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Research Article

Bilinear Form and Two Bäcklund Transformations for the (3+1)-Dimensional Jimbo-Miwa Equation

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With Bell polynomials and symbolic computation, this paper investigates the (3+1)-dimensional Jimbo-Miwa equation, which is one of the equations in the Kadomtsev-Petviashvili hierarchy of integrable systems. We derive a bilinear form and construct a bilinear Bäcklund transformation (BT) for the (3+1)-dimensional Jimbo-Miwa equation, by virtue of which the soliton solutions are obtained. Bell-polynomial-typed BT is also constructed and cast into the bilinear BT.

1. Introduction

Dynamical systems, such as those for the shallow waters [1, 2], plasmas and optical fiber communications [3–6], can often be described by the nonlinear evolution equations (NLEEs) [7–9] and studied by the relevant methods including the inverse scattering [1], Bäcklund transformation (BT) [10–13], and Hirota method [14–16]. Among them, the Hirota method [17, 18] is a direct tool for dealing with certain NLEEs and relevant soliton problems [19, 20]. Based on the bilinear form of a given NLEE, one can obtain the multisoliton solutions [21], bilinear auto-BTs [18], nonlinear superposition formulas, Lax pair, Wronskian formulation [22], and so on [23].

Reflecting the complex nonlinear phenomena in our real world [24–26], higher-dimensional NLEEs with their analytic solutions and integrable properties [27–29] have been of great interest. In fact, some (2 + 1)-dimensional NLEEs have been investigated with different methods, for example, the (2 + 1)-dimensional breaking soliton equation, Kadomtsev-Petviashvili equation, and (2 + 1)-dimensional Kaup-Kupershmidt equation [30–32]. However, for some (3+1)-dimensional NLEEs, the conventional integrability test fails [27], and then a natural problem is whether or not there exists BT for a given (3+1)-dimensional NLEE. Moreover, for the higher-dimensional NLEEs, finding a bilinear BT via the exchange formula is often difficult, even if possible [18, 21].

In this paper, we will study the following (3 + 1)-dimensional Jimbo-Miwa (JM) equation [32]:

$$u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2u_{yt} - 3u_{xz} = 0, \tag{1}$$

where u is a real scalar function with four independent variables x, y, z, and t and the subscripts denote the corresponding partial derivatives. Seen as one of the equations in the Kadomtsev-Petviashvili hierarchy of integrable systems [32, 33], (1) describes certain (3 + 1)-dimensional waves [13, 32] but does not have the Painlevé property [34] as defined in [35]. The soliton [36, 37], periodic [15], rational, and dromion solutions [38, 39] for (1) have been obtained. BTs and analytic solitonic solutions have been given in [13] with the truncated Painlevé expansion at the constant level term.

However, existing literature has not studied the bilinear BT and Bell-polynomial-typed BT of (1) as yet. Therefore, in this paper, by means of the Bell polynomials and Hirota bilinear method, we will obtain two BTs for (1), which are different from those in [13]. In Section 2, we will introduce some concepts on the Bell polynomials and their connection with the bilinear forms. In Section 3, using the Bell-polynomial expressions, we will derive a bilinear form of (1). In Section 4, based on this bilinear form, we will obtain a bilinear BT with soliton solutions and a Bell-polynomial-typed BT. Finally, our conclusions will be given in Section 5.

2. Preliminaries

Suppose that φ is C^∞-function with respect to x, and set $\varphi_{\theta x} = \partial_x^\theta \varphi$ ($\theta = 0, 1, 2, \ldots$). Then the Bell exponential polynomials are given as [40–43]

$$Y_{nx}(\varphi) \equiv Y_n(\varphi_{1x}, \varphi_{2x}, \ldots, \varphi_{nx}) = e^{-\varphi}\partial_x^n e^{\varphi}, \tag{2}$$

where $n = 1, 2, \ldots$.

For example,

$$\begin{aligned} Y_{1x} &= \varphi_{1x}, \\ Y_{2x} &= \varphi_{2x} + \varphi_{1x}^2, \\ Y_{3x} &= \varphi_{3x} + 3\varphi_{1x}\varphi_{2x} + \varphi_{1x}^3, \ldots. \end{aligned} \tag{3}$$

Two-dimensional Bell polynomials are expressed as [40–43]

$$\begin{aligned} &Y_{mx,nt}(\varphi) \equiv Y_{m,n}(\varphi_{1x,0t}, \varphi_{0x,1t}, \ldots, \varphi_{rx,st}, \ldots, \varphi_{mx,nt}) = e^{-\varphi}\partial_x^m \partial_t^n e^{\varphi}, \\ &\varphi_{rx,st} = \partial_x^r \partial_t^s \varphi(x,t), \;\; (m = 1, 2, \ldots;\; r = 0, 1, \ldots, m;\; s = 0, 1, \ldots, n) \end{aligned} \tag{4}$$

with φ hereby being C^∞-function of x and t.

