $l = \max\{l_x, l_z\}$ and $ST = t_{\max}u_{\max}/l$ when $S < 0$ that means the air pollutant concentrations are absorbed from the atmosphere by the chemical reaction.

3. Numerical Methods

We use the finite difference methods for calculating the nondimensional form of the atmospheric diffusion equation. In (9), we get the concentration of C at each time T_{n+1} from T_n when ΔT is a time increment. The solution of concentration at (X, Z, T) is denoted by $C(X_i, Z_j, T_n) = C^n_{i,j}$. The domain is divided by the grid spacing in X-direction and Z-direction which are ΔX and ΔZ, respectively, where $X_i = i\Delta X$ and $Z_j = j\Delta Z$. The approximate solutions are obtained by using the following methods.

3.1. Forward Time Central Space Scheme. The first method, we use the forward difference in transient term that is

$$\frac{\partial C}{\partial T}=\frac{C^{n+1}_{i,j}-C^n_{i,j}}{\Delta T}. \quad (10)$$

Then, the centered difference for the advection and diffusion in X-direction and Z-direction is applied as follows:

$$\frac{\partial C}{\partial X}=\frac{C^n_{i+1,j}-C^n_{i-1,j}}{2\Delta X}, \quad (11)$$

$$\frac{\partial C}{\partial Z}=\frac{C^n_{i,j+1}-C^n_{i,j-1}}{2\Delta Z}, \quad (12)$$

$$\frac{\partial^2 C}{\partial X^2}=\frac{C^n_{i+1,j}-2C^n_{i,j}+C^n_{i-1,j}}{(\Delta X)^2}, \quad (13)$$

$$\frac{\partial^2 C}{\partial Z^2}=\frac{C^n_{i,j+1}-2C^n_{i,j}+C^n_{i,j-1}}{(\Delta Z)^2}, \quad (14)$$

respectively. We substitute (10)-(14) into (9). It will be

$$\begin{aligned}
&\frac{1}{ST}\left(\frac{C_{i,j}^{n+1}-C_{i,j}^{n}}{\Delta T}\right)+U\left(\frac{C_{i+1,j}^{n}-C_{i-1,j}^{n}}{2\Delta X}\right)\\
&\quad+W\left(\frac{C_{i,j+1}^{n}-C_{i,j-1}^{n}}{2\Delta Z}\right)\\
&\quad=D_x\left(\frac{C_{i+1,j}^{n}-2C_{i,j}^{n}+C_{i-1,j}^{n}}{(\Delta X)^2}\right)\\
&\quad+D_z\left(\frac{C_{i,j+1}^{n}-2C_{i,j}^{n}+C_{i,j-1}^{n}}{(\Delta Z)^2}\right)+S.
\end{aligned}\tag{15}$$

Thus, the forward time central space (FTCS) scheme of the nondimensional mathematical model is

$$\begin{aligned}
C_{i,j}^{n+1}&=\left(d_x-A_x\right)C_{i+1,j}^{n}+\left(d_x+A_x\right)C_{i-1,j}^{n}\\
&\quad+\left(1-2d_x-2d_z\right)C_{i,j}^{n}+\left(d_z+A_z\right)C_{i,j-1}^{n}\\
&\quad+\left(d_z-A_z\right)C_{i,j+1}^{n}+ST\left(\Delta T\right)S,
\end{aligned}\tag{16}$$

where $A_x = ST(\Delta T)U/2\Delta X$, $A_z = ST(\Delta T)W/2\Delta Z$, $d_x = ST(\Delta T)D_x/(\Delta X)^2$, $d_z = ST(\Delta T)D_z/(\Delta Z)^2$. The stability of the forward time central space scheme can be investigated by using von Neumann stability analysis. We can obtain that the stability condition is $0 \leqslant 2d_x + A_x + A_z \leqslant 1$.

3.2. Backward Time Central Space Scheme. The second method, we use the backward difference in transient term that is

$$\frac{\partial C}{\partial T}=\frac{C_{i,j}^{n}-C_{i,j}^{n-1}}{\Delta T}.\tag{17}$$

