Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2014, Article ID 974075, 12 pages
http://dx.doi.org/10.1155/2014/974075

Research Article

Centralized Resource Allocation for Connecting Radial and Nonradial Models

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Received 21 October 2013; Revised 8 January 2014; Accepted 8 January 2014; Published 4 March 2014

Academic Editor: Aderemi Oluyinka Adewumi

This paper examines an alternative approach to the centralized resource allocation model that indicates that all the units are under the control of an entity of the centralized decision maker. The proposed approach is a technique for connecting the two basic Radial CRA-BCC and Nonradial CRA-SBM models in an integrated structure called connected CRA-SBM. In the proposed model, exchanging the two parameter amounts can change the location of the analysis between the CRA-BCC and the CRA-SBM models and deal with the weaknesses inherent in such models. By remedying all the weaknesses in one model, the entire units are simply projected on the frontier line and one can obtain suitable benchmarks for each of them. In the offered model, all of the inputs and outputs, respectively, decrease and increase simultaneously. Lastly, numerical examples emphasize the significance of the offered method.

1. Introduction

The Data Envelopment Analysis (DEA) was first presented by Charnes et al. [1] and further developed by Banker et al. [2]. The DEA is a nonparametric LP-based method for evaluating the efficiency score of a number of homogeneous decision-making units (DMUs). There has been literature over the years concerned with the new corrections and developments in both the DEA notion and procedure (see Liu et al. [3], Emrouznejad et al. [4], and Gattoufi et al. [5]).

As a matter of fact, the DEA models are commonly categorized into two types with distinguishing qualities, namely, the radial and nonradial models. In effect, the preliminary works on the radial models were undertaken by Debreu [6] and Farrell [7]. By now, numerous desirable features have been identified in radial measures; for instance, they can generally obtain the relative development in inputs and outputs; moreover, they are potential of estimating the efficiency based on the attainable data or they can provide an obvious economic explanation without considering the prices. Notwithstanding their strengths, these models suffer from the subsequent drawbacks.

(a) They assess the efficiency based on the existing data without considering the decision maker's (DM's) precedence knowledge.

(b) Due to the proportional improvement in these models, they cannot be employed for the cases with inputs such as labors, materials, and capital.

(c) The DM does not have the flexibility to select a reference unit for an inefficient unit.

(d) Finally, they are unable to essentially achieve an efficient goal in the DEA.

At first, analyzing the nonradial models was carried out by Koopmans [8] and Robert Russell [9]. Yet, numerous studies have attempted to explain the nonradial measures for the technical efficiency based on the performance estimation (see Cooper et al. [10], Charnes et al. [11], Cooper et al. [12], Pastor et al. [13], and Cooper et al. [14]). In one major study, Tone [15] employed a new and suitable synthetic procedure to obtain a nonradial measure which was termed the slacks-based measure (SBM) in which both the input and output slacks could be maximized. A number of methods have been also used to describe the nonradial measure, each of which has its own advantages and disadvantages. These models have a number of attractive features; for example, they put aside the supposition of proportional reduction in the inputs and target at earning maximum amounts of contraction in inputs which might abandon the changing rates of the original input resources. Nevertheless, the nonradial models suffer from several major downsides listed below in spite of their safety and efficacy.

(a) When we evaluate changes in the efficiency during the time, the nonzero pattern of the slacks at time period t can meaningfully differ from that of the time period $t + 1$. Therefore, we cannot ascertain whether the pattern is rational or not.

(b) When we miss the primary proportionality, it would be then unsuitable for the investigation.

(c) At last, in models like the SBM model, the optimum slacks would exhibit an acute conflict in catching the positive and zero amounts.

Avkiran et al. [16] proposed the "connected-SBM" model using two scalar parameters which could deal with the above-mentioned weaknesses of the radial and nonradial models. In their approach, they relocated the analysis anywhere between the radial and nonradial models by exchanging the parameter amounts and making an appropriate selection.

In a large longitudinal study by Lozano and Villa [17], there were various conditions in which all the DMUs could be in the possession of the central entity (public or private) so that it prepared the DMUs with the resources essential for earning their inputs. In particular, various applications related to the DEA (e.g., public transportation, police stations, university departments, bank branches, and hospitals) have been taken into account as centralized decision makers that manage and supervise such DMUs. Prior to this, Lozano and Villa [17], attempted to handle the DMUs in a common manner (see Thanassoulis and Dyson [18], Golany [19], Färe et al. [20], Kumar and Sinha [21], Beasley [22], and Athanassopoulos [23, 24]). To this end, it can be said that numerous studies have attempted so far to explain the allocating resources as well as the evaluating goals. Lozano and Villa [17] and Lozano et al. [25] presented the CRA-BCC models wherein the resolving one model in two phases causes the entire DMUs to be reflected on the frontier line. Their study can be majorly mentioned as an unusual item based on the radial model. The assertion behind this model is to consider a situation in which the centralized decision maker decides to minimize the total input consumption or maximize the total output production, simultaneously. Lozano and Villa [26] also proposed the models which deliberated the CRA models when some of the inputs reductions and the outputs remained unchanged. Moreover, a DEA approximation for the emission permits along with a discussion on the desired and undesired levels has been proposed by Lozano et al. [27]. Several studies have produced estimates of the CRA models as well (see Pachkova [28], Liu and Tsai [29], and Fang and Zhang [30]). In another main study, Asmild et al. [31] reported a CRA model for the BCC model in which all the inefficient DMUs had been only reflected on the frontier line. In a randomized controlled study on the CRA, Hosseinzadeh Lotfi et al. [32] described a centralized model for the enhanced Russell model (CRA-ERM) and the SBM model (CRA-SBM) so that all the DMUs could be easily projected onto the frontier line by solving only one model.

Although there have been many suppositions about the idea of the centralized resource allocation, it has not been recognized as a perfect viewpoint yet. When bearing in mind the organizations which function under a multilevel supervision hierarchy, definite resources utilized or the outputs created which cannot be controlled must be recognized using the local DMUs. Consequently, it is pertinent to mention the potential of improving these centrally allocated resources in a central manner for manufacturing purposes.

