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Research Article

Decentralized H_∞ Control for Uncertain Interconnected Systems of Neutral Type via Dynamic Output Feedback

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The design of the dynamic output feedback H_∞ control for uncertain interconnected systems of neutral type is investigated. In the framework of Lyapunov stability theory, a mathematical technique dealing with the nonlinearity on certain matrix variables is developed to obtain the solvability conditions for the anticipated controller. Based on the corresponding LMIs, the anticipated gains for dynamic output feedback can be achieved by solving some algebraic equations. Also, the norm of the transfer function from the disturbance input to the controlled output is less than the given index. A numerical example and the simulation results are given to show the effectiveness of the proposed method.

1. Introduction

With the development of engineering systems, nowadays the systems become more and more complex and large. Therefore, there has been a growing interest in investigating the stability and stabilization problems for the large-scale interconnected systems [1–12]. In [5], Schuler et al. address a design of structured controllers for networks of interconnected multivariable discrete-time subsystems, in which a so-called degree of decentralization is introduced to characterize the sparsity level of the controller. In [6], Chen et al. consider the stabilization and H_∞ disturbance attenuation problem for uncertain interconnected networked systems with both quantised output signal and quantised control inputs signal. A local-output dependent strategy is proposed to update the parameters of quantisers and achieve the H_∞ disturbance attenuation level. In [7], Yan et al. consider the global decentralized stabilization of a class of interconnected systems with known and uncertain interconnections. Based on the Razumikhin-Lyapunov approach, they design a composite sliding surface and analyze the stability of the associated sliding motion, which is governed by a time delayed interconnected system. Not invoking the Lyapunov-Krasovskii functional approach and the Razumikhin Theorem approach, Ye provides a new method to globally stabilize a class of nonlinear large-scale systems with constant time-delay in [8], in which the Nussbaum gain is employed to tackle the unknown high-frequency-gain sign in the considered systems. Hua et al. investigate the model reference adaptive control problem and the exponential stabilization problem for a class of large-scale systems with time-varying delays in [9, 10], respectively. Different from the constraint on the derivatives of time-varying delays in [9, 10], Wu in [11] relaxes the constraint, that is, the derivatives of time-varying delays does not have to be less than one. It is worth pointing out that the nonlinear interconnections are subject to the matched condition in [9, 10] and the time-varying delays only appear in the interconnection in [11].

On the other hand, time delay frequently occurs in many engineering systems, such as the state, input, or related variable of dynamic systems [13, 14]. In particular, when it arises in the state derivative, the considered systems are called as neutral systems [15]. Neutral system is the general form of delay system and contains the same highest order derivatives for the state vector $x(t)$, at both time t and past time(s) $t_s \leq t$. Due to the extensive applications of the neutral systems, in recent years, many efforts have been made for the stability analysis and control problem for neutral systems [16–22]. In [16], Xiong et al. construct a new class of stochastic Lyapunov-Krasovskii functionals to investigate the stability of neutral Markovian jump systems in the case of partly known transition probabilities. In [17], the Lyapunov-Krasovskii functional containing novel triple integral terms is developed to study the robust stabilization for a class of uncertain neutral system with discrete and distributed time delays. Based on the state feedback controller, an improved robust stability and stabilization criteria depending on the allowable maximum delay are derived. In [18], Kwon et al. propose a few delay-dependent stability criteria for uncertain neutral systems with time-varying delays, in which the augmented Lyapunov-Krasovskii functional is constructed and the reciprocal convex optimization approach is introduced. In [19], the delay-dependent exponential stability and stabilisation problems are investigated for a class of special neutral systems with actuator failures. A class of switching laws incorporating the average dwell time method is proposed to robustly stabilise the closed-loop system.

In practice, it is not always possible to have full access to the state variables and only the partial information through a measured output is available [23]. Therefore, it is more realistic in control engineering to design the output feedback control for the considered systems and there is a growing interest in it [24–29]. However, to the authors' best knowledge, there is little literature on designing dynamic output feedback control for interconnected systems of neutral type. This motivates the present study.

In this paper, the H_∞ control problem for uncertain interconnected systems of neutral type is investigated via decentralized dynamic output feedback. Based on the Lyapunov stability theory, we develop a new technique to deal with the nonlinearity problem of certain matrix variables appearing in the solvable conditions of dynamic output feedback H_∞ control. Furthermore, the parameterized characterization of the anticipated controller is achieved, which can be obtained by solving the corresponding LMIs and computing the corresponding algebraic equations. Also, it is guaranteed that the norm of the transfer function from the disturbance input to the controlled output is less than the given index. Finally, the effectiveness of the proposed method is elucidated by a numerical example and the simulation results.

2. Problem Formulation

Consider the following uncertain neutral interconnected systems composed of N subsystems:

$$\begin{aligned}\dot{x}_i(t) - A_{i\eta_i}\dot{x}_i(t-\eta_i(t)) &= [A_i + \Delta A_i(t)]\,x_i(t) \\ &\quad + [A_{i\sigma_i} + \Delta A_{i\sigma_i}(t)]\,x_i(t-\sigma_i(t)) \\ &\quad + B_{i1}\omega_i(t) + \sum_{j=1, j\neq i}^{N}[A_{ij} + \Delta A_{ij}]\,x_j(t-\tau_{ij}(t)), \\ z_i(t) &= C_{i1}x_i(t) + D_{i11}\omega_i(t), \\ x_i(t) = \phi_i(t), &\quad t \in [-l, 0],\ i = 1, 2, \ldots, N,\end{aligned} \tag{1}$$

where $x_i(t) \in \mathfrak{R}^{n_i}$, $z_i(t) \in \mathfrak{R}^{r_i}$, and $\omega_i(t) \in \mathfrak{R}^{p_i}$ are the state, the controlled output, and the disturbance input of the ith subsystem, respectively. A_i, $A_{i\sigma_i}$, $A_{i\eta_i}$, B_{i1}, A_{ij}, C_{i1}, and D_{i11} are known constant matrices of appropriate dimensions. $\phi_i(t)$ is the initial condition. $\sigma_i(t)$, $\eta_i(t)$, and $\tau_{ij}(t)$ are the time-varying delays. Assume that there exist constants f_{i0}, g_{i0}, l_{i0}, f_i, g_i, l_i, and l satisfying

$$\begin{gathered} 0 \le \sigma_i(t) \le f_{i0}, \qquad 0 \le \eta_i(t) \le g_{i0}, \qquad 0 \le \tau_{ij}(t) \le l_{i0}, \\ \dot{\sigma}_i(t) \le f_i < 1, \qquad \dot{\eta}_i(t) \le g_i < 1, \qquad \dot{\tau}_{ij}(t) \le l_i < 1, \\ l = \max\{f_{i0}, g_{i0}, l_{i0}\}, \quad i, j = 1, 2 \ldots, N,\ j \ne i. \end{gathered} \tag{2}$$

Time-varying parametric uncertainties $\Delta A_i(t)$, $\Delta A_{i\sigma_i}(t)$, and $\Delta A_{ij}(t)$ are assumed to satisfy

$$[\Delta A_i(t)\ \ \Delta A_{i\sigma_i}(t)\ \ \Delta A_{ij}(t)] = D_i F_i(t)\,[E_{i1}\ \ E_{i\sigma_i}\ \ L_{ij}], \tag{3}$$

where matrices D_i, E_{i1}, $E_{i\sigma_i}$, and L_{ij} are constant matrices of appropriate dimensions, and $F_i(t)$ is the unknown matrix function satisfying $F_i^T(t)F_i(t) \le I$, for all $t \ge 0$.

Assumption 1 (see [30]). The matrix $A_{i\eta_i} \ne 0$ and $\|A_{i\eta_i}\| < 1$.

