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*Research Article*

# Parameter Dependence of Positive Solutions for Second-Order Singular Neumann Boundary Value Problems with Impulsive Effects

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The author considers the Neumann boundary value problem $-y''(t) + \mathbf{M}y(t) = \lambda\omega(t) f(t, y(t)), t \in J,\ t \neq t_k,\ -\Delta y'|_{t=t_k} = \lambda I_k(t_k, y(t_k)),\ k = 1, 2, \ldots, m,\ y'(0) = y'(1) = 0$ and establishes the dependence results of the solution on the parameter $\lambda$, which cover equations without impulsive effects and are compared with some recent results by Nieto and O'Regan.

## 1. Introduction

Impulsive effects exist widely in many evolution processes in which their states are changed abruptly at certain moments of time. The theory and applications of impulsive differential equations have been emerging as an important area of investigation in recent years [1–6]. There is a vast literature on the existence of solutions by using different methods such as bifurcation theory [7, 8], fixed point theorems in cones [9–12], the method of lower and upper solutions [13, 14], and the theory of critical point theory and variational methods [15–19]. We remark that on second-order impulsive differential equations with parameter only a few results have been obtained; see, for instance, [9, 20–22]. To the best of our knowledge, these papers only consider the existence of positive solutions. However, the corresponding results for the dependence of the solution on the parameter $\lambda$ for second-order impulsive differential equations are not investigated until now. In this paper, we try to solve this kind of problem.

Consider the Neumann boundary value problems

$$\begin{gathered} -y''(t) + \mathbf{M}y(t) = \lambda\omega(t) f(t, y(t)), \\ t \in J, \quad t \neq t_k, \\ -\Delta y'\big|_{t=t_k} = \lambda I_k(t_k, y(t_k)), \quad k = 1, 2, \ldots, m, \\ y'(0) = y'(1) = 0, \end{gathered} \tag{1}$$

where $\mathbf{M} > 0$ is a constant, $\lambda > 0$ is a parameter, $J = [0, 1]$, $\omega$ is a nonnegative measurable function on $(0, 1)$, $\omega \neq 0$ on any open subinterval in $(0, 1)$, which may be singular at $t = 0$ and/or $t = 1$, $t_k$ $(k = 1, 2, \ldots, m)$ (where $m$ is fixed positive integer) are fixed points with $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots < t_m < t_{m+1} = 1$, and $\Delta y'|_{t=t_k} = y'(t_k^+) - x'(t_k^-)$, where $y'(t_k^+)$ and $y'(t_k^-)$ represent the right-hand limit and left-hand limit of $y'(t)$ at $t = t_k$, respectively. In addition, $\omega$, $f$, and $I_k$ satisfy

($H_1$) $\omega \in L^1_{\text{loc}}(0, 1)$;

($H_2$) $f \in C(J \times \mathscr{R}^+, \mathscr{R}^+)$, $I_k \in C(J \times \mathscr{R}^+, \mathscr{R}^+)$, where $\mathscr{R}^+ = [0, +\infty)$, $k = 1, 2, \ldots, m$.

Some special cases of (1) have been investigated. For example, Nieto and O'Regan [17] studied problem (1) with $\lambda = 1$ and $\omega \equiv 1$ for $t \in J$. By using variational methods and critical point theory, the authors proved the existence of solutions of problem (1).

For ease of exposition, we set

$$
\begin{aligned}
f^0 &= \limsup_{y \to 0^+} \max_{t \in J} \frac{f(t,y)}{y}, \\
f^\infty &= \limsup_{y \to \infty} \max_{t \in J} \frac{f(t,y)}{y}, \\
f_0 &= \liminf_{y \to 0^+} \min_{t \in J} \frac{f(t,y)}{y}, \\
f_\infty &= \liminf_{y \to \infty} \min_{t \in J} \frac{f(t,y)}{y}, \\
I^0(k) &= \limsup_{y \to 0^+} \max_{t \in J} \frac{I_k(t,y)}{y}, \\
I^\infty(k) &= \limsup_{y \to \infty} \max_{t \in J} \frac{I_k(t,y)}{y}, \\
I_0(k) &= \liminf_{y \to 0^+} \min_{t \in J} \frac{I_k(t,y)}{y}, \\
I_\infty(k) &= \liminf_{y \to \infty} \min_{t \in J} \frac{I_k(t,y)}{y}, \\
& \qquad k = 1, 2, \ldots, m.
\end{aligned} \tag{2}
$$

Our main results are as follows.