This study seeks to alleviate these problems by introducing alternative new CRA-DEA models while consequently creating new tools ready for use in central management. In fact, the purpose of this paper is to minimize the total input consumption and maximize the total output production of the DMUs at the same time. Moreover, we will demonstrate that the CRA-BCCI and CRA-SBMI models can be joined by retouching the two parameters and selecting them. The objectives of this research are to determine whether we can oversee the appropriateness of slacks (inefficiencies) with the resources essential for the user, and whether we can solve the issue of the mixed slacks amounts in conditions existing at the same time for the DEA with parametric methods.

The remainder of this paper has been divided into five parts. In Section 2, the CRA-BCCI, the CRA-SBMI and the connected-SBM models are presented while Section 3 presents our proposed model and explains it with numerical examples. In Section 4, by considering our procedure in organizations, we extend it into a weighted nonoriented model and following that, the results are also checked out using the numerical examples as well as comparing them with the previous models (Lozano and Villa [17] and Hosseinzadeh Lotfi et al. [32]). In Section 5, we apply our model on gas companies during 2008 [33] for better exploring the models and representing their capabilities. Finally, the discussion and conclusion are provided in Section 6.

2. Preparations

In this section, we discuss two of the renowned CRA-DEA models, namely, the CRA-BCCI and CRA-SBMI proposed by Lozano and Villa [17] and Hosseinzadeh Lotfi et al. [32], respectively. Afterward, we elaborate on some of their attributes for obtaining our targets. Lastly, the connected-SBM model is also deliberated briefly.

Having considered no existing production function for determining the operational performances, a production possibility set (PPS) is mentioned and its frontier is designated for approximating the production function.The PPS is defined as follows:

$$\overline{T_v} = \left\{ (X,Y) : X \geq \sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr} X_j,\ Y \leq \sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr} Y_j, \right.$$
$$\left. \sum_{j=1}^{n} \lambda_{jr} = 1,\ \lambda_{jr} \geq 0,\ r = 1,\dots,n,\quad j = 1,\dots,n \right\}. \tag{1}$$

The following symbolizations are also utilized in this part:

n: the number of DMUs;

m: the number of inputs;

s: the number of outputs;

j, r: the indices for DMUs;

i: the index for inputs;

k: the index for outputs;

x_{ij}: the observed value of input i of DMUr;

y_{kj}: the observed value of output k of DMUr;

λ_r: the vector for projecting DMUr;

θ: the radial constriction of the aggregate input vector;

s_i^-: the nonnegative radial slack along the input dimension i of DMUr;

z_i^-, t_i^-: the nonnegative nonradial slack along the input dimension i of DMUr;

s_i^{-*}: the optimal radial slack to recognize an excess use of input i of DMUr;

z_i^{-*}, t_i^{-*}: the optimal nonradial slack to recognize an excess use of input i of DMUr;

θ^*: the optimal group efficiency by the CRA-BCCI model;

δ^*: the optimal group efficiency by the CRA-SBMI model;

λ_r^*: the optimal vector for projecting DMUr;

x_{ir}^*: the operating point target with the CRA models for input i of DMUr;

y_{kr}^*: the operating point target with the CRA models for output k of DMUr.

2.1. The CRA-BCCI and CRA-SBMI Models. What follows is a succinct description of the CRA-BCCI and CRA-SBMI models in addition to mentioning their properties.

2.1.1. The Input-Oriented CRA-BCCI Model. The input-oriented CRA-BCCI model reported by Lozano and Villa [17] estimates the group efficiency θ^* of DMUs by solving the following linear program:

$$\theta^* = \operatorname{Min} \theta \tag{2}$$

$$\text{s.t.} \quad \sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr} x_{ij} + t_i^- = \theta \sum_{j=1}^{n} x_{ij}, \quad i = 1,\dots,m, \tag{3}$$

$$\sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr} y_{kj} \geq \sum_{j=1}^{n} y_{kj}, \quad k = 1,\dots,s, \tag{4}$$

$$\sum_{j=1}^{n} \lambda_{jr} = 1, \quad r = 1,\dots,n, \tag{5}$$

$$\lambda_{jr} \geq 0, \quad r, j = 1,\dots,n, \tag{6}$$

$$t_i^- \geq 0, \quad i = 1,\dots,m. \tag{7}$$

Note that $\lambda_r = (\lambda_{1r}, \lambda_{2r}, \dots, \lambda_{nr})$ is the vector for projecting DMUr and denotes the intensity vector, while t_i^- also designates the nonradial slacks. Generally, to solve CRA-BCCI, there are two phases. In the first phase, we seek an equiproportional reduction along the entire input measurements and find θ^*. Subsequently, we determine the radial input slacks s_i^{-*} as follows:

$$s_i^{-*} = (1 - \theta^*) \sum_{j=1}^{n} x_{ij}, \quad i = 1,\dots,m. \tag{8}$$

Thus, we have

$$\theta^* = \frac{\sum_{j=1}^{n} x_{ij} - s_i^{-*}}{\sum_{j=1}^{n} x_{ij}}, \quad i = 1,\dots,m. \tag{9}$$

Then, we will maximize $\sum_{i=1}^{m} (t_i^- / \sum_{j=1}^{n} x_{ij})$ according to λ_r and t_i^-, subject to (3) to (7) and $\theta = \theta^*$ in the second phase. Now, if (λ_r^*, t_i^{-*}) is an optimal solution of the second phase, then t_i^{-*} remains as the nonradial slacks after elimination of the radial slacks s_i^{-*}. Hence, the total slacks u_i^{-*} of the CRA-BCCI model will be determined as $u_i^{-*} = t_i^{-*} + s_i^{-*}$. Despite the fact that ever nonradial projections can be presented in association with radial projections, nonradial slacks cannot be projected in the scalar θ^*.

