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Research Article

Adjacent-Compensation Consensus Algorithm in Asynchronously Coupled Form for Second-Order Multiagent Network under Communication Delay

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General asynchronously coupled consensus algorithm associated with adjacent compensations, is proposed to solve the dynamical consensus problem of second-order multiagent network with communication delay under leader-following coordination control framework. Based on frequency-domain analysis, firstly, delay-independent consensus convergence is proved for the second-order multiagent systems with a spanning tree topology that has the leader root and then delay-dependent consensus condition is obtained for the multiagent systems with communication delay under a general leader-following interconnection topology. Simulation illustrates the correctness of the results.

1. Introduction

Coordination control of multiple autonomous dynamic agents has attracted much attention in recent years for its broad application in automated highway systems, air traffic control, congestion control in the Internet, and so on. Consensus problem, which requires the agents' outputs to reach a common value without central or global communication, is one of the most important and fundamental collective types of behavior in distributed coordination control of multiagent systems and has been extensively studied in many research societies in recent years, such as biology, robotics, and sensor networks.

In the past decade, consensus problem has been thoroughly studied for the multiagent systems with diverse agent's dynamics, such as single integrator, double integrator, and high-order integrators. Different consensus algorithms have been designed [1–6], and many conditions have been obtained for the agents converging to the consensus asymptotically.

Due to the information exchange between neighboring agents, communication delay should be investigated in multiagent network. Subjected to communication delays, consensus algorithms are usually divided into synchronously coupled and asynchronously coupled forms. In synchronously coupled consensus algorithm, self-delays introduced for each agent in the coordination part are equal to the corresponding communication delays. Besides, each agent uses its delayed state with the delay value different from the corresponding communication delay or uses its current state to compare it with its delayed neighboring agents' states in the asynchronously coupled consensus algorithm.

Up to now, synchronously coupled consensus algorithm has been extensively studied for multiagent systems, and delay-dependent consensus conditions have been obtained for the system under fixed or switched interconnection topologies based on frequency-domain analysis [2, 7, 8] and Lyapunov function methods [9–15].

Moreover, asynchronously coupled consensus algorithm has been analyzed thoroughly for the first-order and second-order multiagent systems with stationary consensus algorithm by using many different analysis methods including frequency-domain analysis [16, 17], Lyapunov functions [18–20], and the concept of delayed and hierarchical graphs [21–24]. It has been proved that the agents with the stationary consensus algorithms could achieve an asymptotic consensus with arbitrary communication delay by choosing proper control parameters. However, for second-order multiagent systems with the dynamical consensus algorithm composed of the position and the velocity consensus coordination control parts, Yang et al. [25] obtained sufficient consensus conditions for the second-order dynamic agents with time-varying communication delays based on small-μ stability theorem, and C.-L. Liu and F. Liu [26] got sufficient and necessary consensus condition for two coupled second-order dynamic agents with identical time-invariant communication delay by using frequency-domain analysis. In addition, Münz et al. [27] investigated the multiagent systems with agents' dynamics described by strictly stable linear systems under diverse communication delays. By analyzing the convex sets of the frequency-domain feedback matrix, set-valued consensus conditions have been obtained for the system under synchronously coupled and asynchronously coupled consensus algorithms, respectively [27].

In this paper, a new asynchronously coupled consensus algorithm associated with adjacent compensations is proposed for the second-order multiagent systems with communication delay to track a dynamic leader. The consensus algorithm is composed of the position and the velocity consensus coordination control parts, and the compensations are composed of each agent's neighboring agents' delayed state compensation in the normal asynchronously coupled consensus algorithm. To prove the effectiveness of the proposed algorithm, we firstly investigate the second-order multiagent systems with a spanning tree topology that has the leader as a root, and it is proved that the agents can achieve a dynamical consensus without any relationship to the communication delay. Under the general connected topology that is composed of the agents and the leader, delay-dependent sufficient condition is also obtained for the multiagent systems with communication delay.

2. Problem Formulation

2.1. Agent's Dynamics and Interconnection Topology. Second-order agent's dynamic is given by

$$\begin{aligned}\dot{x}_i(t) &= v_i(t),\\ \dot{v}_i(t) &= u_i(t), \quad i = 1, 2, \ldots, n,\end{aligned} \tag{1}$$

where $x_i \in R$, $v_i \in R$, and $u_i \in R$ are the position, velocity, and acceleration, respectively, of the agent i. As we all know, the interconnection topology of multiagent systems (1) is usually described as a digraph. Agents can be considered as the nodes of the digraph, while the information flow between neighboring agents can be regarded as a directed edge between the neighboring nodes in the digraph.