Then, the centered difference for the advection and diffusion in X-direction and Z-direction is utilized as follows:

$$\frac{\partial C}{\partial X}=\frac{C_{i+1,j}^{n}-C_{i-1,j}^{n}}{2\Delta X},\tag{18}$$

$$\frac{\partial C}{\partial Z}=\frac{C_{i,j+1}^{n}-C_{i,j-1}^{n}}{2\Delta Z},\tag{19}$$

$$\frac{\partial^2 C}{\partial X^2}=\frac{C_{i+1,j}^{n}-2C_{i,j}^{n}+C_{i-1,j}^{n}}{(\Delta X)^2},\tag{20}$$

$$\frac{\partial^2 C}{\partial Z^2}=\frac{C_{i,j+1}^{n}-2C_{i,j}^{n}+C_{i,j-1}^{n}}{(\Delta Z)^2},\tag{21}$$

respectively. We substitute (17)-(21) into (9). It obtains that

$$\begin{aligned}
&\frac{1}{ST}\left(\frac{C_{i,j}^{n}-C_{i,j}^{n-1}}{\Delta T}\right)+U\left(\frac{C_{i+1,j}^{n}-C_{i-1,j}^{n}}{2\Delta X}\right)\\
&\quad+W\left(\frac{C_{i,j+1}^{n}-C_{i,j-1}^{n}}{2\Delta Z}\right)\\
&\quad=D_x\left(\frac{C_{i+1,j}^{n}-2C_{i,j}^{n}+C_{i-1,j}^{n}}{(\Delta X)^2}\right)\\
&\quad+D_z\left(\frac{C_{i,j+1}^{n}-2C_{i,j}^{n}+C_{i,j-1}^{n}}{(\Delta Z)^2}\right)+S.
\end{aligned}\tag{22}$$

Therefore, the backward time central space (BTCS) scheme of this research becomes

$$\begin{aligned}
&\left(A_x-d_x\right)C_{i+1,j}^{n+1}+\left(A_x+d_x\right)C_{i-1,j}^{n+1}\\
&\quad+\left(1+2d_x+2d_z\right)C_{i,j}^{n+1}-\left(A_z+d_z\right)C_{i,j-1}^{n+1}\\
&\quad+\left(A_z-d_z\right)C_{i,j+1}^{n+1}=C_{i,j}^{n}+ST\left(\Delta T\right)S.
\end{aligned}\tag{23}$$

The stability of the implicit backward time central space scheme can be investigated by using von Neumann stability analysis. We can obtain that the method is an unconditionally stable method.

4. Numerical Experiment

The two-dimensional atmospheric diffusion equation (9) with a dimension $1,000 \times 1,000\,m^2$ will be considered. The uniform wind velocities and constant diffusion coefficients are introduced. We choose that the wind velocities in x-direction and z-direction are 0.1 and 0.05 m/sec, respectively. The diffusion coefficients in x-direction and z-direction are 4.5×10^{-1} and 4.5×10^{-5} m^2/sec, respectively. The grid spacing is $\Delta x = \Delta z = 25$ m. and the time interval is 20 sec. In this research, we present two cases. The first case considers a point source when the concentration is 0.5 kg/m^3. The second case considers two-point source when the concentration is 0.25 and 0.25 kg/m^3. The air pollutants in (8) are released into our system. These examples are solved by using the forward time central space and the backward time central space schemes in (16) and (23), respectively, with the initial and boundary conditions (3) to (7).

In Figure 1, model of the problem is shown. The physical problem composed of two zones: an industrial zone and an urban zone with the stable wind along the x-axis and z-axis. The point sources are laid along the x-axis. We assume that the primary air pollutants are released from a factory smokestack by a single point source and coupled point sources on industrial zone. The emissions of air pollution are influenced on the urban zone by the rate of air pollutant absorption. In the numerical experiment, the considered domain of solutions is shown in Figure 2.

5. Discussion

The air pollutant emission from multiple point sources above an industrial zone to the urban area is presented. The finite difference techniques introduced two methods for calculating the air pollutant concentrations. Figures 3 and 4 compare the air pollutant concentrations between two cases: a single point source and coupled point sources, respectively. From the both

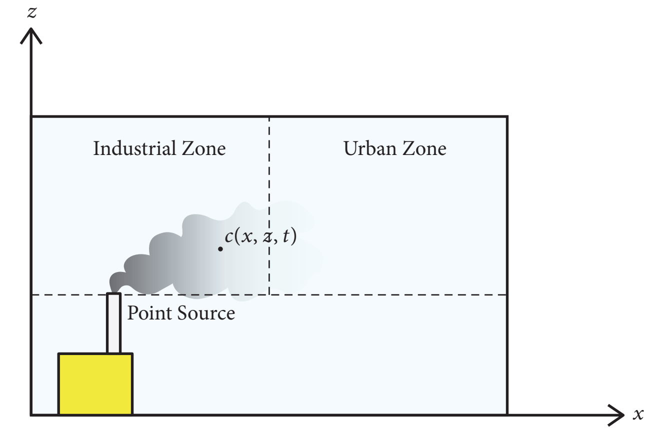


FIGURE 1: Model of the problem.

