Research Article

Adomian Decomposition Method with Modified Bernstein Polynomials for Solving Ordinary and Partial Differential Equations

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In this paper, we used Bernstein polynomials to modify the Adomian decomposition method which can be used to solve linear and nonlinear equations. This scheme is tested for four examples from ordinary and partial differential equations; furthermore, the obtained results demonstrate reliability and activity of the proposed technique. This strategy gives a precise and productive system in comparison with other traditional techniques and the arrangements methodology is extremely straightforward and few emphasis prompts high exact solution. The numerical outcomes showed that the acquired estimated solutions were in appropriate concurrence with the correct solution.

1. Introduction

Adomian decomposition technique was established by George Adomian and has as of late turned into an extremely recognized strategy in connected sciences. The technique does not require any diminutiveness presumptions or linearization to solve the ordinary and partial differential equations and this produces the strategy extremely effective among alternate strategies. Recently, many iteration techniques have been used for solving nonlinear equations from ordinary, partial, and fractional equations [1], like variational iteration method and differential transform method [2], homotopy perturbation, and analysis methods [3]. Numerous works have been tested in various different regions, for example, warmth or mass exchange, incompressible fluid, nonlinear optics and gas elements wonders [4, 5], fractional Maxwell fluid [6, 7], and the Oldroyd-B fluid model [8].

The approximation used polynomials extremely important in scientific experiments where many rely on topics such as the study of statistics different population and the temperatures and others on the approximation theory. In addition, many experiments rely mainly on the approximate measurements and observations to be studied and processed by the appropriate scientific methods in order to reach the results expected from the study.

The Adomian decomposition technique is improved via Chebyshev polynomials in [9, 10], with Legendre polynomials [11] and with Laguerre polynomials [12].

This paper is organized as follows. In Section 2, the basic ideas of the modified Bernstein polynomials are described. Section 3 is devoted to solving a nonlinear differential equations using Adomian decomposition method based on modified Bernstein polynomials, the results and comparisons of the numerical solutions are presented in Section 4, and concluding remarks are given in Section 5.

2. The Modified Bernstein Polynomials

Polynomials are the mathematical technique as these can be characterized, figured, separated, and incorporated effortlessly. The Bernstein premise polynomials are trying to inexact the capacities. Bernstein polynomials are the better guess to a capacity with a couple of terms. These polynomials are utilized as a part of the fields of connected arithmetic and material science and PC helped geometric outlines and are likewise joined with different techniques like Galerkin and

collocation technique to solve some differential and integral equations [13].

Definition 1 (Bernstein basis polynomials). The Bernstein basis polynomials of degree m over the interval $[0, 1]$ are defined by

$$B_{i,m}(x) = \binom{m}{i} x^i (1-x)^{m-i} \qquad (1)$$

where the binomial coefficient is

$$\binom{m}{i} = \frac{m!}{i!\,(m-i)!} \qquad (2)$$

For example, when m=5, then the Bernstein terms are

$$\begin{aligned}
B_{0,5}(x) &= (1-x)^5 \\
B_{1,5}(x) &= 5x(1-x)^4 \\
B_{2,5}(x) &= 10x^2(1-x)^3 \\
B_{3,5}(x) &= 10x^3(1-x)^2 \\
B_{4,5}(x) &= 5x^4(1-x) \\
B_{5,5}(x) &= x^5
\end{aligned} \qquad (3)$$

Definition 2 (Bernstein polynomials). A linear combination of Bernstein basis polynomials

$$B_m(x) = \sum_{i=0}^{m} B_{i,m}(x)\,\beta_i \qquad (4)$$

is called the Bernstein polynomials of degree m, where β_i are the Bernstein coefficients.

Definition 3. Let f be a real valued function defined and bounded on $[0, 1]$; let $B_m(f)$ be the polynomial on $[0, 1]$, defined by

$$B_m(f) = \sum_{i=0}^{m} \binom{m}{i} x^i (1-x)^{m-i} f\left(\frac{i}{m}\right) \qquad (5)$$

where $B_m(f)$ is the m-th Bernstein polynomials for $f(x)$.
    For each function $f : [0, 1] \longrightarrow R$, we have

$$\lim_{m \to \infty} B_m^f(x) = f(x) \qquad (6)$$

Example. If $f(x) = e^x$, $x \in [0, 1]$ then the Bernstein expanded for the function $f(x)$ when m=5 is

$$\begin{aligned}
B_m(f) &= f(0)(1-x)^5 + f\left(\frac{1}{5}\right) 5x(1-x)^4 \\
&\quad + f\left(\frac{2}{5}\right) 10x^2(1-x)^3 \\
&\quad + f\left(\frac{3}{5}\right) 10x^3(1-x)^2 \\
&\quad + f\left(\frac{4}{5}\right) 5x^4(1-x) + f(1)x^5
\end{aligned}$$

$$\begin{aligned}
B_m(f) &= e^0(1-x)^5 + 5e^{1/5}x(1-x)^4 \\
&\quad + 10e^{2/5}x^2(1-x)^3 + 10e^{3/5}x^3(1-x)^2 \\
&\quad + 5e^{4/5}x^4(1-x) + e^1 x^5
\end{aligned} \qquad (7)$$