A weighted digraph $G = (V, E, A)$ of order n consists of a set of vertices $V = \{1, \ldots, n\}$, a set of edges $E \subseteq V \times V$, and a weighted adjacency matrix $A = [a_{ij}] \in R^{n\times n}$ with $a_{ij} \geq 0$. The node indexes belong to a finite index set $\mathcal{I} = \{1, \ldots, n\}$. A directed edge from i to j in G is denoted by $e_{ij} = (i, j) \in E$, which means that the node j can obtain information from the node i. Assume that $a_{ji} > 0 \Leftrightarrow e_{ij} \in E$ and $a_{ii} = 0$ for all $i \in \{1, \ldots, n\}$. The set of neighbors of node i is denoted by $N_i = \{j \in V : (j, i) \in E\}$. The Laplacian matrix of the digraph G is defined as $L = D - A = [l_{ij}] \in R^{n\times n}$, where $D = \operatorname{diag}\{\sum_{j=1}^{n} a_{ij}, i = 1, \ldots, n\}$ is the degree matrix. In the digraph G, a directed path from node i_1 to node i_s is a sequence of ordered edges of the form $(i_1, i_2), \ldots, (i_{s-1}, i_s)$, where $i_j \in V$. A digraph is said to have a spanning tree, if there exists a node such that there is a directed path from this node to every other node.

2.2. Adjacent-Compensation Consensus Algorithm. In this paper, we consider the dynamical consensus problem of the second-order agents (1). Without loss of generality, we analyze the second-order agents (1) following a dynamic leader, and the leader's dynamic is determined by

$$\dot{x}_0 = v_0, \tag{2}$$

where $x_0 \in R$ is the position and $v_0 \in R$ is a constant denoting the desired velocity for all agents (1). Under the leader-following coordination control framework, the agents' states are required to converge to the leader's states asymptotically; that is,

$$\lim_{t\to\infty} x_i(t) = x_0(t), \quad \lim_{t\to\infty} v_i(t) = v_0, \quad \forall i \in \mathcal{I}. \tag{3}$$

We adopt the following consensus algorithm, which consists of the position and the velocity coordination control parts [4]:

$$\begin{aligned}u_i = \frac{\kappa}{d_i + b_i}\Bigg(&\sum_{j\in N_i} a_{ij}\left(\left(x_j(t) - x_i(t)\right) + \gamma\left(v_j(t) - v_i(t)\right)\right)\\ &+ b_i\left(\left(x_0(t) - x_i(t)\right) + \gamma\left(v_0(t) - v_i(t)\right)\right)\Bigg),\\ &i \in \mathcal{I},\end{aligned} \tag{4}$$

where $\kappa > 0$, $\gamma > 0$, N_i denotes the neighbors of the agent i, $a_{ij} > 0$ is the adjacency element of A in the digraph $G = (V, E, A)$, $d_i = \sum_{j\in N_i} a_{ij}$, and b_i is defined as

$$\begin{aligned}b_i &> 0, \quad \text{if agent } i \text{ is connected to the leader},\\ b_i &= 0, \quad \text{otherwise}.\end{aligned} \tag{5}$$

With communication delay, the asynchronously coupled form of the algorithm (4) is given by

$$\begin{aligned}u_i = {}& \frac{\kappa}{d_i + b_i}\\ &\times\Bigg(\sum_{j\in N_i} a_{ij}\left(\left(x_j(t-\tau) - x_i(t)\right) + \gamma\left(v_j(t-\tau) - v_i(t)\right)\right)\\ &+ b_i\left(\left(x_0(t-\tau) - x_i(t)\right) + \gamma\left(v_0 - v_i(t)\right)\right)\Bigg),\\ &i \in \mathcal{I},\end{aligned} \tag{6}$$

where $\tau > 0$ is the communication delay, while the synchronously coupled form is given by

$$\begin{aligned} u_i = {} & \frac{\kappa}{d_i + b_i} \\ & \times \Bigg(\sum_{j \in N_i} a_{ij} \Big(\big(x_j (t - \tau) - x_i (t - \tau) \big) \\ & \qquad + \gamma \big(v_j (t - \tau) - v_i (t - \tau) \big) \Big) \\ & \quad + b_i \left(\left(x_0 (t - \tau) - x_i (t - \tau) \right) \right. \\ & \qquad \left. + \gamma \left(v_0 - v_i (t - \tau) \right) \right) \Bigg), \quad i \in \mathcal{J}. \end{aligned} \tag{7}$$