As a general approach of dealing with the retarded argument in the state derivatives, it is assumed often that either there is no unstable neutral root chain or they can first use derivative feedback to assign the unstable neutral root chain to the left-hand side of the complex plane. Also, since $A_{i\eta_i} \ne 0$, it follows form that that the solution of (1) exists and is unique.

Lemma 2 (see [31]). *Given any constant $\varepsilon > 0$ and matrices D, E, and F with compatible dimensions such that $F^T F < I$ then*

$$2x^T DFEy \le \varepsilon x^T DD^T x + \varepsilon^{-1} y^T E^T Ey, \tag{4}$$

for all $x \in \mathfrak{R}^n$, $y \in \mathfrak{R}^n$.

3. Main Result

3.1. Robust H_∞ Performance Analysis

Theorem 3. *For given $\gamma_i > 0$, consider system* (1) *with* (2) *and* (3). *Under the condition of Assumption 1, system* (1) *is robustly asymptotically stable and satisfies $\|T_{z_i\omega_i}\|_\infty < \gamma_i$, if there exist matrices $P_i > 0$, $Q_{i1} > 0$, $Q_{i2} > 0$, $G_{ij} > 0$, and $G_{ji} > 0$ such that the following LMI holds:*

$$\begin{bmatrix} \Gamma_{11}^i & \Gamma_{12}^i & \Gamma_{13}^i & \Gamma_{14}^i & \Gamma_{15}^i & \Gamma_{16}^i & \Gamma_{17}^i & 0 \\ * & \Gamma_{22}^i & \Gamma_{23}^i & \Gamma_{24}^i & 0 & 0 & 0 & 0 \\ * & * & \Gamma_{33}^i & \Gamma_{34}^i & 0 & 0 & 0 & 0 \\ * & * & * & \Gamma_{44}^i & \Gamma_{45}^i & 0 & 0 & \Gamma_{48}^i \\ * & * & * & * & \Gamma_{55}^i & \Gamma_{56}^i & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{5}$$

where

$$\Gamma_{11}^i = P_iA_i + A_i^TP_i + \frac{1}{1-f_i}Q_{i1} + \frac{1}{1-g_i}Q_{i2} + \frac{1}{1-l_j}\sum_{j=1,j\neq i}^{N} G_{ji} + 2E_{i1}^TE_{i1}, \qquad \Gamma_{12}^i = P_iA_{i\sigma_i} + 2E_{i1}^TE_{i\sigma_i},$$
$$\Gamma_{13}^i = \left[P_iA_{i1} + 2E_{i1}^TL_{i1} \ \cdots \ P_iA_{i\,i-1} + 2E_{i1}^TL_{i\,i-1} \ \ P_iA_{i\,i+1} + 2E_{i1}^TL_{i\,i+1} \ \cdots \ P_iA_{iN} + 2E_{i1}^TL_{iN}\right],$$
$$\Gamma_{14}^i = -A_i^TP_iA_{i\eta_i}, \qquad \Gamma_{15}^i = P_iB_{i1}, \qquad \Gamma_{16}^i = C_{i1}^T, \qquad \Gamma_{17}^i = P_iD_i, \qquad \Gamma_{22}^i = -Q_{i1} + 2E_{i\sigma_i}^TE_{i\sigma_i},$$
$$\Gamma_{24}^i = -A_{i\sigma_i}^TP_iA_{i\eta_i}, \qquad \Gamma_{23}^i = \left[2E_{i\sigma_i}^TL_{i1} \ \cdots \ 2E_{i\sigma_i}^TL_{i\,i-1} \ \ 2E_{i\sigma_i}^TL_{i\,i+1} \ \cdots \ 2E_{i\sigma_i}^TL_{iN}\right], \tag{6}$$
$$\Gamma_{33}^i = \operatorname{diag}\left\{-G_{i1} + 2L_{i1}^TL_{i1}, \ldots, -G_{i\,i-1} + 2L_{i\,i-1}^TL_{i\,i-1}, -G_{i\,i+1} + 2L_{i\,i+1}^TL_{i\,i+1}, \ldots, -G_{iN} + 2L_{iN}^TL_{iN}\right\},$$
$$\Gamma_{34}^i = \left[-A_{i\eta_i}^TP_iA_{i1} \ \cdots \ -A_{i\eta_i}^TP_iA_{i\,i-1} \ \ -A_{i\eta_i}^TP_iA_{i\,i+1} \ \cdots \ -A_{i\eta_i}^TP_iA_{iN}\right]^T, \qquad \Gamma_{44}^i = -Q_{i2},$$
$$\Gamma_{45}^i = -A_{i\eta_i}^TP_iB_{i1}, \qquad \Gamma_{48}^i = -A_{i\eta_i}^TP_iD_i, \qquad \Gamma_{55}^i = -\gamma_i^2I, \qquad \Gamma_{56}^i = D_{i11}^T.$$

Proof. Construct the following Lyapunov-Krasovskii functional candidate of the form

$$\begin{aligned}
V(x_t) &= \sum_{i=1}^{N} V_i(x_t) \\
&= \sum_{i=1}^{N}\Bigg\{\left[x_i(t) - A_{i\eta_i}x_i(t-\eta_i(t))\right]^T \\
&\quad \times P_i\left[x_i(t) - A_{i\eta_i}x_i(t-\eta_i(t))\right] \\
&\quad + \frac{1}{1-f_i}\int_{t-\sigma_i(t)}^{t} x_i^T(s)Q_{i1}x_i(s)\,ds \\
&\quad + \frac{1}{1-g_i}\int_{t-\eta_i(t)}^{t} x_i^T(s)Q_{i2}x_i(s)\,ds \\
&\quad + \frac{1}{1-l_i}\sum_{j=1,j\neq i}^{N}\int_{t-\tau_{ij}(t)}^{t} x_j^T(s)G_{ij}x_j(s)\,ds\Bigg\}.
\end{aligned} \tag{7}$$

The time derivative of $V(x_t)$ along the trajectory of system (1) satisfies

$$\begin{aligned}
\dot V(x_t) &= \sum_{i=1}^{N}\dot V_i(x_t) \le \sum_{i=1}^{N}\dot U_i(x_t) \\
&\le \sum_{i=1}^{N}\Bigg\{2\left(x_i(t) - A_{i\eta_i}x_i(t-\eta_i(t))\right)^T \\
&\quad \times P_i\Bigg[\left(A_i + \Delta A_i(t)\right)x_i(t) \\
&\quad + \left(A_{i\sigma_i} + \Delta A_{i\sigma_i}(t)\right) \\
&\quad \times x_i(t-\sigma_i(t)) + B_{i1}\omega_i(t) \\
&\quad + \sum_{j=1,j\neq i}^{N}\left(A_{ij} + \Delta A_{ij}\right)x_j\left(t-\tau_{ij}(t)\right)\Bigg] \\
&\quad + \frac{1}{1-f_i}x_i^T(t)Q_{i1}x_i(t) \\
&\quad + \frac{1}{1-g_i}x_i^T(t)Q_{i2}x_i(t) \\
&\quad - x_i^T(t-\sigma_i(t))Q_{i1}x_i(t-\sigma_i(t)) \\
&\quad - x_i^T(t-\eta_i(t))Q_{i2}x_i(t-\eta_i(t)) \\
&\quad + \frac{1}{1-l_i}\sum_{j=1,j\neq i}^{N}x_j^T(t)G_{ij}x_j(t) \\
&\quad - \sum_{j=1,j\neq i}^{N}x_j^T\left(t-\tau_{ij}(t)\right)G_{ij}x_j\left(t-\tau_{ij}(t)\right)\Bigg\}.
\end{aligned} \tag{8}$$