2.1.2. The Input-Oriented CRA-SBMI Model. At this time, we demonstrate the input-oriented CRA-SBMI model with the variable returning to the scale supposition of technology proposed by Hosseinzadeh Lotfi et al. [32] which assesses the group efficiency δ^* by solving the following linear program:

$$\begin{aligned}
\delta^* &= \operatorname{Min} \left(1 - \frac{1}{m} \sum_{i=1}^{m} \frac{z_i^-}{\sum_{j=1}^{n} x_{ij}} \right) \\
\text{s.t.} \quad & \sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr} x_{ij} + z_i^- = \sum_{j=1}^{n} x_{ij}, \quad i = 1,\dots,m, \\
& \sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr} y_{kj} \geq \sum_{j=1}^{n} y_{kj}, \quad k = 1,\dots,s, \\
& \sum_{j=1}^{n} \lambda_{jr} = 1, \quad r = 1,\dots,n, \\
& \lambda_{jr} \geq 0, \quad r, j = 1,\dots,n, \\
& z_i^- \geq 0, \quad i = 1,\dots,m,
\end{aligned} \tag{10}$$

where $\lambda_r = (\lambda_{1r}, \lambda_{2r}, \ldots, \lambda_{nr})$ is the vector for projecting DMUr and denotes the intensity vector, and z_i^- also defines the nonradial slacks. At this point, if we consider the case of input-orientated model in accordance with our explanation of the CRA-BCCI model in the previous section and if also (λ_r^*, z_i^{-*}) is one of its optimal solutions, then the CRA-SBMI score δ^* can be rewritten as follows:

$$\delta^* = \frac{1}{m}\sum_{i=1}^{m} \frac{\sum_{j=1}^{n} x_{ij} - z_i^{-*}}{\sum_{j=1}^{n} x_{ij}}. \tag{11}$$

The CRA-SBMI model is nonradial model and δ^* identifies the mean of the decreased rates.

2.1.3. Properties of the Radial and Nonradial Models. The above two models have some properties as follows.

(a) All of the inefficient DMUs are reflected on the efficient frontier to solve only one model rather than solving a model for each DMU independently.

(b) The existent technically efficient DMUs can be reflected onto a certain point on the efficient frontier but they must be reflected onto themselves in the original DEA models.

(c) The CRA-BCCI evaluates just the radial inefficiency in the scalar θ^* while the CRA-SBM model measures just the nonradial inefficiency.

(d) When the CRA-SBMI and the CRA-BCCI models are resolved, by bearing in mind vector $\lambda_r^* = (\lambda_{1r}^*, \lambda_{2r}^*, \ldots, \lambda_{nr}^*)$, for each DMU$r$ as the value of optimal, we can obtain the operating point so that the CRA-SBMI and the CRA-BCCI targets for the inputs and outputs of any such point can be as follows:

$$\begin{aligned} x_{ir}^* &= \sum_{j=1}^{n} \lambda_{jr}^* x_{ij}, \quad r = 1, \ldots, n,\ i = 1, \ldots, m, \\ y_{kr}^* &= \sum_{j=1}^{n} \lambda_{jr}^* y_{kj}, \quad r = 1, \ldots, n,\ k = 1, \ldots, s. \end{aligned} \tag{12}$$

2.2. The Connected-SBM Model. In a large longitudinal study to deal with the weaknesses associated with the radial and nonradial models in the first one, Avkiran et al. [16] presented the connected-SBM model in which there were two nonnegative scalar parameters as L $(0 \le L \le 1)$ and U (≥ 1). By taking into consideration the DMUs, the connected-SBM model with the introduced PPS for the variable returns to the scale supposition of the technology [2] estimates the efficiency ρ^* of DMUo $(o = 1, 2, \ldots, n)$ as follows:

$$\rho^* = \text{Min}\,(1 - f) \tag{13}$$

$$\text{s.t.} \quad f = \frac{1}{m}\sum_{i=1}^{m} \frac{z_i^-}{x_{io}} \tag{14}$$

$$Lf \le \frac{z_i^-}{x_{io}} \le Uf, \quad i = 1, \ldots, m, \tag{15}$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} + z_i^- + v_i^- = x_{io}, \quad i = 1, \ldots, m, \tag{16}$$

$$\sum_{j=1}^{n} \lambda_j y_{kj} \ge y_{ko}, \quad k = 1, \ldots, s, \tag{17}$$

$$\sum_{j=1}^{n} \lambda_j = 1, \tag{18}$$

$$\lambda_j \ge 0, \quad j = 1, \ldots, n, \tag{19}$$

$$z_i^- \ge 0, \quad i = 1, \ldots, m, \tag{20}$$

$$v_i^- \ge 0, \quad i = 1, \ldots, m. \tag{21}$$

In this model, v_i^- designates the nonradial input slacks persuaded to keep with the constraint (15). In addition, f is the mean of the normalized slacks $\{z_i^-/x_{io}\}$ and each normalized slack z_i^-/x_{io} to the range $[Lf, Uf]$ is restricted by the constraint (15). Reciprocally, the average nonradial input inefficiency for the existing DMU explains f. Therefore, the perversion from the mean value f can be restricted according to the lower bound L and upper bound U. The above- considered model has attributes as follows.

(a) If $L = 1$ or $U = 1$, then SBM-I-C (L, U) converts BCC-I.

(b) If $L = 0$ and $U \ge m$, then SBM-I-C $(0, U)$ transforms SBM-I.

(c) Give some freedom to the perversion of the normalized slacks which contain the assumed range around the average.

(d) Lf^* and Uf^* for all the inputs restrict the relative perversion of the reflected input from the original amount.

(e) The parameters L and U can be detracted to a single parameter as $U = m - (m-1)L$.

3. Centralized Resource Allocation Connected Models

In this section, we propose a new model for the abovementioned weaknesses and change the connected-SBM model to deal with computing one instead of n mathematical programming problems and decreasing the aggregate input at the same time. In fact, we intend to present that the DEA model can handle the target setting process by presenting the data envelopment scenario analysis. We call our models as the centralized resource allocation connected models. By bearing in mind the DMUs in the above models, their conforming input and output vectors, and the defined PPS for CRA models, the centralized resource allocation connected-SBMI model (CRA-CSBMI) can be described as follows:

$$\text{Model (1)}: \quad \gamma^* = \text{Min}\,(1-h) \tag{22}$$

$$\text{s.t.} \quad h = \frac{1}{m}\sum_{i=1}^{m}\frac{z_i^-}{\sum_{j=1}^{n} x_{ij}} \tag{23}$$

$$Lh \le \frac{z_i^-}{\sum_{j=1}^{n} x_{ij}} \le Uh, \quad i = 1,\ldots,m, \tag{24}$$

$$\sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}x_{ij} + z_i^- + v_i^- = \sum_{j=1}^{n} x_{ij}, \quad i = 1,\ldots,m, \tag{25}$$

$$\sum_{r=1}^{n}\sum_{j=1}^{n}\lambda_{jr}y_{kj} \ge \sum_{j=1}^{n} y_{kj}, \quad k = 1,\ldots,s, \tag{26}$$