Remark 1. The second-order agents (1) under synchronously coupled consensus algorithm (7) also achieve a dynamical consensus if the communication delay and the control parameters satisfy some certain conditions [8]. With asynchronously coupled consensus algorithm (6), the agents may not converge to an asymptotic consensus or the final consensus state of the dynamic agents (1) is stationary, and consensus conditions depend on the communication delay [25, 26].

So far, consensus algorithms with compensations related to delayed states of neighboring agents, desired target, and dynamical leader have been added to consensus algorithm (6) in order to remain the original control objective. Similar to the compensation-based consensus algorithms in [28], we modify the asynchronously coupled consensus algorithm (6) by introducing the delayed state compensations as follows:

$$\begin{aligned} u_i = {} & \frac{\kappa}{\sum_{j \in N_i} a_{ij} + b_i} \\ & \times \Bigg(\sum_{j \in N_i} a_{ij} \Big(\big(x_j (t - \tau) - x_i (t) \\ & \qquad - \big(x_j (t - 2\tau - \zeta) - x_j (t - \tau - \zeta) \big) \big) \\ & \qquad + \gamma \big(v_j (t - \tau) - v_i (t) \\ & \qquad - \big(v_j (t - 2\tau - \zeta) - v_j (t - \tau - \zeta) \big) \big) \Big) \\ & \quad + b_i \left(\left(x_0 (t - \tau) - x_i (t) \right. \right. \\ & \qquad \left. - \left(x_0 (t - 2\tau - \zeta) - x_0 (t - \tau - \zeta) \right) \right) \\ & \qquad \left. + \gamma \left(v_0 - v_i (t) \right) \right) \Bigg), \quad i \in \mathcal{J}, \end{aligned} \tag{8}$$

where $\zeta > 0$ and the compensations $-(x_j(t-2\tau-\zeta)-x_j(t-\tau-\zeta))$ and $-(v_j(t-2\tau-\zeta)-v_j(t-\tau-\zeta))$ are added to improve the system's performance. Compared with the algorithm in [28], the algorithm (8) in this paper is more general with an adjustable value ζ.

Remark 2. Obviously, the adjustable delay value ζ in algorithm (8) can present different communication-delay robustness, and the best value ζ can be obtained based on numerical simulations.

Besides, we take the following assumption to make the algorithm (8) reasonable.

Assumption 3. Each second-order agent has at least one neighboring agent.

Then, the closed-loop form of second-order dynamic agents (1) with (8) is

$$\begin{aligned} \dot{x}_i = {} & v_i, \\ \dot{v}_i = {} & \frac{\kappa}{d_i + b_i} \\ & \times \Bigg(\sum_{j \in N_i} a_{ij} \Big(\big(x_j (t - \tau) - x_i (t) \\ & \qquad - \big(x_j (t - 2\tau - \zeta) - x_j (t - \tau - \zeta) \big) \big) \\ & \qquad + \gamma \big(v_j (t - \tau) - v_i (t) \\ & \qquad - \big(v_j (t - 2\tau - \zeta) - v_j (t - \tau - \zeta) \big) \big) \Big) \\ & \quad + b_i \left(\left(x_0 (t - \tau) - x_i (t) \right. \right. \\ & \qquad \left. - \left(x_0 (t - 2\tau - \zeta) - x_0 (t - \tau - \zeta) \right) \right) \\ & \qquad \left. + \gamma \left(v_0 - v_i (t) \right) \right) \Bigg), \quad i \in \mathcal{J}. \end{aligned} \tag{9}$$