Based on the Bell polynomials given above, the binary Bell polynomials, namely, $\mathscr{Y}$-polynomials, can be defined as [41]

$$\begin{aligned} \mathscr{Y}_{mx,nt}(v, w) &\equiv Y_{mx,nt}[\varphi(v, w)] = Y_{m,n}(\varphi_{1x,0t}, \varphi_{0x,1t}, \\ &\ldots, \varphi_{rx,st}, \ldots, \varphi_{mx,nt})\Big|_{\varphi_{rx,st} = \begin{cases} v_{rx,st}, & \text{if } r+s \text{ is odd,} \\ w_{rx,st}, & \text{if } r+s \text{ is even,} \end{cases}} \end{aligned} \tag{5}$$

where the vertical line means that the elements on the left-hand side are chosen according to the rule on the right-hand side, while v and w are the functions that replace φ in the corresponding positions of the Bell polynomials. For simplicity, we denote $\mathscr{Y}_{mx,nt}(v, w)$ as $\mathscr{Y}_{mx}(v, w)$ or $\mathscr{Y}_{nt}(v, w)$ if $n = 0$ or $m = 0$, respectively.

As one special kind of $\mathscr{Y}$-polynomials, P-polynomials only possess the even-order partial differential terms and, with $q = w - v$, are defined as [40, 41]

$$\begin{aligned} P_{mx,nt}(q) &\equiv Y_{mx,nt}[\varphi(0, q)] = Y_{m,n}(\varphi_{1x,0t}, \varphi_{0x,1t}, \ldots, \\ &\varphi_{rx,st}, \ldots, \varphi_{mx,nt})\Big|_{\varphi_{rx,st} = \begin{cases} 0, & \text{if } r+s \text{ is odd,} \\ q_{rx,st}, & \text{if } r+s \text{ is even,} \end{cases}} \end{aligned} \tag{6}$$

which vanish unless $n + m$ is even.

According to the above, the lower-order P-polynomials can be given as

$$\begin{aligned} P_0(q) &= 1, \\ P_{2x}(q) &= q_{2x}, \\ P_{x,t}(q) &= q_{xt}, \\ P_{4x}(q) &= q_{4x} + 3q_{2x}^2, \ldots. \end{aligned} \tag{7}$$

For a given pair of exponentials,

$$\begin{aligned} F &= \exp f(x, t), \\ G &= \exp g(x, t), \end{aligned} \tag{8}$$

where f and g are C^∞-functions of x and t, while the Hirota D-operators are defined as [17, 42, 43]

$$\begin{aligned} D_x^m D_t^n F \cdot G &\equiv \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^n F(x, t) \\ &\times G(x', t')\Big|_{x'=x, t'=t}, \end{aligned} \tag{9}$$

where x', t' are the formal variables.

It has been found that there exist some relations between the binary Bell polynomials and the Hirota D-operators [40, 41]. When $v = \ln(F/G)$, $w = \ln FG$, a binary Bell polynomial can be transformed into a bilinear term according to the identity [40, 41]

$$(FG)^{-1} D_x^n D_t^m F \cdot G = \mathscr{Y}_{nx,mt}\left(v = \ln\frac{F}{G}, w = \ln FG\right). \tag{10}$$

Likewise, when $w = 2\ln G$, P-polynomials can be associated with the Hirota D-operators according to the identity [40, 41]

$$\begin{aligned} (G)^{-2} D_x^n D_t^m G \cdot G &= \mathscr{Y}_{nx,mt}(v = 0, w = 2\ln G) \\ &= P_{mx,nt}(q). \end{aligned} \tag{11}$$

3. Bilinear Form

We will next investigate (1), to be written in P-polynomial form with one independent variable. Based on the relation between the binary Bell polynomials and Hirota bilinear operators, namely, identities (10) and (11), (1) can be translated into the corresponding bilinear forms.

Consider the following scale transformations:

$$\begin{aligned} x &\longrightarrow \lambda^k x', \\ y &\longrightarrow \lambda^l y', \\ z &\longrightarrow \lambda^\alpha z', \\ t &\longrightarrow \lambda^\beta t', \\ u &\longrightarrow \lambda^\mu u', \end{aligned} \tag{12}$$

where λ, k, l, α, β, and μ are the real constants. Invariance of (1) under such transformations requires that $\mu = -k$, $\alpha = l + 2k$, and $\beta = 3k$.

Notice that if we require that $\mu = -k$, we have to set $u = cq_x$ in (1) and obtain

$$q_{4x,y} + 3cq_{xy}q_{3x} + 3cq_{2x}q_{2x,y} + 2q_{xyt} - 3q_{2x,z} = 0, \tag{13}$$

where c is an arbitrary constant.