$$\sum_{j=1}^{n}\lambda_{jr} = 1, \quad r = 1,\ldots,n, \tag{27}$$

$$\lambda_{jr} \ge 0, \quad r, j = 1,\ldots,n, \tag{28}$$

$$z_i^- \ge 0, \quad i = 1,\ldots,m, \tag{29}$$

$$v_i^- \ge 0, \quad i = 1,\ldots,m. \tag{30}$$

In this case study, v_i^- designates the nonradial input slacks inferred with regard to the constraint (24). Additionally, h is termed as the average nonradial input inefficiency. Indeed, h can be defined as the mean of the return to normal slacks $\{z_i^- / \sum_{j=1}^{n} x_{ij}\}$ and each normalized slack $z_i^- / \sum_{j=1}^{n} x_{ij}$ to the range $[Lh, Uh]$ is delimited by the constraint (24). Mutually, the average nonradial input inefficiency for the existing DMU gives details h. Hence, perversion from the mean value h can be limited to be consistent with the lower bound L and upper bound U, accordingly.

Once this model is solved, the conforming vector $\lambda_r^* = (\lambda_{1r}^*, \lambda_{2r}^*, \ldots, \lambda_{nr}^*)$ can be determined for each DMUr to achieve the projected point which should be gained. The targets of the inputs and outputs of each of the mentioned points can be computed in a similar manner as follows:

$$\begin{aligned} x_{ir}^* &= \sum_{j=1}^{n}\lambda_{jr}^* x_{ij}, \quad r = 1,\ldots,n,\ i = 1,\ldots,m, \\ y_{kr}^* &= \sum_{j=1}^{n}\lambda_{jr}^* y_{kj}, \quad r = 1,\ldots,n,\ k = 1,\ldots,s. \end{aligned} \tag{31}$$

According to Avkiran et al. [16], it is motivating to discuss the following propositions.

Proposition 1. *If $L = 1$ or $U = 1$, then $\theta^* = \gamma^*$.*

Proof. By considering $L = 1$, we will have $z_i^- / \sum_{j=1}^{n} x_{ij} \ge h = (1/m)\sum_{i=1}^{m}(z_i^- / \sum_{j=1}^{n} x_{ij})$ $(\forall i)$ and then $z_i^- = h\sum_{j=1}^{n} x_{ij}$ because h is the mean of $z_i^- / \sum_{j=1}^{n} x_{ij}$. Now, we can remove the constraint (24) since the right constraint (24) is spontaneously contented.Thus, we can obtain CRA-BCCI by substituting $1-h$ to θ and we have $\theta^* = \gamma^*$. By the same token, if $U = 1$, then $\theta^* = \gamma^*$. □

Proposition 2. *If $L = 0$ and $U \ge m$, then $\rho^* = \gamma^*$.*

Proof. By taking into account $U \ge m$, since h is the mean of $z_i^- / \sum_{j=1}^{n} x_{ij}$ the right constraint (24) is contented and by considering $L = 0$, the left constraint (24) is also spontaneously contented. Moreover, because the objective function is to reduce in z_i^-, the term v_i^- disappears in the constraint (25). Obviously, we can obtain CRA-SBMI and we will have then $\rho^* = \gamma^*$. □

It can be realized by the above two propositions that the CRA-CSBMI model consists of CRA-BCCI and CRA-SBMI as particular states in terms of the values intended for L and U and as if the logical sort of L is $[0, 1]$ and U is $[1, m]$, too.

It is worth mentioning that with these methods not only all of the inefficient DMU*s* will be projected onto the frontier line but also the efficient DMUs are projected onto a certain point on the frontier line. This model differs from the previous model in a number of noteworthy ways.

(a) The decrease in the rth input is wasted while there is a rise in the kth output generated with no restrictions to select the preference quantities.

(b) This model gives some freedom to the perversion of the normalized slacks but it contains the assumed range around the average. While all of the inputs decrease equally in the radial models, they supply the highest value of reduction in all the inputs and also input ingredients are not all reduced in an equal manner for nonradial models.

(c) This model does not require solving the phase-II model in CRA-BCCI because all of the input constraints are binding in every optimal solution.

(d) The feasible region will be narrowed down, if L increases (U decreases) and hence the γ^* will increase.

The aggregate input or output can be minimized or maximized in the CRA models. With regard to the proposed model in this paper, which follows the idea of the CRA models, the benchmark for each DMU can be obtained so that the benchmark can have a lesser value of the total inputs but a bigger value of the total outputs compared to the DMU itself. Therefore, this study can yield results in order to supply an explanation for the suggested model. By this illustration, we demonstrate that the CRA-CSBM is a particular state which differs from the previous work in this field.

As exhibited in Table 1, we assume three DMUs, namely A, B, and C, containing two inputs along with one single constant output wherein DMUs A and B remain efficient while being placed on the frontier line, whereas DMU C

Table 1: The data related to the three DMUs.

DMU	Existing		
	x_1	x_2	y
A	3	10	1
B	4	1	1
C	4	25	1
Total	11	36	3

remains inefficient in the entire conventional DEA model. It should be asserted that although we extracted the original data from Avkiran et al. [16], the idea slightly modified in this study to become more suitable for our method. Hereby, our assumption was that there existed a centralized decision maker (DM) that was potential of overseeing the entire DMUs while it is simultaneously interested to reduce the global input consumption. Besides, we made use of the formula $U = m-(m-1)L$ which could be suitably determined in this case as $U = 2 - L$. Subsequently, by dialing a quantity among 0-1, we modified the appropriateness parameter L from the nonradial to the radial model.

For illustrating the mentioned projected points, we were able to distinguish them after solving the CRA-CSBMI model for $L = 0, 0.2, 0.588, 0.8$ and 1, while utilizing the variable λ^*_{jr}'s (solved by GAMS which is known as a potent software package). Table 2 demonstrates the results which were acquired by analyzing the proposed model in line with the lower L and the upper U amounts.

The feasible region (which is shaped by the dashed lines) is demarcated by L and U values as illustrated in Figure 1. Provided that $0 \leq L \leq 1$ and $U \geq m$ hold, the CRA-BCCI and CRA-SBMI solutions have been contained within the feasible region in any choice of L and U. Moreover, Figure 2 shows the difference existing in projection onto the efficient frontier by the radial CRA-BCCI and the nonradial CRA-SBMI models while setting the proportionality parameter $L = 0$ and $L = 1$ corresponding to [CRA-SBMI] and [CRA-BCCI], in that order. Consistent with the magnitude of L, the normalized slacks $z_i^- / \sum_{j=1}^n x_{ij}$ revealed a tendency to be uniform. To be precise, a slight L allows fairly great variations of the normalized slacks; contrariwise, a huge L limits them to a restricted range.