In view of (3), applying Lemma 2, we obtain the following inequality:

$$\begin{aligned}
&2\left[x_i(t) - A_{i\eta_i}x_i(t-\eta_i(t))\right]^T \\
&\quad \times P_i\Bigg[\Delta A_i(t)x_i(t) + \Delta A_{i\sigma_i}(t)x_i(t-\sigma_i(t)) \\
&\quad + \sum_{j=1,j\neq i}^{N}\Delta A_{ij}x_j\left(t-\tau_{ij}(t)\right)\Bigg] \\
&\le x_i^T(t)P_iD_iD_i^TP_ix_i(t) + 2\alpha_i^T(t)M_i^TM_i\alpha_i(t) \\
&\quad + x_i^T(t-\eta_i(t))A_{i\eta_i}^TP_iD_iD_i^TP_iA_{i\eta_i}x_i(t-\eta_i(t)),
\end{aligned} \tag{9}$$

where

$$\alpha_i(t) = \left[x_i(t)\ \ x_i(t-\sigma_i(t))\ \ x_{i1}(t-\tau_{i1}(t))\ \cdots\ x_{i\,i-1}(t-\tau_{i\,i-1}(t))\ \ x_{i\,i+1}(t-\tau_{i\,i+1}(t))\ \cdots\ x_{iN}(t-\tau_{iN}(t))\ \ x_i(t-\eta_i(t))\right],$$
$$M_i = \left[E_{i1}\ \ E_{i\sigma_i}\ \ L_{i1}\ \cdots\ L_{i\,i-1}\ \ L_{i\,i+1}\ \cdots\ L_{iN}\ \ 0\right]. \quad (10)$$

It follows from (8) and (9) that

$$\dot{V}(x_t) = \sum_{i=1}^{N}\dot{V}_i(x_t) \le \sum_{i=1}^{N}\alpha_i^T(t)\left[\Xi_i + 2M_i^T M_i\right]\alpha_i(t), \quad (11)$$

where

$$M_i = \left[E_{i1}\ \ E_{i\sigma_i}\ \ L_{i1}\ \cdots\ L_{i\,i-1}\ \ L_{i\,i+1}\ \cdots\ L_{iN}\ \ 0\right],$$

$$\Xi_i = \begin{bmatrix} \Xi_{11}^i & P_iA_{i\sigma_i} & \Xi_{13}^i & -A_i^TP_iA_{i\eta_i} \\ * & -Q_{i1} & 0 & -A_{i\sigma_i}^TP_iA_{i\eta_i} \\ * & * & \Xi_{33}^i & \Xi_{34}^i \\ * & * & * & \Xi_{44}^i \end{bmatrix},$$

$$\Xi_{44}^i = -Q_{i2} + A_{i\eta_i}^TP_iD_iD_i^TP_iA_{i\eta_i},$$

$$\Xi_{11}^i = P_iA_i + A_i^TP_i + \frac{1}{1-f_i}Q_{i1} + \frac{1}{1-g_i}Q_{i2} + \frac{1}{1-l_j}\sum_{j=1,j\ne i}^{N}G_{ji} + P_iD_iD_i^TP_i,$$

$$\Xi_{13}^i = \left[P_iA_{i1}\ \cdots\ P_iA_{i\,i-1}\ \ P_iA_{i\,i+1}\ \cdots\ P_iA_{iN}\right],$$

$$\Xi_{33}^i = \operatorname{diag}\left\{-G_{i1},\ldots,-G_{i\,i-1},-G_{i\,i+1},\ldots,-G_{iN}\right\},$$

$$\Xi_{34}^i = \left[-A_{i\eta_i}^TP_iA_{i1}\ \cdots\ -A_{i\eta_i}^TP_iA_{i\,i-1}\ \ -A_{i\eta_i}^TP_iA_{i\,i+1}\ \cdots\ -A_{i\eta_i}^TP_iA_{iN}\right]^T. \quad (12)$$

By the Schur Complement formula, it is easy to see that LMI (5) implies that $\Xi_i+2M_i^TM_i<0$. Then we can obtain that $\dot{V}(t)<0$ for all $\alpha_i(t)\ne 0$ when $\omega_i(t)=0$. Therefore, under the condition of Assumption 1, system (1) is asymptotically stable.

Next, consider the H_∞ performance of system (1) under the zero initial condition. To this end, we introduce the following index:

$$J = \sum_{i=1}^{N}\int_0^\infty\left[z_i^T(t)z_i(t) - \gamma_i^2\omega_i^T(t)\omega_i(t)\right]dt. \quad (13)$$

In view of the zero initial condition, it is easy to obtain that

$$J = \sum_{i=1}^{N}\int_0^\infty\left[z_i^T(t)z_i(t) - \gamma_i^2\omega_i^T(t)\omega_i(t) + \dot{V}_i(x_t)\right]dt + V(x_t)\big|_{t=0} - V(x_t)\big|_{t=\infty}, \quad (14)$$
$$\le \sum_{i=1}^{N}\xi_i^T(t)\left[\Pi_i + 2\overline{M}_i^T\overline{M}_i\right]\xi_i(t),$$

where

$$\Pi_i = \begin{bmatrix} \Pi_{11}^i & P_iA_{i\sigma_i} & \Xi_{13}^i & -A_i^TP_iA_{i\eta_i} & \Pi_{15}^i \\ * & -Q_{i1} & 0 & -A_{i\sigma_i}^TP_iA_{i\eta_i} & 0 \\ * & * & \Xi_{33}^i & \Xi_{34}^i & 0 \\ * & * & * & \Xi_{44}^i & \Pi_{45}^i \\ * & * & * & * & \Pi_{55}^i \end{bmatrix},$$

$$\xi_i = \begin{bmatrix}\alpha_i(t)\\ \omega_i(t)\end{bmatrix}, \qquad \overline{M}_i = \left[M_i\ \ 0\right],$$

$$\Pi_{11}^i = P_iA_i + A_i^TP_i + \frac{1}{1-f_i}Q_{i1} + \frac{1}{1-g_i}Q_{i2} + \frac{1}{1-l_j}\sum_{j=1,j\ne i}^{N}G_{ji} + P_iD_iD_i^TP_i + C_{i1}^TC_{i1}, \quad (15)$$

$$\Pi_{15}^i = P_iB_{i1} + C_{i1}^TD_{i11}, \qquad \Pi_{45}^i = -A_{i\eta_i}^TP_iB_{i1},$$

$$\Pi_{55}^i = -\gamma_i^2I + D_{i11}^TD_{i11}.$$

It is obvious that $\Pi_i + 2\overline{M}_i^T\overline{M}_i < 0$ implies that $J<0$, that is, $\|T_{z_i\omega_i}\|_\infty < \gamma_i$. By the Schur Complement formula, the inequality $\Pi_i + 2\overline{M}_i^T\overline{M}_i < 0$ is equivalent to LMI (5). This completes the proof. □

3.2. H_∞ Output Feedback Synthesis. Consider the following uncertain neutral interconnected systems composed of N subsystems:

$$\begin{aligned}\dot{x}_i(t) - A_{i\eta_i}\dot{x}_i(t-\eta_i(t)) &= \left[A_i + \Delta A_i(t)\right]x_i(t) + \left[A_{i\sigma_i} + \Delta A_{i\sigma_i}(t)\right]x_i(t-\sigma_i(t)) \\ &\quad + B_{i1}\omega_i(t) + \sum_{j=1,j\ne i}^{N}\left[A_{ij} + \Delta A_{ij}\right]x_j\left(t-\tau_{ij}(t)\right) + \left[B_{i2} + \Delta B_{i2}\right]u_i(t),\end{aligned}$$
$$z_i(t) = C_{i1}x_i(t) + D_{i11}\omega_i(t) + D_{i12}u_i(t),$$
$$y_i(t) = C_{i2}x_i(t) + D_{i21}\omega_i(t),$$
$$x_i(t) = \phi_i(t), \quad t\in[-l,0],\ i=1,2,\ldots,N, \quad (16)$$

where $u_i(t) \in \mathfrak{R}^{m_i}$ and $y_i(t) \in \mathfrak{R}^{q_i}$ are the control input and the measurement output. B_{i2}, C_{i2}, D_{i12}, and D_{i21}

are known constant matrices of appropriate dimensions. $\Delta B_{i2}(t)$ is the unknown matrix satisfying $B_{i2}(t) = D_i F_i(t) E_{i2}$, where E_{i2} is the known constant matrix with appropriate dimensions. The other signals are the same with system (1).