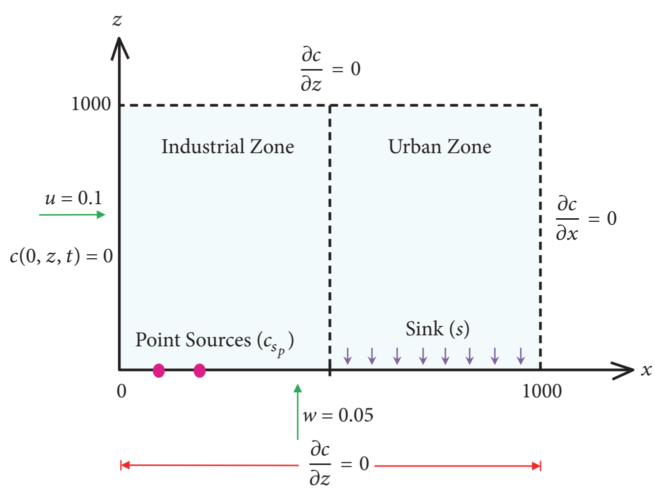


FIGURE 2: Domain of solutions.

figures, it is apparent that the results of the forward time central space scheme are close to the results of the backward time central space scheme, when there is no sink of pollutant absorption ($s = 0$). Figures 5 and 6 illustrate that the sink of pollutant absorption ($s = 10^{-4}$) is added to the base of urban zone. The air pollutant concentration near human living goes down and the two methods also give the close result. In Figures 7 and 8, the computed approximate solutions which are calculated by using the forward time central space and the backward time central space schemes are compared. We can see that the results of added sink case and without sink case are quite similar. These graphs also indicate that the forward time central space scheme gives the computed solutions close to the backward time central space scheme.

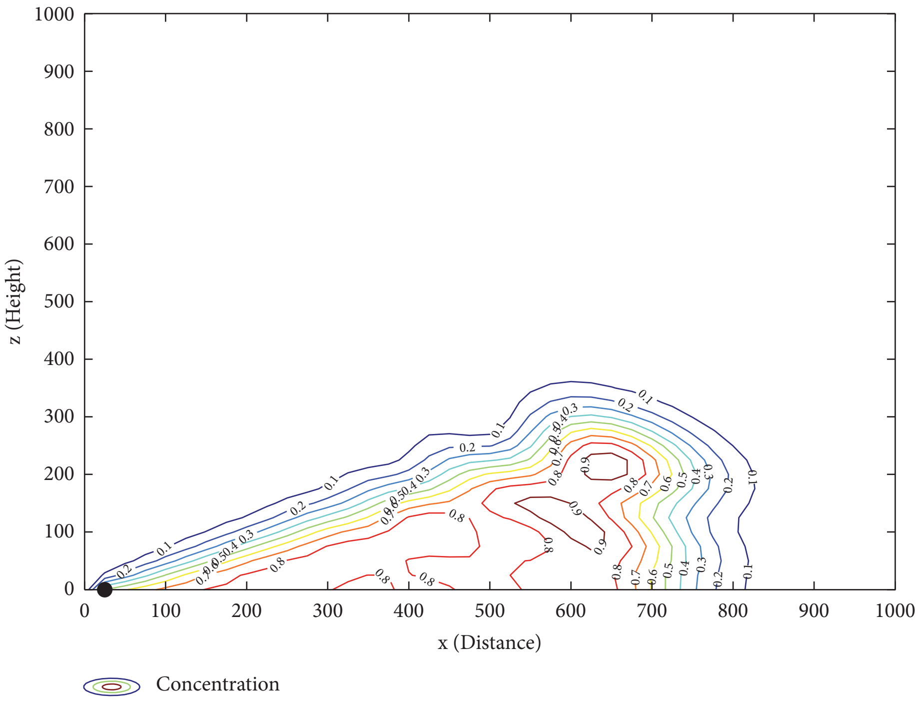
Figures 9 and 10 demonstrate that the air pollutant concentration at the height $z = 25$ m. and $z = 50$ m. are solved by using the forward time central space scheme. The added sink case is less concentration than the without sink case. Therefore, the sink can lower the overall pollutant levels. Figure 11 establishes the various concentrations when we take more sink rate into our system. The comparison of computing time shows that the forward time central space is faster than the backward time central space scheme in Table 1.

TABLE 1: Computing time comparison of forward time central space and backward time central space schemes.