In (1986) [14] Lorentz, prove that if the $2k$-th order derivative $f^{2k}(x)$ is bounded in the interval (0,1) then for each $x \in [0, 1]$

$$B_m^f(x) = f(x) + \sum_{a=2}^{2k-1} \frac{f^{(a)}(x)}{a!\,m^a} T_{m,a}(x) + O\left(\frac{1}{m^k}\right) \qquad (8)$$

where

$$T_{m,a}(x) = \sum_{k} (k - mx)^a \binom{m}{k} x^k (1-x)^{m-k} \qquad (9)$$

Remark (see [15]). Notice that $T_{m,a}(x)$ is the a-th central moment of a random variable with a binomial appropriation with parameters m and x. Clearly, $T_{m,0} = 1$, $T_{m,1} = 0$. It is well known that the sequence $\{T_{m,a}(x)\}$ satisfies the following recurrence:

$$T_{m,a+1}(x) = x(1-x)\left(T'_{m,a}(x) + ma T_{m,a-1}(x)\right) \qquad (10)$$

If we apply (8) to $k = 1; 2; 3$, then we obtain

$$B_m^f(x) = f(x) + O\left(\frac{1}{m}\right)$$

$$B_m^f(x) = f(x) + \frac{x(1-x)f''(x)}{2m} + O\left(\frac{1}{m^2}\right)$$

$$B_m^f(x) = f(x) + \frac{x(1-x)f''(x)}{2m} \qquad (11)$$

$$+ \frac{x(1-x)\left(4(1-2x)f^{(3)}(x) + 3x(1-x)f^{(4)}(x)\right)}{24m^2}$$

$$+ O\left(\frac{1}{m^3}\right)$$

and higher level approximations can be computed.

3. ADM Based on Modified Bernstein Polynomials

Let us consider the following equation:

$$Lu + Nu + Ru = g(x) \qquad (12)$$

where L is an invertible linear term, N represents the nonlinear term, and R is the remaining linear part; from (12) we have

$$Lu = g(x) - Nu - Ru. \qquad (13)$$

Now, applying the inverse factor L^{-1} to both sides of (13) then via the initial conditions we find

$$u = f(x) - L^{-1}Nu - L^{-1}Ru, \qquad (14)$$

where $L^{-1} = \int_0^x (.) \, ds$ and $f(x)$ are the terms having from integrating the rest of the term $g(x)$ and from utilizing the given initial or boundary conditions. The ADM assumes that $N(u)$ (nonlinear term) can be decomposed by an infinite series of polynomials which is expressed in form

$$N(u) = \sum_{n=0}^{\infty} A_n (u_o, u_1, \ldots, u_n) \tag{15}$$

where A_n are the Adomian's polynomials [16] defined as

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[N \left(\sum_{i=0}^{\infty} \lambda^i u_i \right) \right]_{\lambda=0}, \quad n = 0, 1, 2, \ldots \tag{16}$$

We expand the function $g(x)$ by Bernstein series

$$g(x) = \sum_{i=0}^{m} a_i B_i(x) \tag{17}$$

where $B_i(x)$ is the Bernstein polynomials.

Now, using (14) and (17) we have

$$
\begin{aligned}
u_0 &= L^{-1} \left(a_0 B_0(x) + a_1 B_1(x) + a_2 B_2(x) \right. \\
&\quad \left. + \cdots . a_m B_m(x) \right) + \theta(x), \\
u_1 &= -L^{-1} (Ru_0) - L^{-1} (Nu_0), \\
u_2 &= -L^{-1} (Ru_1) - L^{-1} (Nu_1), \\
&\quad \vdots
\end{aligned}
\tag{18}
$$

and so on. These formulas are easy to compute by using Maple 13 software.