Defining $\overline{x}_i = x_i - x_0$, $\overline{v}_i = v_i - v_0$, $i = 1, \dots, n$, we get

$$\begin{aligned} \dot{\overline{x}}_i = {} & \overline{v}_i, \\ \dot{\overline{v}}_i = {} & \frac{\kappa}{d_i + b_i} \\ & \times \Bigg(\sum_{j \in N_i} a_{ij} \Big(\big(\overline{x}_j (t - \tau) - \overline{x}_i (t) \\ & \qquad - \big(\overline{x}_j (t - 2\tau - \zeta) - \overline{x}_j (t - \tau - \zeta) \big) \big) \\ & \qquad + \gamma \big(\overline{v}_j (t - \tau) - \overline{v}_i (t) \\ & \qquad - \big(\overline{v}_j (t - 2\tau - \zeta) - \overline{v}_j (t - \tau - \zeta) \big) \big) \Big) \\ & \quad - b_i \left(\overline{x}_i + \gamma \overline{v}_i \right) \Bigg), \quad i \in \mathcal{J}. \end{aligned} \tag{10}$$

Taking the Laplace transforms of system (10), we get the characteristic equation about $\overline{x}(t) = [x_2, \ldots, x_n]^T$ as follows:

$$\det\left(s^2 I + \kappa(1+\gamma s)(D+B)^{-1} \times \left(D + B - A\left(e^{-s\tau} + e^{-s(\tau+\zeta)} - e^{-s(2\tau+\zeta)}\right)\right)\right) = 0. \tag{11}$$

3. Main Results

First of all, we present consensus criterion for the multiagent systems (9) with a spanning tree topology that has the leader as a root.

Theorem 4. *Assume that the interconnection topology composed of the second-order agents* (9) *and the dynamic leader is a spanning tree. Then, all the agents in system* (9) *asymptotically converge to the leader's state with arbitrary communication delay.*

Proof. When the interconnection topology of n agents and a leader is a spanning tree, the leader must be the root and we assume that the direct edge from agent i to j satisfies $i < j$. Hence, the characteristic equation (11) becomes

$$\det\left(s^2 I + \kappa(1+\gamma s)(D+B)^{-1} \times \left(D + B - A\left(e^{-s\tau} + e^{-s(\tau+\zeta)} - e^{-s(2\tau+\zeta)}\right)\right)\right) = 0, \tag{12}$$

where $D = \operatorname{diag}\{d_i, i = 1, \ldots, n\}$, $B = \operatorname{diag}\{b_i, i = 1, \ldots, n\}$, and

$$A = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ a_{21} & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & 0 \end{bmatrix}. \tag{13}$$

Hence, (12) equals

$$\prod_{i=1}^{n}\left(s^2 + \kappa(1+\gamma s)\right) = 0, \quad i = 1, \ldots, n. \tag{14}$$

The roots of (14) have negative real parts; that is, the roots of the characteristic equation (12) lie on the open left half complex plane. Therefore, the agents in system (9) converge to the leader's states asymptotically. □

Remark 5. To our delight, the system (9) under a special topology can tolerate arbitrary communication delay.

In the following, we investigate the consensus problem of second-order dynamic agents under a general interconnection topology in the leader-following framework.

Theorem 6. *Assume that the agents in system* (9) *without communication delay converge to the leader's states asymptotically and Assumption 3 holds. Let*

$$m(s) = \frac{g_i(s)}{1 + g_i(s)} \frac{\lambda_i}{1 - \lambda_i}, \tag{15}$$

where $g_i(s) = (\kappa(1+\gamma s)/s^2)(1-\lambda_i)$ and λ_i, $i = 1, \ldots, n$ are the eigenvalues of $(D+B)^{-1}A$. Then, all the agents in system (9) *converge to the leader's states asymptotically, if*

$$4\left|(m(j\omega))\sin\left(\frac{\omega\tau}{2}\right)\sin\left(\frac{\omega(\tau+\zeta)}{2}\right)\right| < 1 \tag{16}$$

holds for $\omega \in R$.

Proof. Without communication delay, (11) becomes

$$\det\left(s^2 I + \kappa(1+\gamma s)(D+B)^{-1}(D+B-A)\right) = 0, \tag{17}$$

and the above equation is equivalent to

$$\prod_{i=1}^{n}\left(\left(s^2 + \kappa(1+\gamma s)\right)(1-\lambda_i)\right) = 0. \tag{18}$$

Under the assumption that the agents without communication delay converge to a dynamical consensus asymptotically, the roots of (18) all lie on the open left half complex plane, and $\operatorname{rank}(L + B) = n$; that is, the interconnection topology of n agents and the leader has a spanning tree.