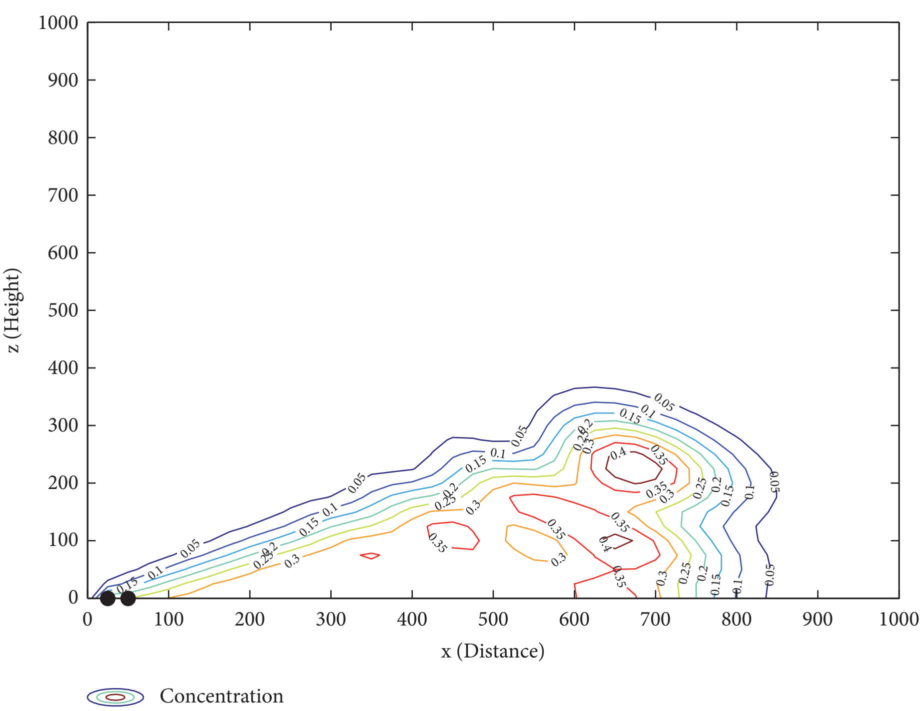
Simulation Time	FTCS (sec.)	BTCS (sec.)
30 minutes	1.49	22.48
1 hour	1.68	42.66
2 hours	2.05	84.18

6. Conclusion

The simple air pollution measurement models which are released air pollutants by a single point source and coupled point sources are proposed. The traditional finite difference methods such as forward time central space and backward time central space schemes can be used to approximate the air pollutant levels for each points and times. The results of this study show that the air pollutant concentrations of forward

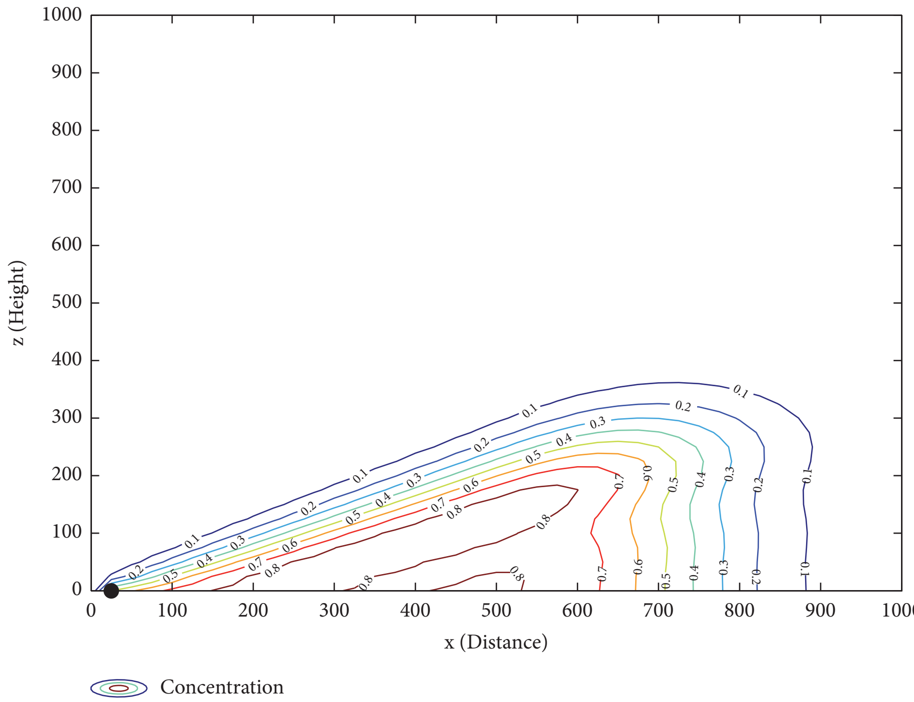


(a)