In this paper, we improve the function $g(x)$ using modified Bernstein series

$$
g(x) = \sum_{i=0}^{m} \binom{m}{i} x^i (1-x)^{m-i} f\left(\frac{i}{m}\right)
$$
$$
- \sum_{a=2}^{2k-1} \frac{f^{(a)}(x)}{a! m^a} T_{m,a}(x)
\tag{19}
$$

And we can approach the derivatives using the Bernstein polynomials

$$\frac{d}{dx} B_{i,m}(x) = m \left(B_{i-1,m-1}(x) - B_{i,m-1}(x) \right), \tag{20}$$

Then (19) becomes

$$
g(x) = \sum_{i=0}^{m} \binom{m}{i} x^i (1-x)^{m-i} f\left(\frac{i}{m}\right)
$$
$$
- \sum_{a=2}^{2k-1} \frac{\left(d^{(a)}/dx^{(a)}\right) B_{i,m}(x)}{a! m^a} T_{m,a}(x)
\tag{21}
$$

Now, using (18) and (21) we have

$$
\begin{aligned}
u_0 &= L^{-1} \left(B_{i,m}(x) \right) + \theta(x), \\
u_1 &= -L^{-1} (Ru_0) - L^{-1} (Nu_0), \\
u_2 &= -L^{-1} (Ru_1) - L^{-1} (Nu_1), \\
&\quad \vdots
\end{aligned}
\tag{22}
$$

The above equation is governing equation of ADM using modified Bernstein polynomials. The obtained approximate solution, $\omega_V(x) = \sum_{i=0}^{V} u_i$, by (22) has a comparison with the classic approximation solution and the correct solution.

4. Numerical Results

In this section, we solve ordinary and partial differential equations by ADM based on Bernstein polynomials and we compare with ADM based on classical Bernstein polynomial.

Example 1. Consider the ordinary equation

$$
\frac{d^2 y}{dt^2} + t \frac{dy}{dt} + t^2 y^3 = \left(2 + 6t^2\right) e^{t^2} + t^2 e^{3t^2},
$$
$$
y(0) = 1, \tag{23}
$$
$$
\frac{dy}{dt}(0) = 0,
$$

with the exact solution $(t) = e^{t^2}$. Using (12) we have

$$Ly + Ny + Ry = g(x) \tag{24}$$

where $L = d^2/dt^2, Ry = t(d/dt), Ny = t^2 y^3$, and $g(t) = (2 + 6t^2)e^{t^2} + t^2 e^{3t^2}$.

The Adomian polynomials for representing the nonlinear term Ny are

$$
\begin{aligned}
A_0 &= t^2 y_0^3, \\
A_1 &= t^2 \left(3 y_0^2 y_1\right), \\
A_2 &= t^2 \left(3 y_0^2 y_2 + 3 y_0 y_1^2\right), \\
&\quad \vdots
\end{aligned}
\tag{25}
$$

Now $L^{-1} = \int_0^t \int_0^t (.) dt \, dt$; then using (5) the classical Bernstein polynomials of $g(t)$ when v=m=6 are

$$
\begin{aligned}
g_b(t) &= 2 + 1.547324t + 9.290164t^2 + 7.83289t^3 \\
&\quad + 9.751887t^4 + 7.659668t^5 + 3.749864t^6
\end{aligned}
\tag{26}
$$

And modified Bernstein polynomials (21) of $g(t)$ with k=2 are

$$
\begin{aligned}
g_{mb}(t) &= 2 - 0.001037t + 6.922082t^2 + 1.997441t^3 \\
&\quad + 6.737662t^4 + 11.051121t^5 + 13.124523t^6
\end{aligned}
\tag{27}
$$

TABLE 1: Comparison of absolute errors using y_6 when m=v=6 and k=2.

t	$\left|y_{exact} - y_b\right|$	$\left|y_{exact} - y_{mb}\right|$
0	0	0
0.01	2.580000 E-7	2.000000 E-9
0.02	2.068000 E-6	2.900000 E-8
0.03	6.992000 E-6	1.420000 E-7
0.04	1.660300 E-5	4.450000 E-7
0.05	3.249600 E-5	1.074000 E-6
0.06	5.629000 E-5	2.207000 E-6
0.07	8.962800 E-5	4.057000 E-6
0.08	1.341850 E-4	6.872000 E-6
0.09	1.916690 E-4	1.093000 E-5
0.1	2.638350 E-4	1.654900 E-5
MSE	1.369383957 E-8	4.632475500 E-11

(a)