Consider the following output feedback controller for system (16):

$$\begin{aligned} \dot{\hat{x}}_i(t) &= A_{iK}\hat{x}_i(t) + B_{iK}y_i(t), \\ u_{iK}(t) &= C_{iK}\hat{x}_i(t), \end{aligned} \tag{17}$$

where $\hat{x}_i(t) \in \mathfrak{R}^{n_i \times n_i}$ is the controller state, and A_{iK}, B_{iK}, and C_{ik} are the gains to be designed.

Then the closed-loop system composed of system (16) with the controller (17) can be written as

$$\begin{aligned} &\dot{\overline{x}}_i(t) - \overline{A}_{i\eta_i}\dot{x}_i(t-\eta_i(t)) \\ &\quad = \left[\overline{A}_i + \Delta\overline{A}_i(t)\right] x_i(t) \\ &\qquad + \left[\overline{A}_{i\sigma_i} + \Delta\overline{A}_{i\sigma_i}(t)\right] x_i(t-\sigma_i(t)) + \overline{B}_{i1}\omega_i(t) \\ &\qquad + \sum_{j=1, j\neq i}^{N} \left[\overline{A}_{ij} + \Delta\overline{A}_{ij}\right] \overline{x}_j\left(t-\tau_{ij}(t)\right), \\ &\quad \overline{z}_i(t) = \overline{C}_{i1}x_i(t) + \overline{D}_{i11}\omega_i(t), \end{aligned} \tag{18}$$

where

$$\begin{gathered} \overline{A}_i = \begin{bmatrix} A_i & B_{i2}C_{ik} \\ B_{iK}C_{i2} & A_{iK} \end{bmatrix}, \qquad \overline{A}_{i\sigma_i} = \begin{bmatrix} A_{i\sigma_i} & 0 \\ 0 & 0 \end{bmatrix}, \\ \overline{A}_{ij} = \begin{bmatrix} A_{ij} & 0 \\ 0 & 0 \end{bmatrix}, \qquad \overline{A}_{i\eta_i} = \begin{bmatrix} A_{i\eta_i} & 0 \\ 0 & 0 \end{bmatrix}, \\ \overline{A}_i(t) = \begin{bmatrix} \Delta A_i(t) & \Delta B_{i2}C_{iK} \\ 0 & 0 \end{bmatrix} = \overline{D}_i F_i(t)\overline{E}_{i1} \\ = \begin{bmatrix} D_i \\ 0 \end{bmatrix} F_i(t)\left[E_{i1} \quad E_{i2}C_{iK}\right], \\ \Delta\overline{A}_{i\sigma_i}(t) = \begin{bmatrix} \Delta A_{i\sigma_i}(t) & 0 \\ 0 & 0 \end{bmatrix} = \overline{D}_i F_i(t)\overline{E}_{i\sigma_i} \\ = \begin{bmatrix} D_i \\ 0 \end{bmatrix} F_i(t)\left[E_{i\sigma_i} \quad 0\right], \\ \Delta\overline{A}_{ij}(t) = \begin{bmatrix} \Delta A_{ij}(t) & 0 \\ 0 & 0 \end{bmatrix} = \overline{D}_i F_i(t)\overline{L}_{ij} \\ = \begin{bmatrix} D_i \\ 0 \end{bmatrix} F_i(t)\left[L_{ij} \quad 0\right], \\ \overline{B}_{i1} = \begin{bmatrix} B_{i1} \\ B_{iK}D_{i21} \end{bmatrix}, \qquad \overline{C}_{i1} = \left[C_{i1} \quad D_{i12}C_{iK}\right], \\ \overline{x}_i(t) = \begin{bmatrix} x_i(t) \\ \hat{x}_i(t) \end{bmatrix}, \qquad \overline{z}_i(t) = z_i(t). \end{gathered} \tag{19}$$

The following theorem presents the solving method of the dynamic H_∞ output feedback controller gains for uncertain neutral interconnected systems (16).

Theorem 4. *For given $\gamma_i > 0$, consider system* (16) *with* (2) *and* (3). *Under the condition of Assumption 1, if there exist matrices $X_i > 0$, $Y_i > 0$, $Q_{i1} > 0$, $Q_{i2} > 0$, $G_{ij} > 0$, $G_{ji} > 0$ and invertible matrices N_i, matrices $\hat{A}_i$, $\hat{B}_i$, $\hat{C}_i$, such that $\Psi_i = \left[\begin{smallmatrix} X_i & I \\ * & Y_i \end{smallmatrix}\right] > 0$ and the following LMI holds,*

$$\Omega_i = \begin{bmatrix} \Omega_{11}^i & \Omega_{12}^i & \Omega_{13}^i & \Omega_{14}^i & \Omega_{15}^i & \Omega_{16}^i & \Omega_{17}^i & \Omega_{18}^i & \Omega_{19}^i & \Omega_{110}^i \\ * & \Omega_{22}^i & \Omega_{23}^i & \Omega_{24}^i & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Omega_{33}^i & \Omega_{34}^i & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Omega_{44}^i & \Omega_{45}^i & \Omega_{46}^i & 0 & 0 & 0 & 0 \\ * & * & * & * & \Omega_{55}^i & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Omega_{66}^i & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Omega_{77}^i & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Omega_{88}^i & 0 & 0 \\ * & * & * & * & * & * & * & * & \Omega_{99}^i & 0 \\ * & * & * & * & * & * & * & * & * & -\frac{1}{2}I \end{bmatrix} < 0 \tag{20}$$

then there exists a dynamic output feedback controller such that the closed-loop system (18) *is asymptotically stable and satisfies $\|T_{z_i\omega_i}\| < \gamma_i$ with $A_{iK} = N_i^{-1}(\hat{A}_i - Y_iA_iX_i - N_iB_{iK}C_{i2}X_i - Y_iB_{i2}C_{iK}M_i^T)M_i^{-T}$, $B_{iK} = N_i^{-1}\hat{B}_i$, $C_{iK} = \hat{C}_iM_i^{-T}$, where*