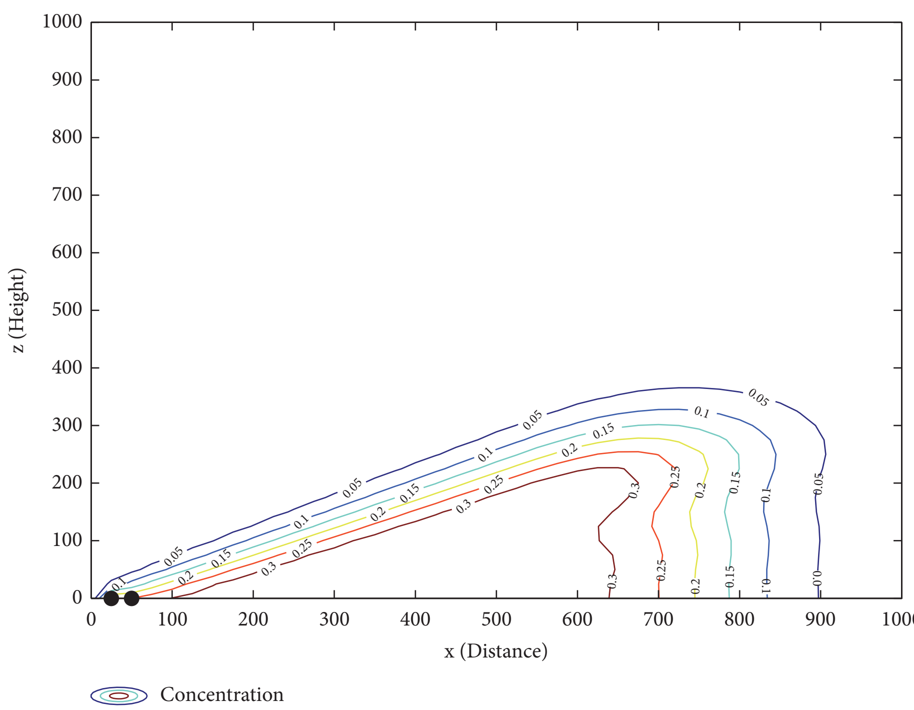


(b)

FIGURE 3: The air pollutant concentration levels after 2 hours passed which are computed by the forward time central space scheme ($s = 0$): (a) one-point source and (b) two-point source.

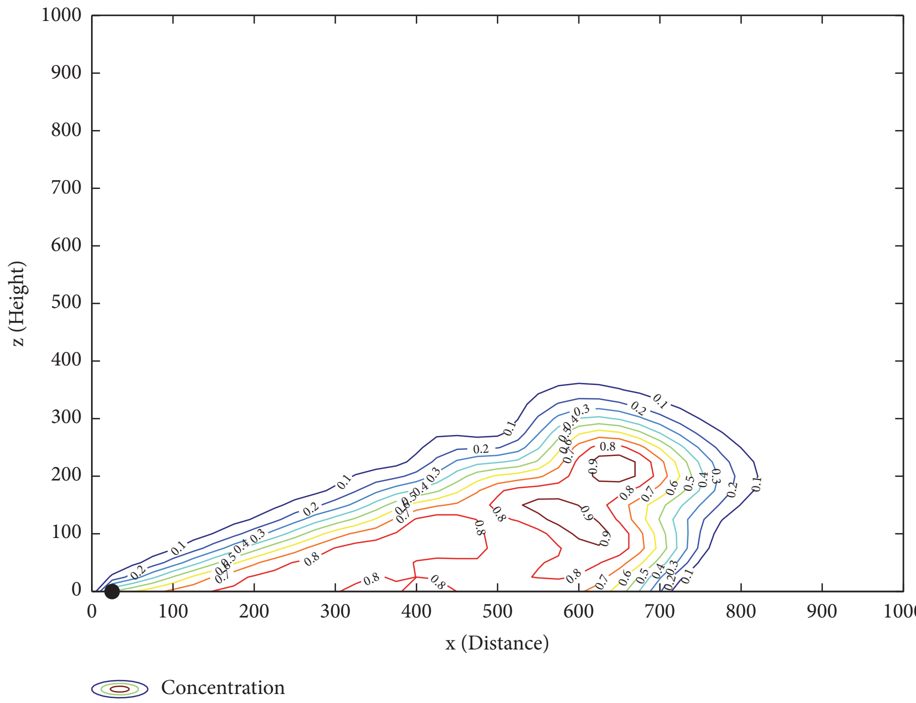


(a)