**Theorem 1.** *Assume that* $(H_1)$ *and* $(H_2)$ *hold. Then the following two conclusions hold:*

$(\mathrm{H}_3)$ *if* $f^0 = 0$, $I^0(k) = 0$, *and* $f_\infty = \infty$, $I_\infty(k) = \infty$, $k = 1, 2, \ldots, m$, *then for every* $\lambda > 0$ *problem* (1) *has a positive solution* $y_\lambda(t)$ *satisfying* $\lim_{\lambda \to 0^+} \|y_\lambda\|_{PC^1} = \infty$;

$(\mathrm{H}_4)$ *if* $f_0 = \infty$, $I_0(k) = \infty$, *and* $f^\infty = 0$, $I^\infty(k) = 0$, $k = 1, 2, \ldots, m$, *then for every* $\lambda > 0$ *problem* (1) *has a positive solution* $y_\lambda(t)$ *satisfying* $\lim_{\lambda \to 0^+} \|y_\lambda\|_{PC^1} = 0$.

*Remark 2.* Assume that $(\mathrm{H}_1)$ and $(\mathrm{H}_2)$ hold. Furthermore, suppose that $f_\infty = \infty$ or $I_\infty(k) = \infty$, $k = 1, 2, \ldots, m$, in $(\mathrm{H}_3)$ or $f_0 = \infty$ or $I_0(k) = \infty$, $k = 1, 2, \ldots, m$, in $(\mathrm{H}_4)$. Then the conclusions of Theorem 1 also hold.

*Remark 3.* It follows from the conditions of Theorem 1 that we develop some ideas of Guo and Lakshmikantham essentially; for detail, see Theorem 2.3.7 in [23].

*Remark 4.* For simplicity we only consider Neumann boundary conditions since all the results obtained in this paper can also be adapted with minor changes to the other boundary conditions.

## 2. Preliminaries

Let $J' = J \setminus \{t_1, t_2, \ldots, t_m\}$, $k = 1, 2, \ldots, m$, and

$$
\begin{aligned}
PC^1[0,1] = \Big\{ y \in C[0,1] : \; & y' \big|_{(t_k, t_{k+1})} \in C(t_k, t_{k+1}), \\
& y'(t_k^-), y'(t_k^+) \text{ exists}, \\
& k = 1, 2, \ldots, m \Big\}.
\end{aligned} \tag{3}
$$

Then $PC^1[0,1]$ is a real Banach space with norm

$$
\|y\|_{PC^1} = \max\left\{ \|y\|_\infty, \|y'\|_\infty \right\}, \tag{4}
$$

where $\|y\|_\infty = \sup_{t \in J} |y(t)|$, $\|y'\|_\infty = \sup_{t \in J} |y'(t)|$.

A function $y \in PC^1[0,1] \cap C^2(J')$ is called a solution of problem (1) if it satisfies (1).

In our main results, we will make use of the following lemmas.

**Lemma 5.** *If* $(H_1)$ *and* $(H_2)$ *hold, then problem* (1) *has a unique solution* $y$ *given by*

$$
\begin{aligned}
y(t) = {} & \lambda \int_0^1 G(t,s)\, \omega(s)\, f(s, y(s))\, ds \\
& + \lambda \sum_{k=1}^m G(t, t_k)\, I_k(t_k, y(t_k)),
\end{aligned} \tag{5}
$$