At $L = 0$, both DMU A and DMU C were, respectively, reflected onto DMU B and DMU A while DMU B was reflected onto itself which were CRA-SBMI solution's points. Figure 2 exhibits the results relevant to this case. The normalized deviations $z_1^{-*} / \sum_{j=1}^n x_{1j}$ and $z_2^{-*} / \sum_{j=1}^n x_{2j}$ are considered as the quantities 0 and 0.66. It is maintained that the proposed model at $L = 0.2$ would indicate that DMU A could be reflected onto itself while DMU B is done so onto a portion of the frontier line; in addition, DMU C could be reflected onto DMU B. Then, $z_1^{-*} / \sum_{j=1}^n x_{1j}$ is augmented to 0.05, whereas $z_2^{-*} / \sum_{j=1}^n x_{2j}$ is reduced to 0.51. Moreover, DMU C was reflected onto DMU A at $L = 0.588$, whereas DMU A was projected onto itself and DMU B was done so onto a nonextreme point related to the frontier line. Similarly, DMU B and DMU C were projected at $L = 0.8$ onto DMU A,

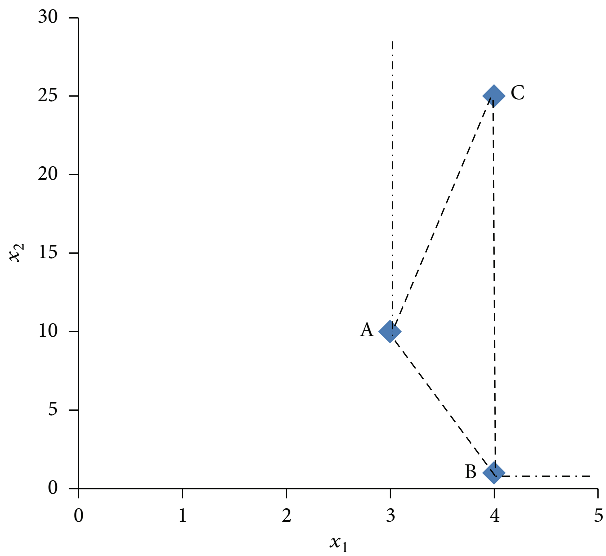

Figure 1: Feasible region bounded by L and U.

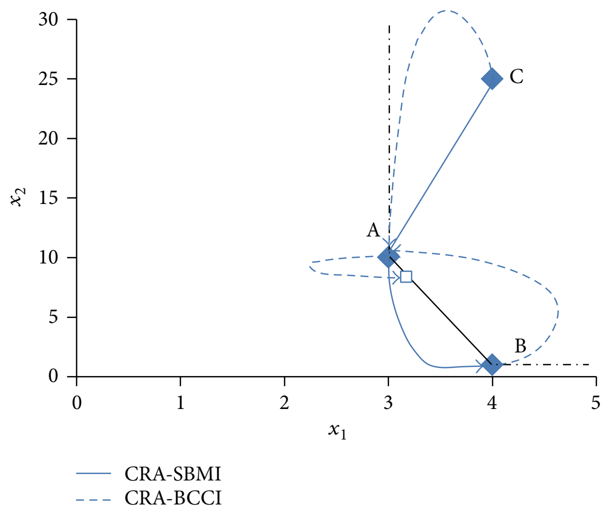

Figure 2: CRA-BCCI and CRA-SBMI projections onto the frontier line.

whereas DMU A was done so onto a nonextreme point related to the frontier line. Lastly, DMUs B and C were reflected onto the DMU A at $L = 1$, whereas DMU A was reflected onto $x_1^* = 3.04$ and $x_2^* = 9.60$; as shown in Figure 2, these latter are considered as the points related to the CRA-BCCI solution. Because the normalized deviations $z_1^{-*} / \sum_{j=1}^n x_{1j}$ and $z_2^{-*} / \sum_{j=1}^n x_{2j}$ are the equal value of 0.17, the two DMUs are correspondingly decreased by 17%. The point of interest is that the group efficiency score (γ^*) upsurges from 0.66 (CRA-SBMI) to 0.82 (CRA-BCCI); in other words, the quantity γ^* upsurges in L while it diminishes in U.

We need to notice that by assuming the model proposed by Avkiran et al. [16], for this example, only DMU C projects onto the frontier line while it is potential of acquiring diverse projections for DMU C. Conversely, in our proposed model,

Table 2: The results related to the three DMUs.

L		0			0.2			0.588			0.8			1		
γ^*		0.66			0.71			0.77			0.80			0.82		
$h^*(1-\gamma^*)$		0.34			0.29			0.23			0.20			0.18		
$z_1^{\ *}$		0			0.62			1.42			1.72			1.95		
$z_2^{\ *}$		24			18.36			11.18			8.47			6.40		
$z_1^{-*}/\sum_{j=1}^{n} x_{1j}$		0			0.05			0.12			0.15			0.17		
$z_2^{-*}/\sum_{j=1}^{n} x_{2j}$		0.66			0.51			0.31			0.23			0.17		
Projections		x_1^*	x_2^*	y^*	x_1^*	x_2^*	y^*	x_1^*	x_2^*	y^*	x_1^*	x_2^*	y^*	x_1^*	x_2^*	y^*
DMU	A	4	1	1	3	10	1	3	10	1	3.2	7.5	1	3.04	9.60	1
	B	4	1	1	3.3	6.6	1	3	10	1	3	10	1	3	10	1
	C	3	10	1	4	1	1	3.5	4.8	1	3	10	1	3	10	1
Total		11	12	3	10.3	17.6	3	9.5	24.8	3	9.2	27.5	3	9.04	29.6	3

the inefficient DMUs were able to project onto the frontier line; in addition, the efficient DMUs could project onto the other part of the frontier line.

4. Extension

The proposed approach can be developed into numerous statuses such as being output-oriented, having different settings of the lower bounds or the upper bounds, and being nonoriented. The next step is to improve our procedure to a weighted nonoriented model.