With communication delay, (11) can be rewritten as

$$\det\left(s^2 I + \kappa(1+\gamma s) \times \left(I - (D+B)^{-1}A\left(e^{-s\tau} + e^{-s(\tau+\zeta)} - e^{-s(2\tau+\zeta)}\right)\right)\right) = 0, \tag{19}$$

which equals

$$\prod_{i=1}^{n}\left(s^2 + \kappa(1+\gamma s)\left(1 - \lambda_i\left(e^{-s\tau} + e^{-s(\tau+\zeta)} - e^{-s(2\tau+\zeta)}\right)\right)\right) = 0, \quad i = 1, \ldots, n. \tag{20}$$

Letting $f_i(s) = s^2 + \kappa(1+\gamma s)(1 - \lambda_i(e^{-s\tau} + e^{-s(\tau+\zeta)} - e^{-s(2\tau+\zeta)}))$, we get $f(0) = \kappa(1-\lambda_i) \neq 0$. Thus, (20) can be rewritten as

$$1 + \kappa\frac{(1+\gamma s)}{s^2}(1-\lambda_i) \times \left(1 + \frac{\lambda_i}{1-\lambda_i}\left(1 - e^{-s\tau}\right)\left(1 - e^{-s(\tau+\zeta)}\right)\right) = 0, \quad i = 1, \ldots, n. \tag{21}$$

The above equation is equivalent to

$$1 + m(s)\left(1 - e^{-s\tau}\right)\left(1 - e^{-s(\tau+\zeta)}\right) = 0, \quad i = 1, \ldots, n, \tag{22}$$

where $m(s)$ is defined in (15). Obviously, $(1 - e^{-s\tau})$, $(1 - e^{-s(\tau+\zeta)})$, and $m(s)$ have no poles in the open right half complex plane.

By computing, we obtain

$$\left|m(j\omega)\left(1 - e^{-j\omega\tau}\right)\left(1 - e^{-j\omega(\tau+\zeta)}\right)\right| = 4\left|m(j\omega)\sin\left(\frac{\omega\tau}{2}\right)\sin\left(\frac{\omega(\tau+\zeta)}{2}\right)\right|. \tag{23}$$

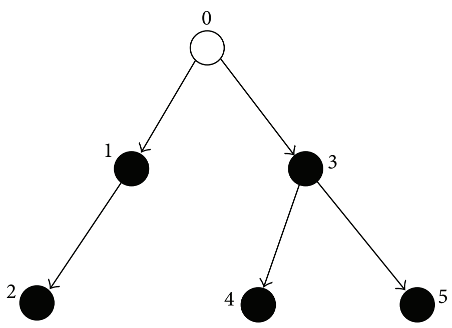

FIGURE 1: A spanning tree topology.

From condition (16),

$$\left|m(j\omega)\left(1-\mathrm{e}^{-j\omega\tau}\right)\left(1-\mathrm{e}^{-j\omega(\tau+\zeta)}\right)\right|<1 \tag{24}$$

holds for all $\omega \in R$.

Therefore, $1+m(s)(1-\mathrm{e}^{-s\tau})(1-\mathrm{e}^{-s(\tau+\zeta)})$ is nonsingular for $\mathrm{Re}(s) \geq 0$; that is, the roots of the characteristic equation (11) all lie on the open left half complex plane. Hence, the agents in the system (9) converge to the leader's states asymptotically. Theorem 6 is proved. □

4. Simulation

Example 7. Consider a multiagent network with five second-order agents and a leader given by (9), and the interconnection topology described in Figure 1 is a spanning tree. The weights of the directed edges are $b_1 = 0.4$, $a_{21} = 0.1$, $a_{43} = 0.8$, $b_3 = 1.5$, and $a_{53} = 0.2$. Choose the control parameters as $\kappa = 1.2$ and $\gamma = 0.8$. Then, the agents in the system (9) can reach a dynamical consensus asymptotically under arbitrary communication delay (see Figure 2).

It should be noted that the dynamical consensus convergence for the agents with synchronously coupled consensus algorithm under the same control parameters and interconnection topology must depend on communication delay strictly.