In order to express (13) with P-polynomials, we choose $c = 1$. Then

$$\left[P_{3x,y}(q) + 2P_{yt}(q) - 3P_{x,z}(q)\right]_x = 0, \quad (14)$$

whose corresponding bilinear form is

$$\left(D_x^3 D_y + 2D_y D_t - 3D_x D_z\right) G \cdot G = 0. \quad (15)$$

Therefore, we get the bilinear form of (1), which is (14) with P-polynomials or (15) with the bilinear operators. We note that (15) is the same as that in [15], but the method that we used is different from that in [15].

4. Bell-Polynomial-Typed BT and Bilinear BT with Soliton Solutions

To construct a BT, we express (1) with P-polynomials:

$$E(q) = P_{3x,y}(q) + 2P_{yt}(q) - 3P_{x,z}(q). \quad (16)$$

Based on $E(q)$ we will derive the Bell-polynomial-typed BT under the homogenous constraints between the primary and replica fields instead of using exchange formulae.

Using the Bell polynomials, we have

$$\begin{aligned} c(v,w) &= E\left(q'\right) - E(q)\big|_{q=2\ln G,\ q'=2\ln F} = E(w+v) \\ &- E(w-v)\big|_{w=\ln FG,\ v=\ln(F/G)} = (w+v)_{3x,y} \\ &+ 3(w+v)_{xy}(w+v)_{2x} + 2(w+v)_{yt} - 3(w+v)_{xz} \\ &- \left[(w-v)_{3x,y} + 3(w-v)_{xy}(w-v)_{2x}\right. \\ &\left.+ 2(w-v)_{yt} - 3(w-v)_{xz}\right] = 2v_{3x,y} + 6w_{xy}v_{xx} \\ &+ 6v_{xy}w_{xx} + 4v_{yt} - 6v_{xz}. \end{aligned} \quad (17)$$

Note that

$$\begin{aligned} \mathscr{Y}_{2x,y} &= v_{2x,y} + v_y w_{2x} + 2v_x w_{xy} + v_y v_x^2, \\ \mathscr{Y}_{3x} &= v_{3x} + 3v_x w_{2x} + v_x^3. \end{aligned} \quad (18)$$

Therefore, substituting (18) into (17), we have

$$\begin{aligned} \frac{c(v,w)}{2} &= v_{3x,y} + 3w_{xy}v_{xx} + 3v_{xy}w_{xx} + 2v_{yt} - 3v_{xz} \\ &= \left[av_{3x,y} + (1-a)v_{3x,y}\right] + 2v_{yt} - 3v_{xz} \\ &\quad + 3w_{xy}v_{2x} + 3v_{xy}w_{2x} \\ &= \left(a\mathscr{Y}_{2x,y}\right)_x - a\left(v_y w_{2x} + 2v_x w_{xy} + v_y v_x^2\right)_x \\ &\quad + (1-a)\left(\mathscr{Y}_{3x}\right)_y \\ &\quad - (1-a)\left(3v_x w_{2x} + v_x^3\right)_y + 2\left(\mathscr{Y}_t\right)_y \\ &\quad - 3\left(\mathscr{Y}_z\right)_x + 3w_{xy}v_{2x} + 3v_{xy}w_{2x} \\ &= \left(a\mathscr{Y}_{2x,y} - 3\mathscr{Y}_z\right)_x + \left[(1-a)\mathscr{Y}_{3x} + 2\mathscr{Y}_t\right]_y \\ &\quad + A, \end{aligned} \quad (19)$$

where

$$\begin{aligned} A &= 3w_{xy}v_{2x} + 3v_{xy}w_{2x} \\ &\quad - a\left(v_y w_{2x} + 2v_x w_{xy} + v_y v_x^2\right)_x \\ &\quad - (1-a)\left(3v_x w_{2x} + v_x^3\right)_y, \end{aligned} \quad (20)$$

and a is an arbitrary constant.

Further computation shows that

$$\begin{aligned} A &= 2av_{xy}w_{2x} + (a-3)v_x w_{2x,y} + (2a-3)v_x^2 v_{xy} \\ &\quad - (2a-3)v_{2x}w_{xy} - av_y w_{3x} - 2av_x v_y v_{2x} \\ &= 2a\left(v_x\mathscr{Y}_{2x}\right)_y + \left(-2a - \frac{3}{2}\right)v_x\left(\mathscr{Y}_{2x}\right)_y \\ &\quad - av_y\left(\mathscr{Y}_{2x}\right)_x \\ &\quad + \left[\left(a - \frac{3}{2}\right)v_x w_{2x,y} + (-2a+3)v_{2x}w_{xy}\right]. \end{aligned} \quad (21)$$