(b)
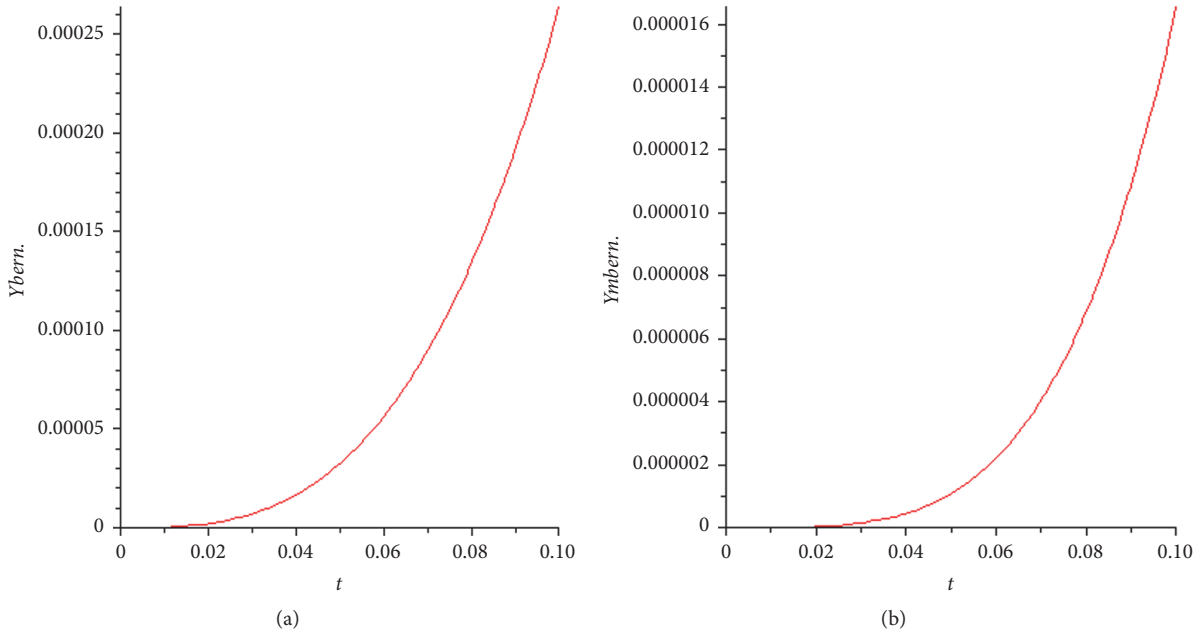
FIGURE 1: The absolute error between ADM with modified Bernstein polynomials and the exact solution when m=v=6 and k=2.

By (22), we have

$$y_0 = L^{-1}\left(g_{mb}\left(t\right)\right) + y\left(0\right) + \frac{dy}{dt}\left(0\right)t = 1 + t^2$$

$$- 0.000173t^3 + 0.57684t^4 + 0.099872t^5$$

$$+ 0.224589t^6 + 0.263122t^7 + 0.234367t^8,$$

$$y_1 = -L^{-1}\left(t\frac{d}{dt}y_0\right) - L^{-1}\left(A_0\right) = -0.25t^4$$

$$+ 0.000026t^5 - 0.176912t^6 - 0.011877t^7 + \cdots,$$

$$y_2 = -L^{-1}\left(t\frac{d}{dt}y_1\right) - L^{-1}\left(A_1\right) = 0.033333t^6$$

$$- 0.000003t^7 + 0.032348t^8 + 0.011536t^9 + \cdots,$$

$$\vdots$$

(28)

And we obtain

$$y_{mb}\left(t\right) = \sum_{i=0}^{6} y_i$$

$$= 1 + t^2 - 0.000173t^3 + 0.32684t^4 + \cdots.$$

(29)

The absolute error of $y_{mb}(t)$ and $y_b(t)$ is presented in Table 1 and Figure 1.

Figure 1 presents the absolute error of ADM with Bernstein polynomial in (a) and ADM with modified Bernstein

Table 2: Comparison of absolute errors using y_{10} when m=16, v=4, and k=2.

t	$\lvert y_{exact} - y_b \rvert$	$\lvert y_{exact} - y_{mb} \rvert$
0	0	0
0.1	2.061126 E-6	3.074560 E-7
0.2	1.145239 E-5	1.058636 E-5
0.3	1.315732 E-5	5.114716 E-5
0.4	2.165603 E-4	1.331415 E-4
0.5	8.646500 E-4	2.420463 E-4
0.6	2.311555 E-3	3.299021 E-4
0.7	4.905204 E-3	3.231831 E-4
0.8	8.829753 E-3	1.540876 E-4
0.9	1.391114 E-2	1.870564 E-4
1	1.946226 E-2	6.088701 E-4
MSE	6.804632 E-5	7.217798344 E-8

polynomial in (b) at m=v=6 and k=2. The absolute errors generated using the ADM with Bernstein polynomial are of 10^{-4} while the errors yielded from ADM with modified Bernstein polynomial are of 10^{-5}.