Example 8. Consider a multiagent network with five second-order agents and a leader given by (9), and the interconnection topology of the agents is described in Figure 3. Obviously, the topology satisfies Assumption 3. The weights of the directed edges are $b_1 = 1.7$, $a_{13} = 0.5$, $a_{21} = 1.3$, $a_{34} = 2.1$, $a_{35} = 1.8$, $b_4 = 1.8$, $a_{41} = 0.7$, $a_{32} = 1.2$, and $a_{54} = 1.2$. Choose the control parameters as $\kappa = 1.5$, $\gamma = 0.8$, and $\zeta = 0.8$(s); we obtain $\tau < 0.527$(s) based on condition (16); that is, the agents in the system (9) can reach a dynamical consensus asymptotically if $\tau < 0.527$(s) (see Figure 4). By simulation, it is found that the agents' states oscillate when $\tau = 0.6$(s) (see Figure 5). Moreover, we found that the largest communication delay, which the agents with synchronously coupled algorithm can tolerate, is $\tau = 0.345$(s).

To illustrate the function of adjustable value ζ, we choose diverse ζ and get an algorithm with the best delay robustness.

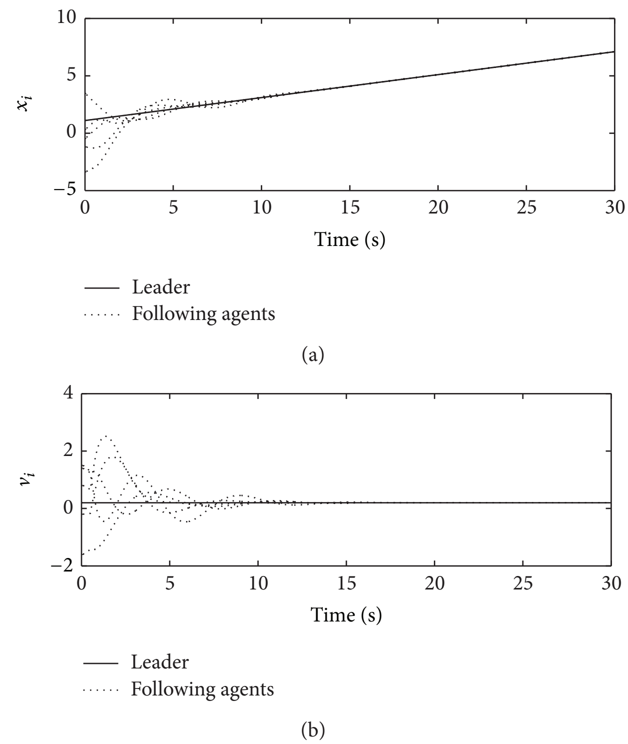

FIGURE 2: Delay-independent dynamical consensus under a spanning tree.

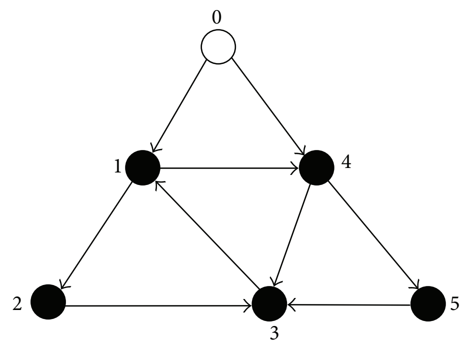

FIGURE 3: Interconnection topology of five agents and one leader.

Figure 6 shows the largest communication delay our proposed algorithm can bear under different ζ. When $\zeta = 3$(s), the largest communication delay the agents in system (9) can bear is $\tau_{\max} = 1.47$(s), which is much larger than that the synchronously coupled algorithm can achieve.

5. Conclusion

In this paper, we consider the dynamical consensus problem of the second-order multiagent systems under communication delay and adopt the leader-following consensus algorithm which is composed of the position and the velocity consensus coordination control parts. By introducing neighboring agents' delayed state compensation into the normal