$$
\begin{aligned}
&M_i = (I - X_iY_i)\, N_i^{-T},\\
&\Omega_{11}^i = \begin{bmatrix} A_iX_i + X_iA_i^T + B_{i2}\widehat{C}_i + \widehat{C}_i^T B_{i2}^T & \widehat{A}_i^T + A_i \\ * & Y_iA_i + A_i^TY_i + \widehat{B}_iC_{i2} + C_{i2}^T\widehat{B}_i^T \end{bmatrix},\\
&\Omega_{12}^i = \begin{bmatrix} A_{i\sigma_i} + 2X_iE_{i1}^TE_{i\sigma_i} + 2\widehat{C}_i^TE_{i2}^TE_{i\sigma_i} & 0 \\ Y_iA_{i\sigma_i} + 2E_{i1}^TE_{i\sigma_i} & 0 \end{bmatrix},\\
&\Omega_{13}^i = \left[\begin{matrix} A_{i1} + 2X_iE_{i1}^TL_{i1} + 2\widehat{C}_i^TE_{i2}^TL_{i1} & 0 & \cdots & A_{i\,i-1} + 2X_iE_{i1}^TL_{i\,i-1} + 2\widehat{C}_i^TE_{i2}^TL_{i\,i-1} & 0 \\ Y_iA_{i1} + 2E_{i1}^TL_{i1} & 0 & \cdots & Y_iA_{i\,i-1} + 2E_{i1}^TL_{i\,i-1} & 0 \end{matrix} \right.\\
&\qquad \left. \begin{matrix} A_{i\,i+1} + 2X_iE_{i1}^TL_{i\,i+1} + 2\widehat{C}_i^TE_{i2}^TL_{i\,i+1} & 0 & \cdots & A_{iN} + 2X_iE_{i1}^TL_{iN} + 2\widehat{C}_i^TE_{i2}^TL_{iN} & 0 \\ Y_iA_{i\,i+1} + 2E_{i1}^TL_{i\,i+1} & 0 & \cdots & Y_iA_{iN} + 2E_{i1}^TL_{iN} & 0 \end{matrix} \right],\\
&\Omega_{14}^i = -\begin{bmatrix} \widehat{A}_i^TA_{i\eta_i} & 0 \\ A_i^TY_iA_{i\eta_i} + C_{i2}^T\widehat{B}_i^TA_{i\eta_i} & 0 \end{bmatrix}, \qquad \Omega_{15}^i = \begin{bmatrix} B_{i1} & X_iC_{i1}^T + \widehat{C}_i^TD_{i12}^T \\ Y_iB_{i1} + \widehat{B}_iD_{i21} & C_{i1}^T \end{bmatrix},\\
&\Omega_{16}^i = \begin{bmatrix} D_i & 0 \\ Y_iD_i & 0 \end{bmatrix}, \qquad \Omega_{17}^i = \Psi_i, \qquad \Omega_{18}^i = \Big[\underbrace{\Psi_i \ \cdots \ \Psi_i \ \Psi_i \ \cdots \ \Psi_i}_{N-1} \Big], \qquad \Omega_{19}^i = \Psi_i,\\
&\Omega_{110}^i = \begin{bmatrix} X_iE_{i1}^T + \widehat{C}_i^TE_{i2}^T \\ E_{i1}^T \end{bmatrix}, \qquad \Omega_{22}^i = -Q_{i1} + \begin{bmatrix} 2E_{i\sigma_i}^TE_{i\sigma_i} & 0 \\ 0 & 0 \end{bmatrix}, \qquad \Omega_{24}^i = -\begin{bmatrix} A_{i\sigma_i}^TY_iA_{i\eta_i} & 0 \\ 0 & 0 \end{bmatrix},\\
&\Omega_{23}^i = \begin{bmatrix} 2E_{i\sigma_i}^TL_{i1} & 0 & \cdots & 2E_{i\sigma_i}^TL_{i\,i-1} & 0 & 2E_{i\sigma_i}^TL_{i\,i+1} & 0 & \cdots & 2E_{i\sigma_i}^TL_{iN} & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix},\\
&\Omega_{33}^i = \operatorname{diag}\Bigg\{ -G_{i1} + \begin{bmatrix} 2L_{i1}^TL_{i1} & 0 \\ 0 & 0 \end{bmatrix}, \ldots, -G_{i\,i-1} + \begin{bmatrix} 2L_{i\,i-1}^TL_{i\,i-1} & 0 \\ 0 & 0 \end{bmatrix},\\
&\qquad -G_{i\,i+1} + \begin{bmatrix} 2L_{i\,i+1}^TL_{i\,i+1} & 0 \\ 0 & 0 \end{bmatrix}, \ldots, -G_{iN} + \begin{bmatrix} 2L_{iN}^TL_{iN} & 0 \\ 0 & 0 \end{bmatrix} \Bigg\},\\
&\Omega_{34}^i = -\begin{bmatrix} A_{i\eta_i}^TY_iA_{i1} & 0 & \cdots & A_{i\eta_i}^TY_iA_{i\,i-1} & 0 & A_{i\eta_i}^TY_iA_{i\,i+1} & 0 & \cdots & A_{i\eta_i}^TY_iA_{iN} & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix},\\
&\Omega_{44}^i = -Q_{i2}, \qquad \Omega_{45}^i = -\begin{bmatrix} A_{i\eta_i}^TY_iB_{i1} + A_{i\eta_i}^T\widehat{B}_iD_{i21} & 0 \\ 0 & 0 \end{bmatrix}, \qquad \Omega_{46}^i = -\begin{bmatrix} 0 & A_{i\eta_i}^TY_iD_i \\ 0 & 0 \end{bmatrix},\\
&\Omega_{55}^i = \begin{bmatrix} -\gamma_i^2I & D_{i11}^T \\ * & -I \end{bmatrix}, \qquad \Omega_{66}^i = \begin{bmatrix} -I & \\ * & -I \end{bmatrix}, \qquad \Omega_{77}^i = Q_{i1} - (1 - f_i)\begin{bmatrix} 2I & Y_i \\ * & N_i + N_i^T \end{bmatrix},\\
&\Omega_{88}^i = \operatorname{diag}\Bigg\{ G_{1i} - (1 - l_1)\begin{bmatrix} 2I & Y_i \\ * & N_i + N_i^T \end{bmatrix}, \ldots, G_{i-1\,i} - (1 - l_{i-1})\begin{bmatrix} 2I & Y_i \\ * & N_i + N_i^T \end{bmatrix},\\
&\qquad G_{i+1\,i} - (1 - l_{i+1})\begin{bmatrix} 2I & Y_i \\ * & N_i + N_i^T \end{bmatrix}, \ldots, G_{Ni} - (1 - l_N)\begin{bmatrix} 2I & Y_i \\ * & N_i + N_i^T \end{bmatrix} \Bigg\},\\
&\Omega_{99}^i = Q_{i2} - (1 - g_i)\begin{bmatrix} 2I & Y_i \\ * & N_i + N_i^T \end{bmatrix}.
\end{aligned}
\tag{21}
$$

Applying Theorem 3 to the closed-loop system (18), then system (18) is robustly asymptotically stable and satisfies $\|T_{z_i\omega_i}\|_\infty < \gamma_i$ under the condition of Assumption 1, if there exist matrices $P_i > 0$, $Q_{i1} > 0$, $Q_{i2} > 0$, $G_{ij} > 0$, and $G_{ji} > 0$ such that the LMI (5) holds, where A_i, $A_{i\sigma_i}$, $A_{i\eta_i}$, B_{i1}, A_{ij}, C_{i1}, D_{i11}, D_i, $E_{i\sigma_i}$, and L_{ij} are substituted with $\overline{A}_i$, $\overline{A}_{i\sigma_i}$, $\overline{A}_{i\eta_i}$, $\overline{B}_{i1}$, $\overline{A}_{ij}$, $\overline{C}_{i1}$, $\overline{D}_{i11}$, $\overline{D}_i$, $\overline{E}_{i\sigma_i}$, and $\overline{L}_{ij}$, respectively.