Example 2. Consider the ordinary equation

$$\frac{d^2 y}{dt^2} + y \frac{dy}{dt} = t \sin\left(2t^2\right) - 4t^2 \sin\left(t^2\right) + 2 \cos\left(t^2\right),$$

$$0 \le t \le 1, \quad (30)$$

$$y(0) = 0,$$

$$\frac{dy}{dt}(0) = 0,$$

with the exact solution $(t) = \sin(t^2)$.

Here, $L = d^2/dt^2$, $Ny = y(dy/dt)$, and $g(t) = t \sin(2t^2) - 4t^2 \sin(t^2) + 2 \cos(t^2)$.

The Adomian polynomials for represent the nonlinear term Nu are

$$A_0 = y_0 \frac{d}{dt} y_0,$$

$$A_1 = y_1 \frac{d}{dt} y_0 + y_0 \frac{d}{dt} y_1,$$

$$(31)$$

$$A_2 = y_2 \frac{d}{dt} y_0 + y_1 \frac{d}{dt} y_1 + y_0 \frac{d}{dt} y_1,$$

$$\vdots$$

Then using (5) the classical Bernstein polynomials of $g(t)$ when v=4 and m=16 is

$$g_b(t) = 20.00659171t + 0.2233190t^2 + 0.098085t^3$$

$$- 3.39540t^4 + \cdots \quad (32)$$

And modified Bernstein polynomials (21) of g(t) with k=2 are

$$g_{mb}(t) = 2 - 0.007366t + 0.2188855t^2 + 1.389751t^3$$

$$- 4.50213t^4 + \cdots \quad (33)$$

By (22), we have

$$y_{mb}(t) = \sum_{i=0}^{4} y_i$$

$$= t^2 - 0.001228t^3 + 0.018241t^4 - 0.030513t^5$$

$$+ \cdots, \quad (34)$$

The absolute error of $y_{mb}(t)$ and $y_b(t)$ is presented in Table 2 and Figure 2.

Figure 2 presents the absolute error of ADM with Bernstein polynomial in (a) and ADM with modified Bernstein polynomial in (b) at m=v=10 and k=3. The absolute errors generated using the ADM with Bernstein polynomial are of 10^{-2} while the errors yielded from ADM with modified Bernstein polynomial are of 10^{-4}.

Example 3. Consider the ordinary equation

$$\frac{dy}{dt} - ty + y^2 = e^{t^2},$$

$$y(0) = 1, \quad (35)$$

with the exact solution $(t) = e^{t^2/2}$.

Here $L = d/dt$, $Ru = -ty$, $Ny = y^2$, and $g(t) = e^{t^2}$.

Then using (5) the classical Bernstein polynomials of $g(t)$ when v=8 and m=12 is

$$g_b(t) = 1 + 0.083623t + 0.939177t^2 + 0.197729t^3$$

$$+ 0.331572t^4 + \cdots, \quad (36)$$

TABLE 3: Comparison of absolute errors using y_{10} when m=12, v=8, and k=3.

t	$\|y_{exact} - y_b\|$	$\|y_{exact} - y_{mb}\|$
0	0	0
0.01	4.134000 E-6	1.750000 E-7
0.02	1.635300 E-5	6.400000 E-7
0.03	3.639700 E-5	1.314000 E-6
0.04	6.402400 E-5	2.123000 E-6
0.05	9.900900 E-5	2.999000 E-6
0.06	1.411460 E-4	3.883000 E-6
0.07	1.902410 E-4	4.720000 E-6
0.08	2.461180 E-4	5.463000 E-6
0.09	3.086100 E-4	6.069000 E-6
0.1	3.775690 E-4	6.501000 E-6
MSE	3.699974901 E-8	1.619641710 E-11

(a)