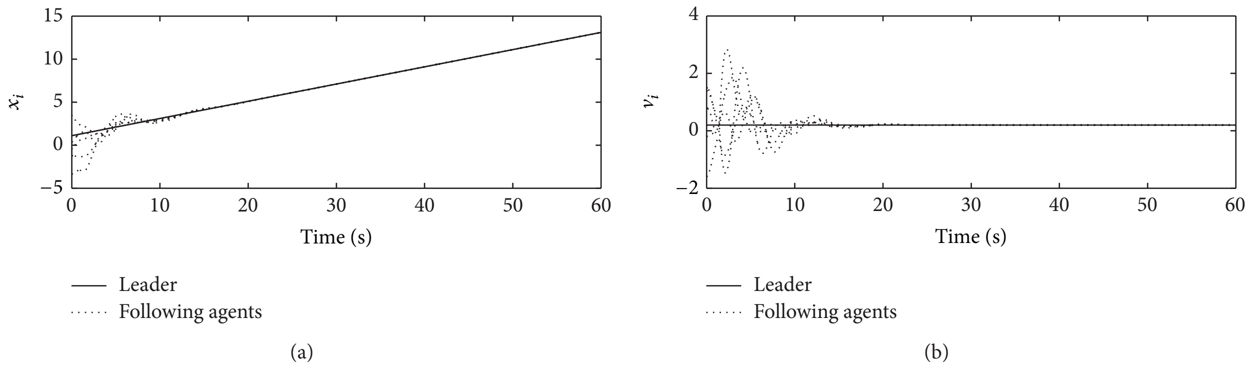

FIGURE 4: Delay-dependent dynamical consensus convergence.

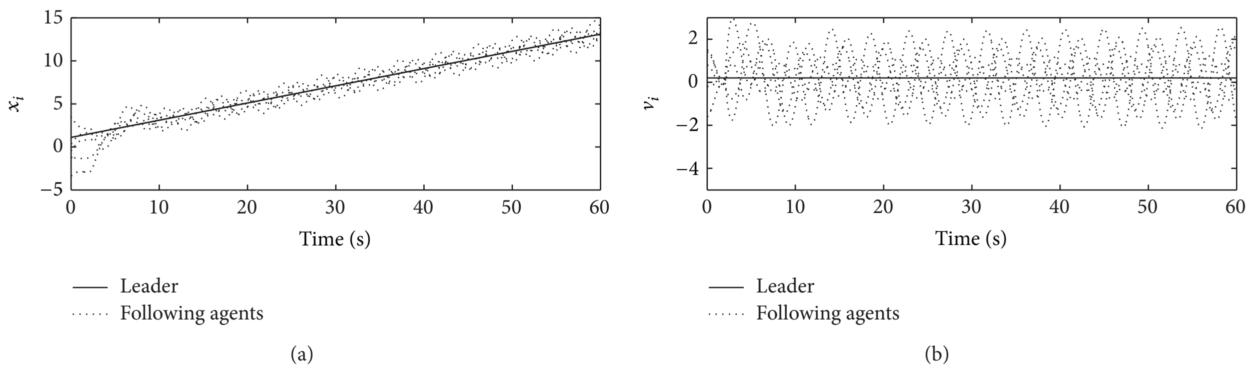

FIGURE 5: Oscillation of agents' states.

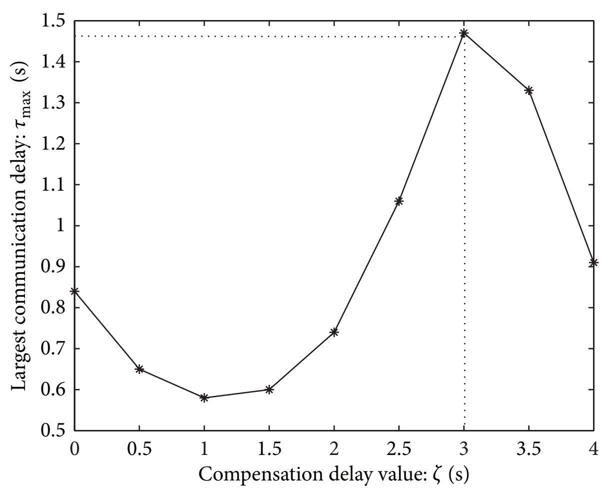

FIGURE 6: Bound of communication delay with different ζ.

asynchronously coupled consensus algorithm, the second-order agents can achieve a dynamical consensus as the synchronously coupled algorithm. Firstly, we consider second-order dynamic agents under a spanning tree topology that has the leader as the root and use frequency-domain analysis to prove the delay-independent consensus convergence. Then, we investigate the consensus seeking under the general interconnection topology in the leader-following coordination control framework. Delay-dependent consensus condition is obtained for the second-order agents converging to the leader's states asymptotically. It should be noted that choosing proper adjustable delay introduced in our proposed algorithm can obtain different delay robustness. In a word, our proposed algorithm can tolerate much higher communication delay than synchronously coupled consensus algorithm.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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