Firstly, decompose matrix P_i and its inverse as

$$P_i = \begin{bmatrix} Y_i & N_i \\ * & W_i \end{bmatrix}, \qquad P_i^{-1} = \begin{bmatrix} X_i & M_i \\ * & Z_i \end{bmatrix}, \tag{22}$$

where $Y_i, X_i \in \mathfrak{R}^{n_i}$ are positive definite matrices, and M_i and N_i are invertible matrices. According to $P_i^{-1}P_i = I$, we have

$$M_i N_i^T = I - X_i Y_i. \tag{23}$$

Define $F_{i1} = \begin{bmatrix} X_i & I \\ M_i^T & 0 \end{bmatrix}$, $F_{i2} = \begin{bmatrix} I & Y_i \\ 0 & N_i^T \end{bmatrix}$, then it follows that

$$P_i F_{i1} = F_{i2}, \qquad F_{i1}^T P_i F_{i1} = F_{i2}^T F_{i1} = \begin{bmatrix} X_i & I \\ * & Y_i \end{bmatrix} > 0. \tag{24}$$

Next, pre- and postmultiply the substitute of LMI (5) by the matrix

$$\operatorname{diag}\left\{F_{i1}^T, I, I, I, I, I, I\right\} \tag{25}$$

and its transpose, respectively. By the Schur Complement formula, the following LMI can be obtained:

$$\Phi_i = \begin{bmatrix}
\Phi_{11}^i & \Phi_{12}^i & \Phi_{13}^i & \Phi_{14}^i & \Phi_{15}^i & \Phi_{16}^i & \Phi_{17}^i & 0 & \Phi_{19}^i & \Phi_{110}^i & \Phi_{111}^i & \Phi_{112}^i \\
* & \Phi_{22}^i & \Phi_{23}^i & \Phi_{24}^i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & \Phi_{33}^i & \Phi_{34}^i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & \Phi_{44}^i & \Phi_{45}^i & 0 & 0 & \Phi_{48}^i & 0 & 0 & 0 & 0 \\
* & * & * & * & \Phi_{55}^i & \Phi_{56}^i & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -I & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -I & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -I & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & \Phi_{99}^i & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & \Phi_{1010}^i & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & \Phi_{1111}^i & 0 \\
* & * & * & * & * & * & * & * & * & * & * & -\frac{1}{2}I
\end{bmatrix} < 0, \tag{26}$$

where

$$\Phi_{11}^i = F_{i1}^T P_i \overline{A}_i F_{i1} + F_{i1}^T \overline{A}_i^T P_i F_{i1},$$

$$\Phi_{12}^i = F_{i1}^T P_i \overline{A}_{i\sigma_i} + 2F_{i1}^T \overline{E}_{i1}^T \overline{E}_{i\sigma_i},$$

$$\Phi_{13}^i = \left[\, F_{i1}^T P_i \overline{A}_{i1} + 2F_{i1}^T \overline{E}_{i1}^T \overline{L}_{i1} \ \cdots \ F_{i1}^T P_i \overline{A}_{i\,i-1} + 2F_{i1}^T \overline{E}_{i1}^T \overline{L}_{i\,i-1} \right.$$
$$\left. F_{i1}^T P_i \overline{A}_{i\,i+1} + 2F_{i1}^T \overline{E}_{i1}^T \overline{L}_{i\,i+1} \ \cdots \ F_{i1}^T P_i \overline{A}_{iN} + 2F_{i1}^T \overline{E}_{i1}^T \overline{L}_{iN} \,\right],$$

$$\Phi_{14}^i = -F_{i1}^T \overline{A}_i^T P_i \overline{A}_{i\eta_i}, \qquad \Phi_{15}^i = F_{i1}^T P_i \overline{B}_{i1},$$

$$\Phi_{16}^i = F_{i1}^T \overline{C}_{i1}^T, \qquad \Phi_{17}^i = F_{i1}^T P_i \overline{D}_i,$$

$$\Phi_{19}^i = F_{i1}^T, \qquad \Phi_{110}^i = F_{i1}^T,$$

$$\Phi_{111}^i = F_{i1}^T, \qquad \Phi_{112}^i = F_{i1}^T \overline{E}_{i1}^T,$$

$$\Phi_{22}^i = -Q_{i1} + 2\overline{E}_{i\sigma_i}^T \overline{E}_{i\sigma_i}, \qquad \Phi_{24}^i = -\overline{A}_{i\sigma_i}^T P_i \overline{A}_{i\eta_i},$$

$$\Phi_{23}^i = \left[2\overline{E}_{i\sigma_i}^T \overline{L}_{i1} \ \cdots \ 2\overline{E}_{i\sigma_i}^T \overline{L}_{i\,i-1} \ 2\overline{E}_{i\sigma_i}^T \overline{L}_{i\,i+1} \ \cdots \ 2\overline{E}_{i\sigma_i}^T \overline{L}_{iN}\right],$$

$$\Phi_{33}^i = \operatorname{diag}\left\{-G_{i1} + 2\overline{L}_{i1}^T \overline{L}_{i1}, \ \ldots, -G_{i\,i-1} + 2\overline{L}_{i\,i-1}^T \overline{L}_{i\,i-1}, \right.$$
$$\left. -G_{i\,i+1} + 2\overline{L}_{i\,i+1}^T \overline{L}_{i\,i+1}, \ldots, -G_{iN} + 2\overline{L}_{iN}^T \overline{L}_{iN}\right\},$$

$$\Phi_{34}^i = \left[-\overline{A}_{i\eta_i}^T P_i \overline{A}_{i1} \ \cdots \ -\overline{A}_{i\eta_i}^T P_i \overline{A}_{i\,i-1} \ -\overline{A}_{i\eta_i}^T P_i \overline{A}_{i\,i+1} \ \cdots \ -\overline{A}_{i\eta_i}^T P_i \overline{A}_{iN}\right]^T,$$

$$\Phi_{44}^i = -Q_{i2}, \qquad \Phi_{45}^i = -\overline{A}_{i\eta_i}^T P_i \overline{B}_{i1},$$

$$\Phi_{48}^i = -\overline{A}_{i\eta_i}^T P_i \overline{D}_i, \qquad \Phi_{55}^i = -\gamma_i^2 I,$$

$$\Gamma_{56}^i = \overline{D}_{i11}^T, \qquad \Phi_{99}^i = -\left(1 - f_i\right) Q_{i1}^{-1},$$

$$\Phi_{1111}^i = -\left(1 - g_i\right) Q_{i2},$$

$$\Phi_{1010}^i = \operatorname{diag}\left\{-\left(1 - l_1\right) G_{1i}, \ldots, -\left(1 - l_{i-1}\right) G_{i-1\,i}, \right.$$
$$\left. -\left(1 - l_{i+1}\right) G_{i+1\,i}, \ldots, -\left(1 - l_N\right) G_{Ni}\right\}. \tag{27}$$

By Lemma 2, we have

$$\begin{aligned} &-F_{i2}^{T}Q_{i1}^{-1}F_{i2}-Q_{i1}\leq -F_{i2}^{T}-F_{i2},\\ &-F_{i2}^{T}Q_{i2}^{-1}F_{i2}-Q_{i2}\leq -F_{i2}^{T}-F_{i2},\\ &-F_{i2}^{T}G_{ji}^{-1}F_{i2}-G_{ji}\leq -F_{i2}^{T}-F_{i2},\\ &i,j=1,2,\ldots,N,\ \ j\neq i. \end{aligned} \tag{28}$$

Pre- and postmultiplying the inequality (26) by the matrix

$$\operatorname{diag}\left\{I,I,I,I,I,I,I,F_{i2}^{T},F_{i2}^{T},F_{i2}^{T},I\right\}, \tag{29}$$

and its transpose, respectively, and utilizing (28), and denoting

$$\begin{aligned} \widehat{A}_i &= Y_iA_iX_i+N_iB_{iK}C_{i2}X_i+Y_iB_{i2}C_{iK}M_i^T+N_iA_{iK}M_i^T,\\ \widehat{B}_i &= N_iB_{iK}, \qquad \widehat{C}_i = C_{iK}M_i^T, \end{aligned} \tag{30}$$

one can obtain Theorem 4 immediately. This completes the proof.