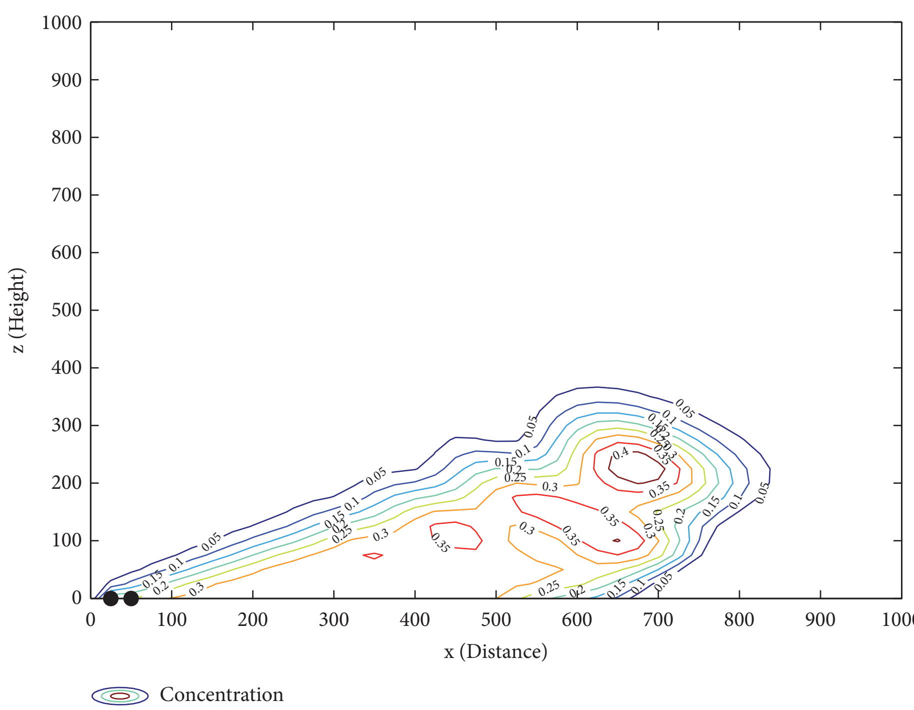


(b)

FIGURE 4: The air pollutant concentration levels after 2 hours passed which are computed by the backward time central space scheme ($s = 0$): (a) one-point source and (b) two-point source.

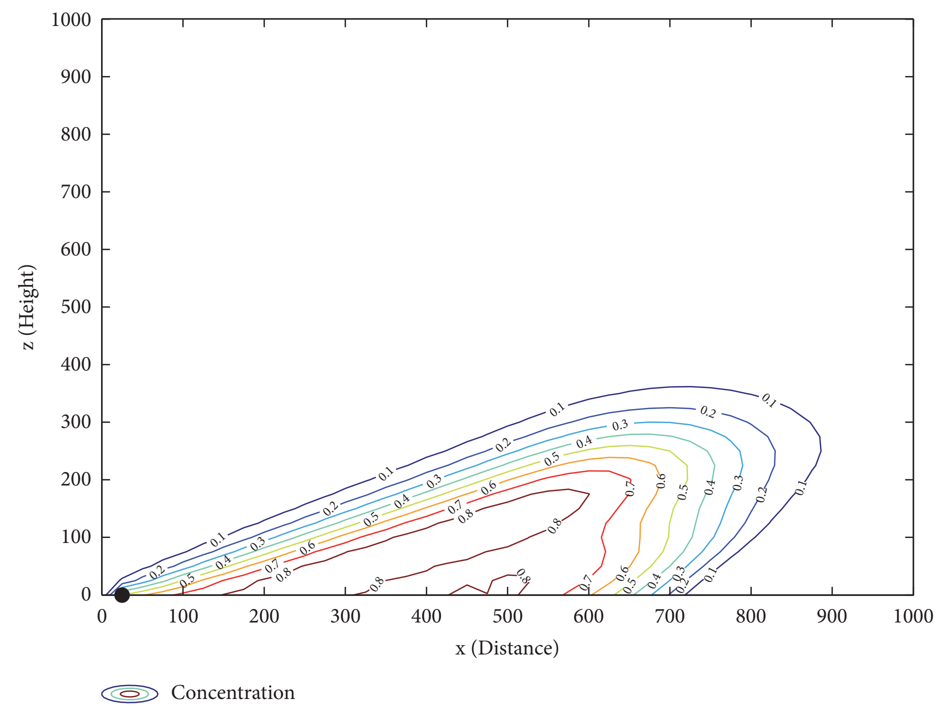


(a)