(b)
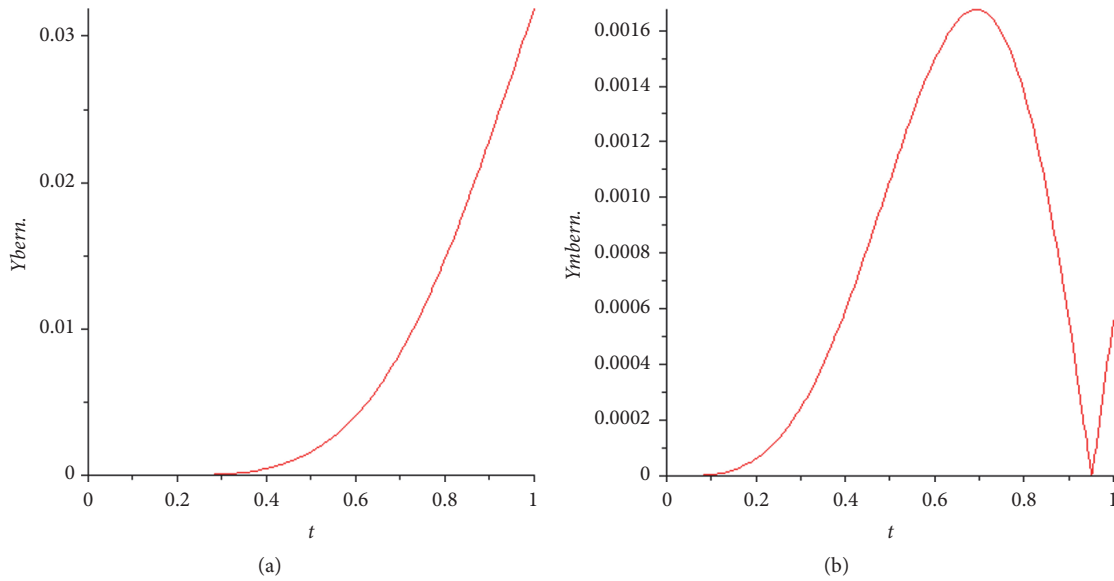
FIGURE 2: The absolute error between ADM with modified Bernstein polynomials and the exact solution when m=v=10 and k=3.

and modified Bernstein polynomials (21) of $g(t)$ with k=3 are

$$g_{mb}(t) = 1 + 0.003794t + 0.957094t^2 + 0.103509t^3$$
$$+ 0.410289t^4 + \cdots . \tag{37}$$

By (22), we have

$$u_{mb}(t) = \sum_{i=0}^{8} u_i$$
$$= 1 + 0.501897t^2 - 0.015567t^3 + 0.159135t^4 \tag{38}$$
$$+ \cdots ,$$

The absolute error of $u_{mb}(t)$ and $u_b(t)$ is presented in Table 3 and Figure 3.

Figure 3 presents the absolute error of ADM with Bernstein polynomial in (a) and ADM with modified Bernstein

polynomial in (b) at m=12, v=8, and k=3. The absolute errors generated using the ADM with Bernstein polynomial are of 10^{-4} while the errors yielded from ADM with modified Bernstein polynomial are of 10^{-6}.

Example 4. Consider the partial differential equation

$$\frac{\partial y}{\partial t} + y\frac{\partial y}{\partial x} - v\frac{\partial^2 y}{\partial x^2} = x\left(2t\cos\left(t^2\right) + \sin^2\left(t^2\right)\right),$$
$$y(x, 0) = 0, \tag{39}$$

Using (12) we have

$$Ly + Ny + Ry = g(x, t) \tag{40}$$

where $L = d/dt$, $Ry = -v(\partial^2 y/\partial x^2)$, $Ny = y(\partial y/\partial x)$, and $g(x, t) = x(2t\cos(t^2) + \sin^2(t^2))$.
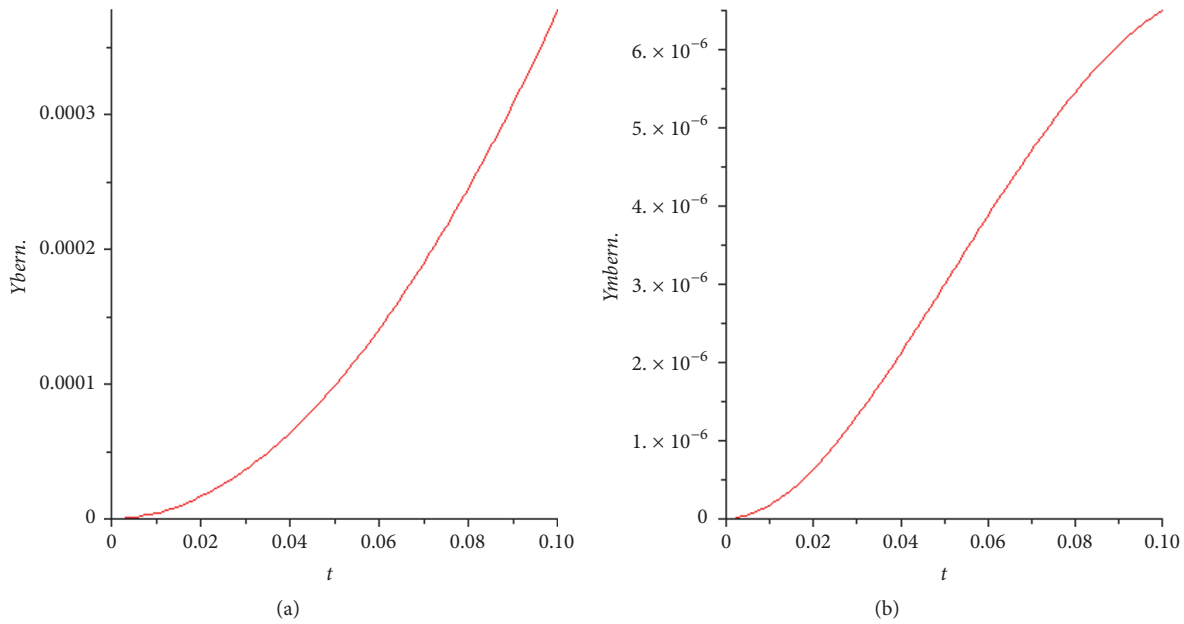
(a)