Having considered the DMUs and their matching input and output vectors, the extended centralized resource allocation connected-SBM model (ECRA-CSBM) is reported as follows:

$$\text{Model (2)}: \quad \varphi^* = \text{Min}\, \frac{1-h}{1+g} \tag{32}$$

$$\text{s.t.} \quad h = \frac{1}{m}\sum_{i=1}^{m} \frac{w_i z_i^-}{\sum_{j=1}^{n} x_{ij}} \tag{33}$$

$$g = \frac{1}{s}\sum_{k=1}^{s} \frac{w_k z_k^+}{\sum_{j=1}^{n} y_{kj}} \tag{34}$$

$$L^- h \le \frac{w_i z_i^-}{\sum_{j=1}^{n} x_{ij}} \le U^- h, \quad i = 1,\dots,m, \tag{35}$$

$$L^+ g \le \frac{w_k z_k^+}{\sum_{j=1}^{n} y_{kj}} \le U^+ g, \quad k = 1,\dots,s, \tag{36}$$

$$\sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr} x_{ij} + z_i^- + v_i^- = \sum_{j=1}^{n} x_{ij}, \quad i = 1,\dots,m, \tag{37}$$

$$\sum_{r=1}^{n}\sum_{j=1}^{n} \lambda_{jr} y_{kj} - z_k^+ - v_k^+ = \sum_{j=1}^{n} y_{kj}, \quad k = 1,\dots,s, \tag{38}$$

$$\sum_{j=1}^{n} \lambda_{jr} = 1, \quad j = 1,\dots,n, \tag{39}$$

$$\sum_{i=1}^{m} w_i = m, \tag{40}$$

$$\sum_{k=1}^{s} w_k = k, \tag{41}$$

$$z_i^- \ge 0, \quad v_i^- \ge 0, \quad w_i^- \ge 0, \quad i = 1,\dots,m, \tag{42}$$

$$z_k^+ \ge 0, \quad v_k^+ \ge 0, \quad w_k^+ \ge 0, \quad k = 1,\dots,s, \tag{43}$$

$$\lambda_{jr} \ge 0, \quad r, j = 1,\dots,n, \tag{44}$$

where v_k^+ denotes the nonradial output slacks concluded in connection with the constraint (36). The constraint (36) has the same properties as the constraint (24), which was observed in the input-oriented model. Moreover, the input and output take different amounts of L and U; the perversion from the mean value h can be restricted according to the lower bound L^- and the upper bound U^-. Moreover, the perversion from the mean value g can be restricted according to the lower bound L^+ and upper bound U^+. Additionally, the weights (w_i^- and w_k^+) are provided in an exogenous manner and the average, the lower, and the upper bounds will be affected by an input or an output item with a great weight. In fact, w_i^- and w_k^+ are the preference coefficients for the decreases and increases of the total consumption and the production of input i and output k, respectively.

It is significant to mention the following properties of the above model.

(a) Unlike the previous model, this model is not a linear programming problem but can be simply converted into a linear programming problem by using Charnes and Cooper [34] conversions (see Hosseinzadeh Lotfi et al. [32] for particularity) in the first step and then choosing two appropriate independent w_i^- and w_k^+ as the preference coefficients for the decreases and increases of the total consumption and the production of input i and output k, respectively.

(b) All input and output constraints are binding in each optimal solution.

(c) For any DMUr, the points $x_{ir}^* = \sum_{j=1}^{n} \lambda_{jr}^* x_{ij}$, $\forall i = 1, \ldots, m$ and $y_{kr}^* = \sum_{j=1}^{n} \lambda_{jr}^* y_{kj}$, $\forall k = 1, \ldots, s$ are the Pareto efficient.

(d) The φ^* obtained from the objective function is $0 < \varphi^* \le 1$.

A numerical model of this has been provided by Lozano and Villa's study [17] wherein seven DMUs with an input could engender an output. We need to bear in mind the proposed model as in the cases of $L^- = 0$, $U^- = 2$, $L^+ = 0$, and $U^+ = 1$. Further, we were able to acquire the projecting point, in order to illustrate it afterward solving our developed model as well as utilizing the variable λ_{jr}^*'s (which was solved using the GAMS known as a potent software package). Figures 3, 4, and 5 present the graphical interpretation related to the CRA-BCCI, CRA-SBMI, and our proposed model while Table 3 exhibits the original DMUs along with the ones to which they are reflected by making use of the CRA-BCCI, CRA-SBMI, and the proposed model. As it can be discerned, our developed method finds alternative projection points in comparison with developed procedure which finds alternative projection points compared with previous procedures.

As it is apparent in Lozano and Villa's model [17], the entire DMUs are reflected onto DMU 2; the exception goes with the inefficient DMU 3. Moreover, DMU 3 is reflected onto a nonextreme point related to the frontier line. However, DMUs 1, 4, 5, 6, and 7 in the approach proposed by Hosseinzadeh Lotfi et al. [32] are projected onto the identical projections point (4.13, 8.26) of the frontier line. Likewise, DMUs 4, 5, and 6 are reflected onto the equivalent nonextreme point (5.16, 10.32) of the frontier line.

Stimulatingly, the findings obtained from the proposed model reveal that the entire DMUs were reflected onto the DMU 3; in other words, they were reflected onto the identical hyperplan of the frontier line. When establishing a comparison between these results, it is observed that the quantity of the global output production to the global input decrease of illustration DMUs are respectively, 1.95 and 2 for the Lozano and Villa's approach [17] and the Hosseinzadeh Lotfi et al.'s approach [32]. Yet, this proportion equals 2 in the proposed approach which resembles that of the Hosseinzadeh Lotfi et al.'s approach [32]. By this, we can conclude that the propounded method is precisely suitable.

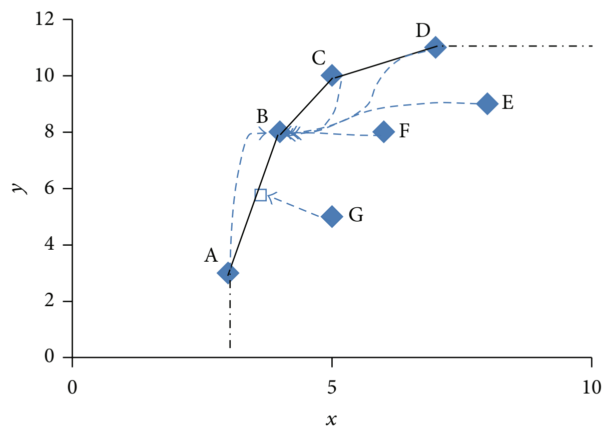

Figure 3: The CRA-BCCI projections onto the frontier line.