Hereby, if we choose $a = 3/2$ and set $\mathscr{Y}_{2x} = \sigma$, then

$$\begin{aligned} A &= 2a\left(v_x\mathscr{Y}_{2x}\right)_y = 3\tau v_{xy}, \\ c(v,w) &\\ &= 2\left\{\left(a\mathscr{Y}_{2x,y} - 3\mathscr{Y}_z\right)_x + \left[(1-a)\mathscr{Y}_{3x} + 2\mathscr{Y}_t\right]_y\right\} \\ &\quad + 6\sigma v_{xy}. \end{aligned} \quad (22)$$

Hence,

$$\begin{aligned} \frac{c(v,w)}{2} &= \left[\frac{3}{2}\mathscr{Y}_{2x,y} - 3\mathscr{Y}_z + (3-b)\sigma\mathscr{Y}_y\right]_x \\ &\quad + \left(-\frac{1}{2}\mathscr{Y}_{3x} + 2\mathscr{Y}_t + b\sigma\mathscr{Y}_x\right)_y. \end{aligned} \quad (23)$$

Moreover, a decomposition of (23) leads to the following Bell-polynomial-typed BT:

$$\begin{aligned} \mathscr{Y}_{2x} &= \sigma, \\ \frac{3}{2}\mathscr{Y}_{2x,y} - 3\mathscr{Y}_z + (3-b)\sigma\mathscr{Y}_y &= \tau, \\ -\frac{1}{2}\mathscr{Y}_{3x} + 2\mathscr{Y}_t + b\sigma\mathscr{Y}_x &= \delta, \end{aligned} \quad (24)$$

where b, σ, τ, and δ are the arbitrary constants.

Using the connection between the Bell polynomials and bilinear operators, we give a bilinear BT between G and G' as

$$\begin{aligned} D_x^2 G' \cdot G &= \sigma G' \cdot G, \\ \left[\frac{3}{2}D_x^2 D_y - 3D_z + (3-b)\sigma D_y\right] G' \cdot G &= \tau G' \cdot G, \\ \left(-\frac{1}{2}D_x^3 + 2D_t + b\sigma D_x\right) G' \cdot G &= \delta G' \cdot G. \end{aligned} \quad (25)$$

As an application, we derive the one-soliton solutions from a trivial solution, by virtue of the bilinear BT, that is, (25). Taking $\tau = 0$, $\delta = 0$, and $G' = 1$ in (25), we get

$$G_{xx} = \sigma G, \tag{26}$$

$$\frac{3}{2}G_{xxy} - 3G_z + (3-b)\,\sigma G_y = 0, \tag{27}$$

$$-\frac{1}{2}G_{3x} + 2G_t + b\sigma G_x = 0. \tag{28}$$

Substituting (26) into (28), we have

$$\left(b - \frac{1}{2}\right) G_{3x} + 2G_t = 0, \tag{29}$$

and hence we take

$$G = e^{\xi} + e^{-\xi}, \tag{30}$$

where $\xi = kx+ly+mz+\omega t+\xi^{(0)}$ and $\xi^{(0)}$ is a nonzero constant. On the other hand, we can choose $\sigma = k^2$ and substitute it into (26) and (29), which implies that

$$\omega = \left(\frac{1}{4} - \frac{b}{2}\right) k^3. \tag{31}$$

Similarly, substituting (26) into (27) leads to

$$\left(\frac{9}{2} - b\right) k^2 G_y = 3G_z. \tag{32}$$

Solving (32), we obtain

$$m = \left(\frac{3}{2} - \frac{b}{3}\right) k^2 l. \tag{33}$$

Finally, we can present the one-soliton solutions of (1) as

$$u = 2\,(\ln G)_x = 2k \tanh \xi, \tag{34}$$

where $\xi = kx+ly+(3/2-b/3)k^2lz+(1/4-b/2)k^3t+\xi^{(0)}$, while the parameters k, l, b, and $\xi^{(0)}$ are all arbitrary constants.

5. Discussions and Conclusions

We have investigated the (3 + 1)-dimensional Jimbo-Miwa equation, that is, (1). With the aid of the Bell polynomials and Hirota bilinear operators, we have derived bilinear form (16) of (1) and then constructed a new BT, that is, (25), with the Bell polynomials and symbolic computation. The bilinear form and BT are important integrable property for the nonlinear evolution equations. Moreover, a BT often can be cast into the Lax pair for integrable equations. It may be possible to construct the bilinear BTs for the (3 + 1)-dimensional Jimbo-Miwa equation via the exchange formulae; however, the computation is tedious. Bell-polynomial-typed BTs (24) have been constructed hereby and then cast into bilinear BTs (25), which help us avoid the difficulties in using the exchange formulae. As an application, one-soliton solutions (34) have been obtained via BT (25). The existence of solution obtained via solving this BT indicates that (24) or (25) are genuine ones.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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