(b)

FIGURE 3: The absolute error between ADM with modified Bernstein polynomials and the exact solution when m=12, v=8, and k=3.

The Adomian polynomials for Ny are

$$A_0 = y_o \frac{\partial y_o}{\partial x},$$

$$A_1 = y_o \frac{\partial y_1}{\partial x} + y_1 \frac{\partial y_o}{\partial x},$$

$$A_2 = y_o \frac{\partial y_2}{\partial x} + y_2 \frac{\partial y_o}{\partial x} + y_1 \frac{\partial y_1}{\partial x},$$

$$\vdots$$

(41)

Then using (5) the classical Bernstein polynomials of $g(x,t)$ when v=m=6 is

$$g_b(x,t) = 2.003859xt + 0.103475xt^2 + 0.149954xt^3$$
$$- 0.27935xt^4 + \cdots,$$

(42)

and modified Bernstein polynomials (21) of $g(x,t)$ with k=2 are

$$g_{mb}(x,t) = 1.986611xt + 0.045744xt^2$$
$$+ 0.504281xt^3 - 0.25375xt^4 + \cdots.$$

(43)

By (22) with $v = 1$, we have

$$y_0 = L^{-1}(g_{mb}(x,t)) + y(x,0) = 0.993306xt^2$$
$$+ 0.015248xt^3 + 0.12607xt^4 - 0.050750xt^5 \ldots,$$

$$y_1 = -L^{-1}\left(-v\frac{\partial^2 y_0}{\partial x^2}\right) - L^{-1}(A_0) = -0.197331xt^5$$
$$- 0.005049xt^6 - 0.035812xt^7 + 0.012122xt^8$$
$$+ \cdots,$$

$$y_2 = -L^{-1}\left(-v\frac{\partial^2 y_1}{\partial x^2}\right) - L^{-1}(A_1) = 0.049003xt^8$$
$$+ 0.001783xt^9 + 0.012105xt^{10} - 0.003795xt^{11}$$
$$+ \cdots,$$

$$\vdots$$

(44)

And we obtain

$$y_{mb}(x,t) = \sum_{i=0}^{6} y_i$$
$$= 0.993306xt^2 + 0.015248xt^3$$
$$+ 0.126070xt^4 - 0.248081xt^5$$
$$- 0.083749xt^6 + \cdots,$$

(45)

The absolute error of $y_{mb}(x,t)$ and $y_b(x,t)$ is presented in Table 4 and Figure 4 with the exact solution $y(x,t) = x\sin(t^2)$.

Also Figure 4 presents the absolute error of ADM with Bernstein polynomial in (a) and ADM with modified Bernstein polynomial in (c) at m=v=6 and k=2. The absolute errors generated using the ADM with Bernstein polynomial are of 10^{-3} while the errors yielded from ADM with modified Bernstein polynomial are of 10^{-4}.

5. Conclusions

In this paper, we show that utilizing modified Bernstein polynomials is smartly thought to modify the performance

TABLE 4: Comparison of absolute errors using y_6 when m=v=6, k=2, and x=0.1.