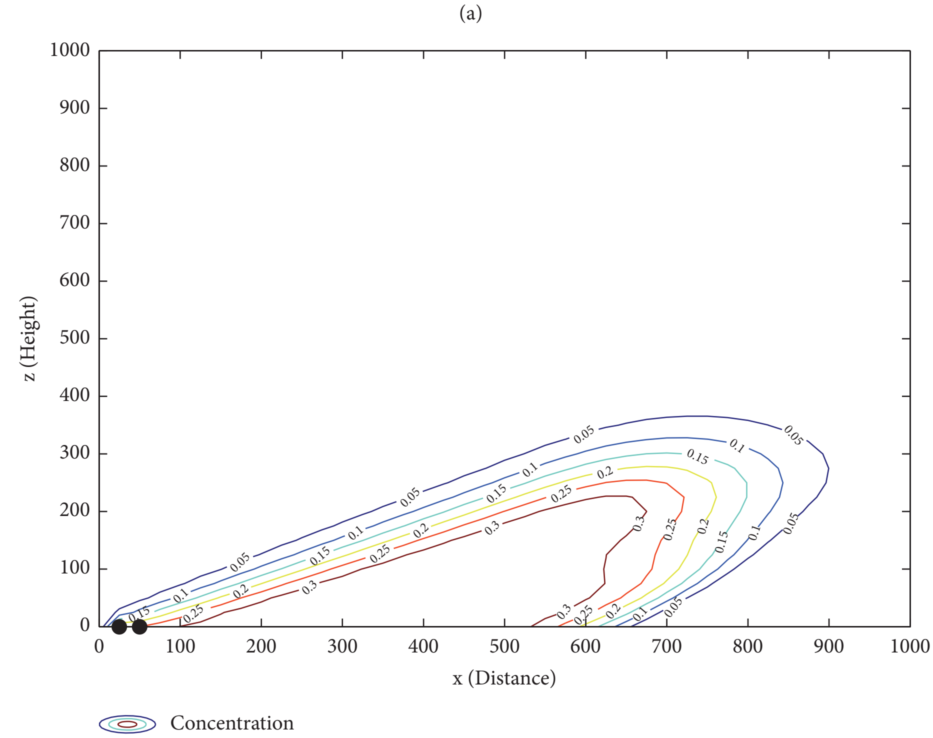


(b)

FIGURE 5: The air pollutant concentration levels after 2 hours passed which are computed by the forward time central space scheme ($s = 10^{-4}$): (a) one-point source and (b) two-point source.



(a)



(b)

Figure 6: The air pollutant concentration levels after 2 hours passed which are computed by the backward time central space scheme ($s = 10^{-4}$): (a) one-point source and (b) two-point source.

time central space are close to the air pollutant concentrations of backward time central space. In the case of a coupled point sources problem, the overall concentration levels of air pollution are less than a single point source problem. Therefore, the influence of multiple point sources and the variable rate of sink are also considered. It obtains that the higher sink rate does decrease pollutant levels around human living. The both finite difference methods are used to compute the numerical solutions of air pollution by MATLAB. The forward time central space has advantages that the method

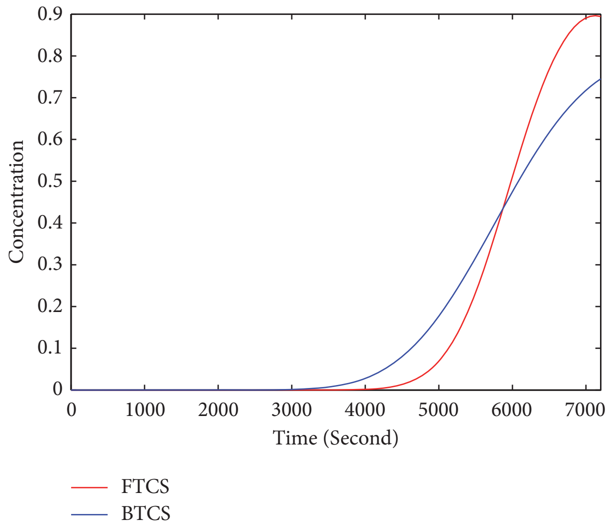


Figure 7: The air pollutant concentration between the forward time central space and the backward time central space schemes ($s = 0$) at $z = 0$ m. and $x = 600$ m.

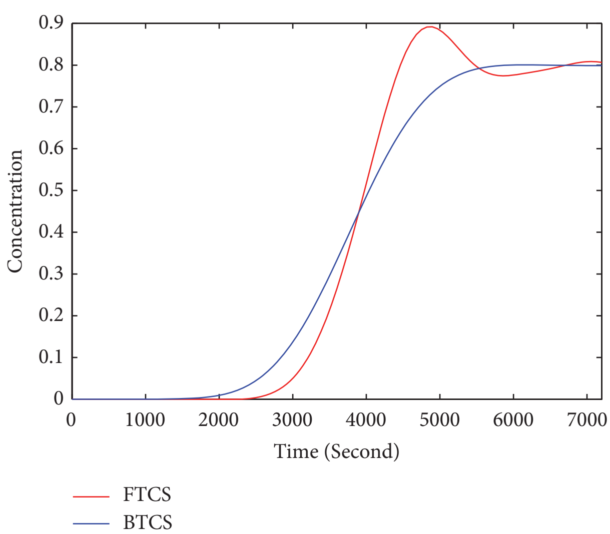


Figure 8: The air pollutant concentration between the forward time central space and the backward time central space schemes ($s = 10^{-4}$) at $z = 0$ m. and $x = 400$ m.