Algorithm 5. Given any solution of the LMI (20) in Theorem 4, a corresponding controller of the form (17) will be constructed as follows.

(i) Utilizing the two positive definite solutions X_i, Y_i and the invertible matrix N_i; compute the invertible M_i satisfying (23).

(ii) Utilizing the matrices M_i and N_i obtained above; compute the gains A_{iK}, B_{iK}, and C_{iK} according to (30).

4. Illustrative Example

Consider system (16) composed of a three-order subsystem and a two-order subsystem with the following parameters:

$$\begin{aligned}
&A_1=\begin{bmatrix}0.3 & -1.50.8 & \\ -0.9 & -23.5 & 5.6\\ 0.5 & 0.9 & -25.3\end{bmatrix}, \qquad B_{11}=\begin{bmatrix}-0.1 & -0.2\\ -0.3 & 0.2\\ 0.1 & -0.1\end{bmatrix},\\
&B_{12}=\begin{bmatrix}0.2 & 0.5\\ -0.1 & -0.7\\ -0.1 & 0.2\end{bmatrix}, \qquad A_{1\sigma_1}=\begin{bmatrix}-0.1 & 0.3 & -0.1\\ 0.1 & -0.2 & -0.3\\ 0.2 & 0.4 & 0.2\end{bmatrix},\\
&A_{1\eta_1}=\begin{bmatrix}0.1 & -0.3 & -0.1\\ 0.1 & 0.5 & -0.1\\ 0.2 & 0.1 & -0.5\end{bmatrix}, \qquad A_{12}=\begin{bmatrix}-0.1 & 0.1\\ -0.1 & 0.2\\ -0.6 & -0.4\end{bmatrix},\\
&D_1=\begin{bmatrix}0.01 & 0.5 & -0.01\\ -0.1 & 0 & 0\\ 0 & 0.1 & 0\end{bmatrix}, \qquad E_{11}=\begin{bmatrix}-0.1 & -0.1 & 0.1\\ -0.1 & 0.2 & 0.1\\ 0.1 & -0.1 & -0.2\end{bmatrix},\\
&L_{12}=\begin{bmatrix}-0.01 & 0.1\\ 0.01 & -0.2\\ 0.01 & -0.2\end{bmatrix}, \qquad E_{1\sigma_1}=\begin{bmatrix}0.1 & 0.1 & -0.1\\ -0.1 & -0.2 & -0.1\\ -0.1 & -0.1 & 0.1\end{bmatrix},\\
&E_{12}=\begin{bmatrix}-0.1 & -0.3\\ -0.1 & 0.1\\ -0.4 & 0.2\end{bmatrix}, \qquad C_{11}=\begin{bmatrix}-0.4 & -0.1 & 0.1\\ -0.1 & -0.2 & 0.3\end{bmatrix},\\
&D_{111}=\begin{bmatrix}-0.1 & 0.1\\ 0.01 & -0.1\end{bmatrix}, \qquad D_{112}=\begin{bmatrix}0.1 & -0.1\\ -0.1 & 0.3\end{bmatrix},\\
&C_{12}=\begin{bmatrix}-0.1 & -0.1 & -0.1\\ 0.5 & 0.3 & -1.4\end{bmatrix}, \qquad D_{121}=\begin{bmatrix}-0.2 & -0.1\\ 0.1 & 0.1\end{bmatrix},\\
&\sigma_1(t)=0.1(2+\sin(t)), \qquad \eta_1(t)=0.2(1+\cos(t)),\\
&A_2=\begin{bmatrix}-15.1 & 0.1\\ -0.7 & -5.4\end{bmatrix}, \qquad A_{2\sigma_2}=\begin{bmatrix}-0.6 & -0.3\\ -0.4 & 0.1\end{bmatrix},\\
&A_{2\eta_2}=\begin{bmatrix}-0.2 & 0.2\\ 0.1 & -0.1\end{bmatrix}, \qquad B_{21}=\begin{bmatrix}-0.1\\ -0.1\end{bmatrix},\\
&A_{21}=\begin{bmatrix}0.1 & -0.1 & -0.1\\ -0.1 & 0.1 & 0.1\end{bmatrix}, \qquad B_{22}=\begin{bmatrix}0.5\\ 0.1\end{bmatrix},\\
&D_2=\begin{bmatrix}0.1 & 0.4\\ -0.1 & 0.1\end{bmatrix}, \qquad E_{21}=\begin{bmatrix}-0.1 & 0.1\\ 0.2 & -0.1\end{bmatrix},\\
&E_{2\sigma_2}=\begin{bmatrix}-0.1 & -0.1\\ -0.1 & 0.1\end{bmatrix}, \qquad E_{22}=\begin{bmatrix}-0.1\\ 0.1\end{bmatrix},\\
&L_{21}=\begin{bmatrix}-0.01 & 0.01 & -0.01\\ -0.01 & 0.02 & 0.01\end{bmatrix}, \qquad C_{21}=\begin{bmatrix}-0.1 & 0.8\end{bmatrix},\\
&D_{211}=0.13, \qquad D_{221}=-0.53,\\
&C_{22}=\begin{bmatrix}0.1 & 0.5\end{bmatrix}, \qquad D_{212}=0.16,\\
&\sigma_2(t)=0.2(1+\cos(t)),\\
&\eta_2(t)=0.1(2+\cos(t)),\\
&\tau_{21}(t)=0.2(2+\sin(t)),\\
&\tau_{12}(t)=0.1(1+\cos(t)),\\
&\gamma_1=0.5, \qquad \gamma_2=0.3.
\end{aligned} \tag{31}$$

Using the above parameters and applying Matlab Software to solving LMI (20), we can obtain the following results:

$$\begin{aligned}
X_1&=\begin{bmatrix}0.0947 & 0.5569 & -0.1173\\ 0.5569 & 6.1762 & 0.5942\\ -0.1173 & 0.5942 & 7.2277\end{bmatrix},\\
Y_1&=\begin{bmatrix}0.2448 & -0.0612 & 0.2138\\ -0.0612 & 0.1830 & -0.0255\\ 0.2138 & -0.0255 & 0.2034\end{bmatrix},
\end{aligned}$$

$$N_1 = \begin{bmatrix} 1.2558 & -0.0000 & 0.0000 \\ -0.0000 & 1.2558 & -0.0000 \\ 0.0000 & -0.0000 & 1.2558 \end{bmatrix} \times 10^5,$$

$$X_2 = \begin{bmatrix} 4.4043 & 0.4772 \\ 0.4772 & 1.4044 \end{bmatrix}, \qquad Y_2 = \begin{bmatrix} 1.4996 & 1.4523 \\ 1.4523 & 8.4219 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} 1.2075 & 0.0001 \\ 0.0001 & 1.2078 \end{bmatrix} \times 10^5,$$

$$\widehat{A}_1 = \begin{bmatrix} -1.4345 & -2.7721 & 5.7617 \\ -1.2489 & -9.1781 & 1.1119 \\ 0.3361 & 3.1163 & -5.0939 \end{bmatrix},$$

$$\widehat{C}_1 = \begin{bmatrix} -1.6597 & -1.9990 & -0.3384 \\ -1.7543 & 0.5932 & 1.5744 \end{bmatrix},$$

$$\widehat{B}_1 = \begin{bmatrix} -1.8297 & -4.1087 \\ 0.3341 & -0.3560 \\ -0.9167 & -0.6854 \end{bmatrix}, \qquad \widehat{A}_2 = \begin{bmatrix} 0.5829 & -1.2895 \\ 1.5056 & -5.9999 \end{bmatrix},$$

$$\widehat{B}_2 = \begin{bmatrix} -1.1894 \\ -2.1534 \end{bmatrix}, \qquad \widehat{C}_2 = \begin{bmatrix} -10.0067 & -3.6127 \end{bmatrix}. \tag{32}$$