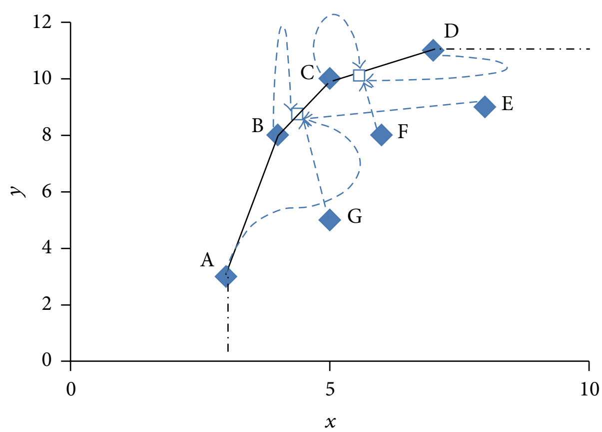

Figure 4: The CRA-SBMI projections onto the frontier line.

5. An Application

We need to assume an empirical example on gas companies during 2008 [33] for better investigating the models and demonstrating their competences. The National Gas Company (NGC) of Iran encompasses 14 large branches which are situated in 13 provinces of Iran. While every branch functions independently in its province, the companies are entirely categorized under the NGC. Distributing the gas to domestic and industrial customers is reported as the branches' foremost task that is these 14 companies are responsible for distributing the service to end users (domestic and industrial). These companies have utilized seven variables from the dataset with three inputs (budget, number of staff, comprehensive cost (operational cost and labour cost)), and four outputs (number of customers, length of gas network (km), delivered volumes (m^3), and sold-out gas (Rials)), which are labeled as x_1, x_2 and x_3 for inputs and y_1, y_2, y_3 and y_4 for outputs in Table 4.

It is distinguished in DEA that those DMUs are more efficient which consume less input to yield more output; therefore, they are typically taken into account as benchmarks [19, 35]. Traditional DEA models can be employed for acquiring an appropriate benchmark for each DMU. The point to highlight is that the DMU benchmark consumes a lesser sum of inputs while yielding a bigger quantity of outputs in comparison with the DMU itself. Indeed the mentioned models separately decline (elevate) the inputs (outputs) of each DMU. Nevertheless, the results differ if centralized models are used. The reason is that such models are unable to decline (elevate) the inputs (outputs) of the individual DMUs.

It should be highlighted that the centralized models reduce (elevate) the aggregate inputs (outputs). Then the benchmark gained by our proposed method for each DMU

Table 3: Data and results of the seven DMUs with one-input and one-output.

DMU	Existing		Lozano and Villa's approach		Hosseinzadeh Lotfi et al.'s approach		Proposed approach	
	x	y	x^*	y^*	x^*	y^*	x^*	y^*
1	3	3	4	8	4.13	8.26	5	10
2	4	8	4	8	4.13	8.26	5	10
3	5	5	3.6	6	4.13	8.26	5	10
4	5	10	4	8	5.16	10.32	5	10
5	6	8	4	8	5.16	10.32	5	10
6	7	11	4	8	5.16	10.32	5	10
7	8	9	4	8	4.13	8.26	5	10
Total	38	54	27.6	54	32	64	35	70

Table 4: Data related to 14 gas companies.

DMU	x_1	x_2	x_3	y_1	y_2	y_3	y_4
1	177,430	401	528,325	801	41,675	77,564	201,529
2	221,338	1,094	1,186,905	803	34,960	44,136	840,446
3	267,806	1,079	1,323,325	251	24,461	27,690	832,616
4	160,912	444	648,685	816	23,744	45,882	251,770
5	177,214	801	909,539	654	36,409	72,676	443,507
6	146,325	686	545,115	177	18,000	19,839	341,585
7	195,138	687	790,348	695	31,221	40,154	233,822
8	108,146	152	236,722	606	23,889	37,770	118,943
9	165,663	494	523,899	652	25,163	28,402	179,315
10	195,728	503	428,566	959	43,440	63,701	195,303
11	87,050	343	298,696	221	9,689	17,334	106,037
12	124,313	129	198,598	565	21,032	30,242	61,836
13	67,545	117	131,649	152	10,398	14,139	46,233
14	47,208	165	228,730	211	9,391	13,505	42,094
Total	2,141,816	7,095	7,979,102	7,563	353,472	533,034	3,895,036

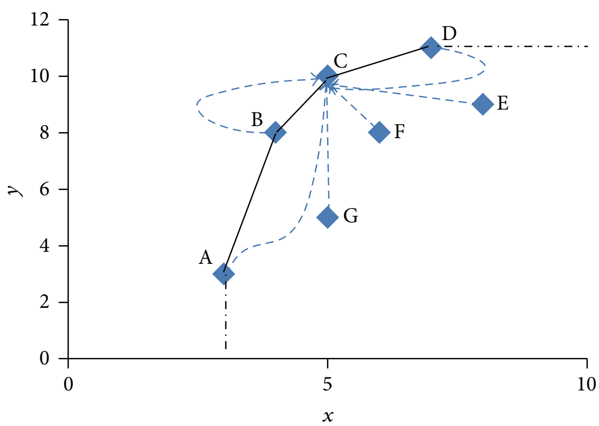

Figure 5: The CRA-CSBM projections onto the frontier line.

reveals a slighter amount of aggregate inputs and a corresponding larger amount of aggregate outputs than the DMU itself because of the fact that the proposed approach lends its basis to the centralized models. We need to take into consideration each of the mentioned branches as a DMU with the purpose of attaining proper benchmarks for the branches and then solve the proposed model (1) for the DMUs in the case of $L = 0.588$. In details, the relevant results are provided in Table 5.

If we take into account branch 5 as an example, the first, second, and third inputs related to this branch are a smaller amount than the relevant inputs of the benchmark shown for this DMU, according to Table 5. Considering the related outputs, it should be also mentioned that only the benchmark's first output is a smaller amount than the output of the DMU itself; this is not in effect a desirable quantity from the DEA viewpoint. At this point, let us consider branch 14 while its relevant results are unlike: the entire outputs related to the mentioned branch are smaller than the outputs of the benchmark given for the branch, with the approximation used, which is theoretically more desired. Still, the branch's first and third inputs are smaller than the inputs related to the benchmark; in fact, since the objective in DEA is to detect a benchmark for a DMU having a slighter input and a larger output than the DMU itself, this amount is not desired from the DEA perspective. As for branch 9, it is observed that its

Table 5: Benchmarks for each branch by using the proposed model.