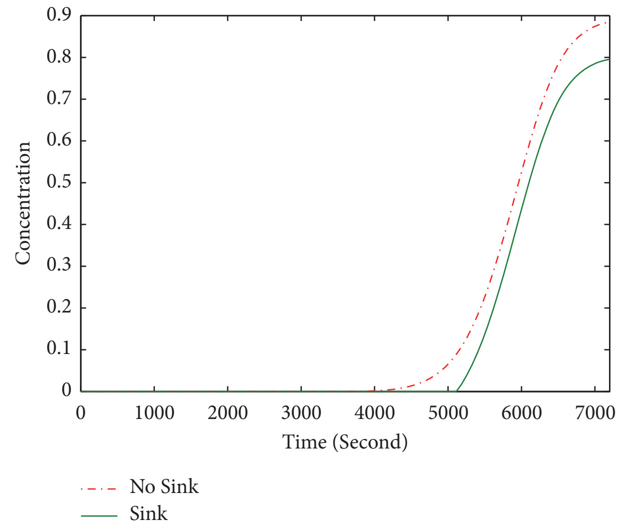


Figure 9: The air pollutant concentration between 2 cases: added sink and without sink (computed by the forward time central space scheme) at $z = 25$ m. and $x = 600$ m.

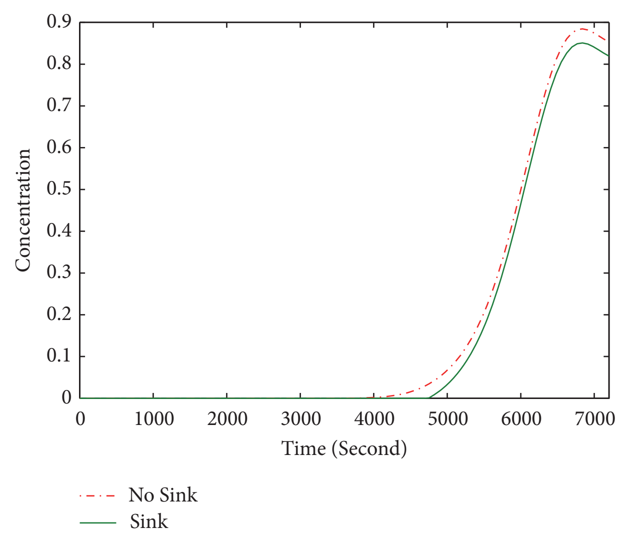


Figure 10: The air pollutant concentration between 2 cases: added sink and without sink (computed by the forward time central space scheme) at $z = 50$ m. and $x = 600$ m.

gives less computing time than the backward time central space computing time. On the other hand, the forward time central space also has disadvantages that are the limitation of the grid spacing due to the stability condition.

Data Availability

The calculated air pollution measurement data used to support the findings of this study are available from the corresponding author upon request.

Disclosure

A part of the research has been presented as an oral presentation in ACFPTO 2016.

Conflicts of Interest

The authors declare no conflicts of interest.

Acknowledgments

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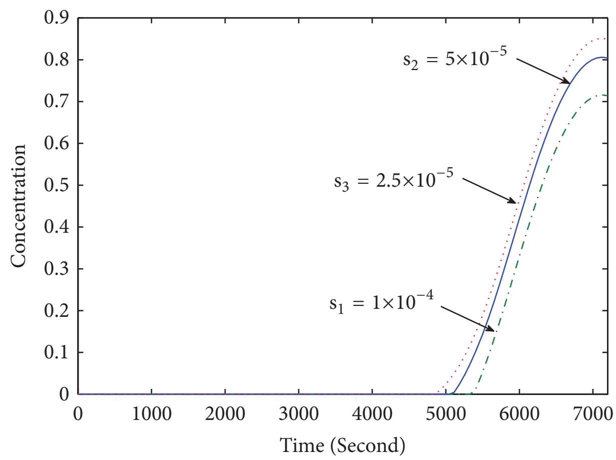

Figure 11: The air pollutant concentration with the variant values of sink rate (computed by the forward time central space scheme) at $z = 0$ m. and $x = 600$ m.

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