Using the obtained solutions X_1, Y_1, N_1, X_2, Y_2, and N_2 to solve (23), we have

$$M_1 = \begin{bmatrix} 0.0825 & -0.0079 & 0.0014 \\ 0.0091 & -0.0064 & -0.0066 \\ -0.1178 & 0.0054 & -0.0342 \end{bmatrix} \times 10^{-4},$$
$$M_2 = \begin{bmatrix} -0.5215 & -0.8623 \\ -0.2281 & -0.9538 \end{bmatrix} \times 10^{-4}. \tag{33}$$

Using the above solutions M_1, N_1, M_2, and N_2 to compute A_{1K}, B_{1K}, C_{1K}, A_{2K}, B_{2K}, and C_{2K} according to (30), the following results are obtained:

$$A_{1K} = \begin{bmatrix} 5.6151 & 65.5141 & -14.4781 \\ -33.1939 & -343.0572 & 85.8446 \\ 3.9271 & 29.4768 & -64.3063 \end{bmatrix},$$

$$B_{1K} = \begin{bmatrix} -0.1457 & -0.3272 \\ 0.0266 & -0.0283 \\ -0.0730 & -0.0546 \end{bmatrix} \times 10^{-4},$$

$$C_{1K} = \begin{bmatrix} 0.0683 & 2.8751 & 0.3203 \\ -0.4408 & -2.2598 & 0.6995 \end{bmatrix} \times 10^6,$$

$$A_{2K} = \begin{bmatrix} -25.1345 & 3.8472 \\ -24.1933 & -1.0036 \end{bmatrix}, \qquad B_{2K} = \begin{bmatrix} -0.0985 \\ -0.1783 \end{bmatrix} \times 10^{-4},$$

$$C_{2K} = \begin{bmatrix} 2.1380 & -0.1326 \end{bmatrix} \times 10^5. \tag{34}$$

When $F_1(t) = \operatorname{diag}\{\sin(t), \sin(t), \sin(t)\}$ and $F_2(t) = \operatorname{diag}\{\cos(t), \cos(t)\}$, the simulation results are shown in Figures 1–4 based on the above parameters. From Figures 1 and 2, one can see that the uncertain interconnected systems of neutral type (16) without controllers are not convergent. From Figures 3 and 4, one can see that the uncertain interconnected systems of neutral type (16) are indeed well stabilized.

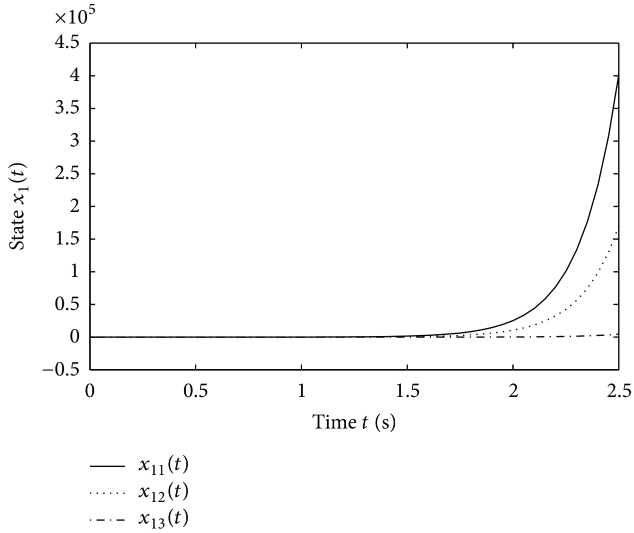

FIGURE 1: State response of the first open-loop subsystem.

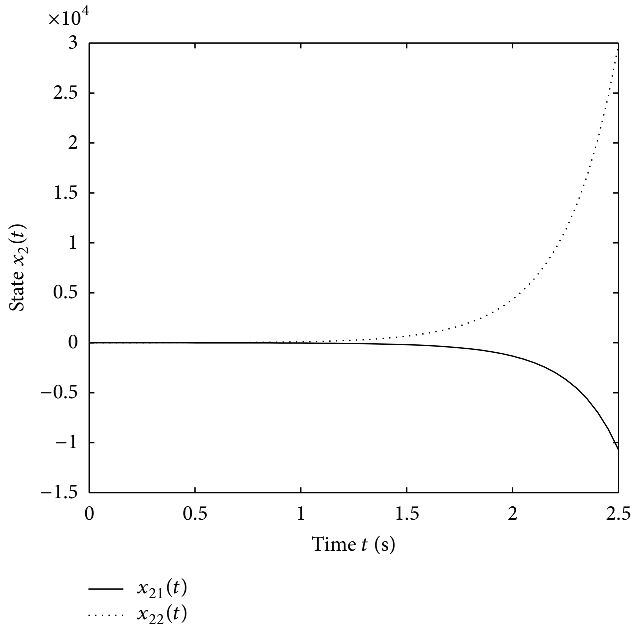

FIGURE 2: State response of the second open-loop subsystem.

5. Conclusion

The H_∞ decentralized control problem via output feedback for uncertain neutral interconnected systems with time-varying delays is complex and challenging. Developing a novel mathematical technique for treating the nonlinear interconnection variable matrices, a sufficient condition of existing anticipated controller is obtained in terms of LMIs based on Lyapunov stability theory, which not only depends on the sizes of delays but also on the information of derivatives. The illustrative example shows that the results obtained in this paper are effective.

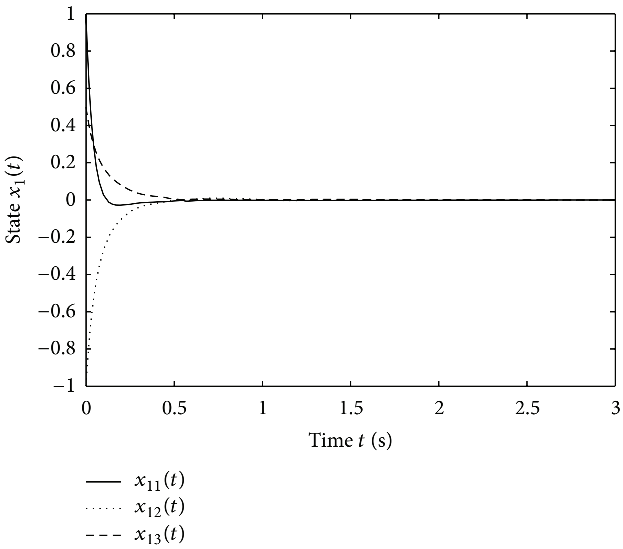

FIGURE 3: State response of the first closed-loop subsystem.

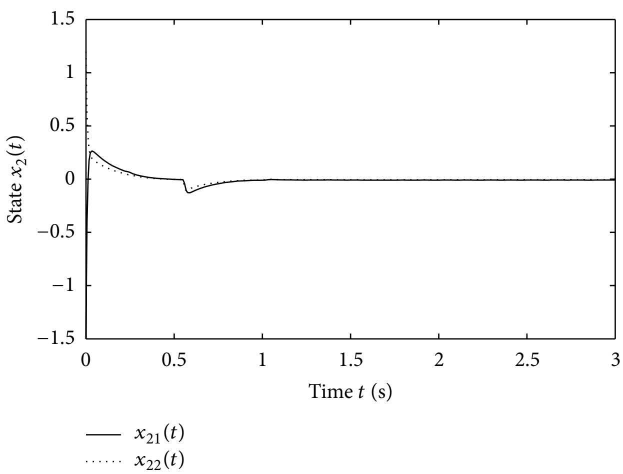

FIGURE 4: State response of the second closed-loop subsystem.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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