| t	$\lvert y_{exact} - y_b \rvert$	$\lvert y_{exact} - y_{mb} \rvert$
0	0	0
0.1	5.512102 E-6	4.149093 E-6
0.2	3.401160 E-5	1.849504 E-6
0.3	8.851342 E-5	2.838128 E-5
0.4	1.404426 E-5	9.121566 E-5
0.5	1.256008 E-4	1.620426 E-4
0.6	4.250973 E-5	1.880417 E-4
0.7	4.360788 E-4	1.086168 E-4
0.8	1.050197 E-3	1.020235 E-4
0.9	1.718854 E-3	3.752218 E-4
1	2.024768 E-3	4.863297 E-4
MSE	8.393551358 E-7	4.702782622 E-8
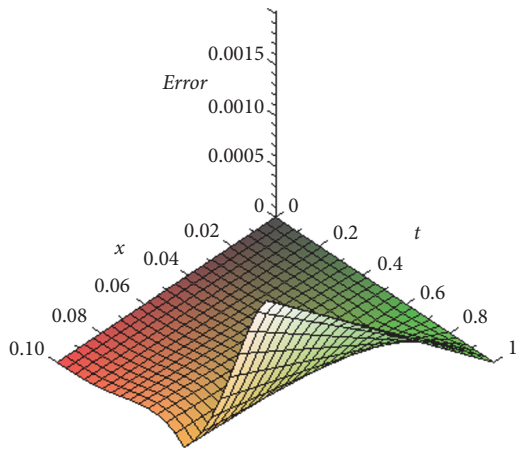
(a) The absolute error for classical Bernstein polynomials
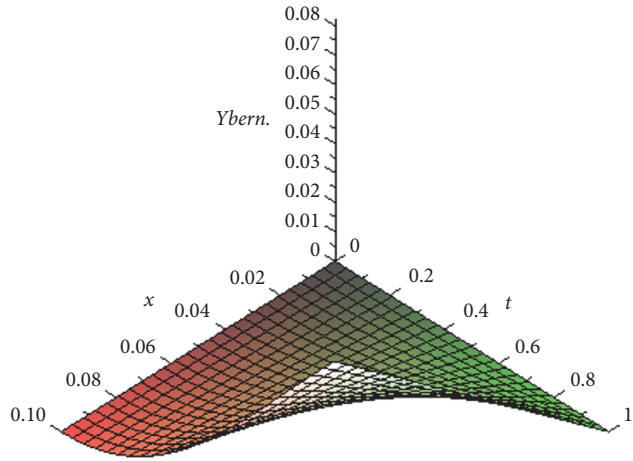
(b) The numerical solution for classical Bernstein polynomials
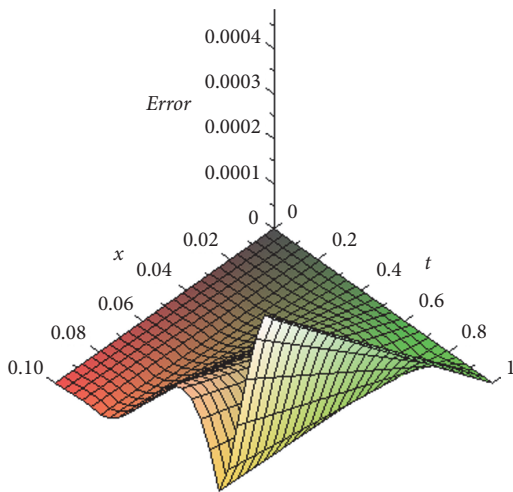
(c) The absolute error for modified Bernstein polynomials
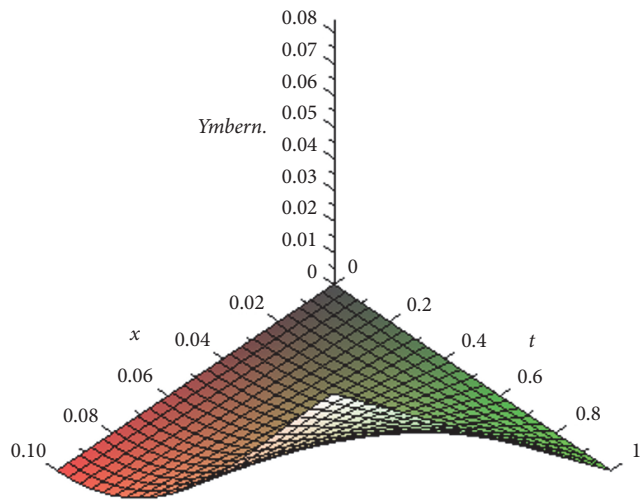
(d) The numerical solution for modified Bernstein polynomials

FIGURE 4: The absolute error between ADM with classical and modified Bernstein polynomials and the exact solution using y_6 when m=v=6, k=2, and x=0.1.

of the Adomian decomposition technique. The fundamental preferred standpoint of this strategy is that it can be used specifically for all sort of differential and integral equations.

We utilize modified Bernstein extensions of the nonlinear term to get more exact outcomes. Figures empower us to consider the difference between utilizing two strategies graphically. Tables are additionally given to demonstrate the variety of the outright mistakes for bigger estimation, to be specific for bigger m. We observed from the numerical outcomes in Tables 1–4 and Figures 1–4 that the ADM with modification Bernstein polynomials gives more exact and robust numerical solution than the classical Bernstein polynomials. Every one of the calculations was done with the guide of Maple 13 programming.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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