DMU	x_1^*	x_2^*	x_3^*	y_1^*	y_2^*	y_3^*	y_4^*
1	108,146.06	152.00	236,722.47	606.00	23,889.00	37,770.00	118,943.22
2	108,146.06	152.00	236,722.47	606.00	23,889.00	37,770.00	118,943.22
3	177,213.97	801.00	909,538.52	654.00	36,408.98	72,675.95	443,506.77
4	84,935.82	245.52	322,248.05	402.22	17,844.93	29,515.80	121,358.87
5	108,146.06	152.00	236,722.47	606.00	23,889.00	37,770.00	118,943.22
6	69,319.49	160.28	231,630.37	354.33	14,651.62	22,309.59	69,978.98
7	108,146.06	152.00	236,722.47	606.00	23,889.00	37,770.00	118,943.22
8	187,249.86	810.31	900,754.21	743.67	31,625.93	42,218.86	623,162.83
9	187,249.86	810.31	900,754.21	743.67	31,625.93	42,218.86	623,162.83
10	182,913.27	787.18	876,891.51	728.93	30,989.15	41,456.01	603,280.41
11	108,146.06	152.00	236,722.47	606.00	23,889.00	37,770.00	118,943.22
12	108,146.06	152.00	236,722.47	606.00	23,889.00	37,770.00	118,943.22
13	187,249.86	810.31	900,754.21	743.67	31,625.93	42,218.86	623,162.83
14	72,320.18	159.64	232,023.91	373.78	15,365.53	23,504.44	73,763.150
Total	1,797,328.68	5,496.57	6,694,929.80	8,380.27	353,473	542,738.35	3,895,037

entire inputs are smaller than the benchmark's inputs which are given for the branch and again this is not desired from the DEA perspective; however, the entire outputs related to this branch are smaller than the ones for the benchmark presented for the branch, with the approximation utilized, which is theoretically desired.

It can be discerned that our proposed approach lends its basis to the centralized models wherein not all the inputs (outputs) are probably reduced (elevated). Indeed, it is possible to reduce/elevate a DMU's inputs/outputs only in case the traditional DEA models are exploited.

In practice, the centralized models aim at reducing the aggregate inputs while elevating the aggregate outputs. It is observed that the benchmarks' aggregate inputs/outputs are reduced/elevated in comparison with the corresponding branches and this in turn is said to be desirable. The quantities of the reduction in the provided benchmarks' aggregate inputs, compared to the respective branches, are 1,797,328.68, 5,496.57 and 6,694,929.80 for the first, second, and third inputs, respectively. As for the aggregate outputs, the ones related to the benchmarks show a rise of 8,380.27, 353,473, 542,738.35, and 3,895,037 for the first, second, third, and fourth outputs, respectively; in effect, this can be desirable while taking into account the properties associated with the centralized models.

It should be noticed that branches 8, 9, and 13 entail identical projection points on the efficient frontier; therefore, they entail the similar benchmark, too. Moreover, the projection points related to branches 1, 2, 5, 7, 11, and 12 coincide with the ones related to branch 8, implying the fact that they entail the similar benchmark. Besides, the remainder of the branches is reflected onto the other part of the frontier line.

At this instant, with the purpose of comparing the approach introduced in this paper with the ones presented in the previous models, we would be able to attain the benchmarks consistent with the branches by the approach utilized by the CRA-BCCI model [17] and CRA-SBMI model [32] which stand as two extreme cases of the CRA-CSBM model.

Furthermore, in case of employing the connected-SBM [16] for attaining the benchmark for every branch, the acquired results will indicate that the benchmark presented for every branch contains a slighter quantity of inputs as well as a larger quantity of outputs than the branch itself. Nonetheless, neither the CRA-BCCI and CRA-SBMI nor the model proposed in this paper has such characteristic; it is impossible to declare that the entire input/output components corresponding to each branch are slighter/larger than the ones for each single branch. We can merely affirm that the aggregate input/output related to the benchmark for every branch is slighter/larger than that of the branch itself.

6. Discussions and Conclusions

There is a need for a different approach which can project the entire units simultaneously because of the fact that by now the conventional DEA models set the targets independently for each DMU. In brief, an intraorganizational scenario was addressed in this paper wherein the entire units could be categorized under the supervision of a centralized decision maker. This decision maker required them to be efficient and considered the total input consumption and total output production. A new DEA model is introduced in this paper for centralized resource allocation.

The foremost finding here is illustrating a scheme for integrating the radial and nonradial CRA-DEA models with the intention of controlling the proportionality of slacks. Such a control will supply the decision maker with a superior chance of taking the appropriate benchmarks which stand as superior reflections of the expected patterns of the potential improvements. Two parameters are used by the CRA-CSBM model to limit the variations related to the normalized slacks in a certain range. The renowned CRA-BCC and

CAR-SBM models are considered as the two extreme cases of the mentioned model. Likewise, the sensitivity analysis by altering the CRA-CSBM model's parameters can allow incapacitating the latent problem of the mixed value slacks that would then confuse other analyses which rely on slacks.

We are able to project the entire DMUs in our method onto the efficient frontier. This can be achieved through solving only one model. Furthermore, in comparison with the preceding methods we can attain unalike benchmarks for all the DMUs by exploiting the given approximation. We also indicated by presenting an applied example that more appropriate benchmarks can be commonly acquired by the suggested approximation than by the preceding ones. Besides, the appropriate benchmarks can be suitably presented for the entire DMUs through solving one model for the case wherein the amount of DMUs is big. Moreover, for obtaining an insight into the way this approach functions, a graphical interpretation of two-dimensional cases are also provided.

To put it briefly, researchers and practitioners in the DEA field can currently match the estimated contraction of the resources and expansion of the outputs in a certain production system for establishing more realistic benchmarks. The proposed approach here is fairly simple which is also able to be simply extended in different directions. In effect, we are able to project only the inefficient DMUs by splitting the efficient and inefficient DMUs. The proposed models can also be employed for discretionary and nondiscretionary data.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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