*where*

$$
G(t,s) = \begin{cases} \dfrac{\cosh\left(\sqrt{\mathbf{M}}(1-t)\right)\cosh\left(\sqrt{\mathbf{M}}s\right)}{\sqrt{\mathbf{M}}\sinh\left(\sqrt{\mathbf{M}}\right)}, & 0 \le s \le t \le 1, \\[2ex] \dfrac{\cosh\left(\sqrt{\mathbf{M}}t\right)\cosh\left(\sqrt{\mathbf{M}}(1-s)\right)}{\sqrt{\mathbf{M}}\sinh\left(\sqrt{\mathbf{M}}\right)}, & 0 \le t \le s \le 1. \end{cases} \tag{6}
$$

*Proof.* The proof is similar to that of Lemma 2.4 in [24]. □

By (6), we can prove that $G(t,s)$ has the following property:

$$
\frac{1}{\sqrt{\mathbf{M}}\sinh\left(\sqrt{\mathbf{M}}\right)} = \alpha \le G(t,s) \le \beta = \frac{\cosh^2\left(\sqrt{\mathbf{M}}\right)}{\sqrt{\mathbf{M}}\sinh\left(\sqrt{\mathbf{M}}\right)}, \quad \forall t, s \in J. \tag{7}
$$

Define a cone in $PC^1[0,1]$ by

$$
K = \left\{ y \in PC^1[0,1] : y \ge 0,\ y(t) \ge \delta \|y\|_{PC^1},\ t \in J \right\}, \tag{8}
$$

where

$$
\delta = \frac{1}{\cosh^2\left(\sqrt{\mathbf{M}}\right)}. \tag{9}
$$

It is easy to see $K$ is a closed convex cone of $PC^1[0,1]$.

Define an operator $T_\lambda : K \to \mathrm{PC}^1[0,1]$ by

$$(T_\lambda y)(t) = \lambda \int_0^1 G(t,s)\,\omega(s)\, f(s,y(s))\,ds + \lambda \sum_{k=1}^{m} G(t,t_k)\, I_k(t_k, y(t_k)). \tag{10}$$

From (10), we know that $y \in \mathrm{PC}^1[0,1]$ is a solution of problem (1) if and only if $y$ is a fixed point of operator $T_\lambda$.

**Lemma 6.** *Suppose that* $(H_1)$ *and* $(H_2)$ *hold. Then* $T_\lambda(K) \subset K$ *and* $T_\lambda : K \to K$ *is completely continuous.*

*Proof.* For $y \in K$, it follows from (7) and (10) that

$$\begin{aligned}(Ty)(t) &= \lambda \int_0^1 G(t,s)\,\omega(s)\, f(s,y(s))\,ds + \lambda \sum_{k=1}^{m} G(t,t_k)\, I_k(t_k, y(t_k)) \\ &\le \lambda \frac{\cosh^2(\sqrt{\mathbf{M}})}{\sqrt{\mathbf{M}}\sinh(\sqrt{\mathbf{M}})} \times \left[\int_0^1 \omega(s)\, f(s,y(s))\,ds + \sum_{k=1}^{m} I_k(t_k, y(t_k))\right], \\ & \qquad t \in J.\end{aligned} \tag{11}$$

It follows from (7), (10), and (11) that

$$\begin{aligned}(Ty)(t) &= \lambda \int_0^1 G(t,s)\,\omega(s)\, f(s,y(s))\,ds + \lambda \sum_{k=1}^{m} G(t,t_k)\, I_k(t_k, y(t_k)) \\ &\ge \frac{1}{\sqrt{\mathbf{M}}\sinh(\sqrt{\mathbf{M}})}\lambda \times \left[\int_0^1 \omega(s)\, f(s,y(s))\,ds + \sum_{k=1}^{m} I_k(t_k, y(t_k))\right] \\ &\ge \delta \frac{\cosh^2(\sqrt{\mathbf{M}})}{\sqrt{\mathbf{M}}\sinh(\sqrt{\mathbf{M}})}\lambda \times \left[\int_0^1 \omega(s)\, f(s,y(s))\,ds + \sum_{k=1}^{m} I_k(t_k, y(t_k))\right] \\ &\ge \delta \|Ty\|.\end{aligned} \tag{12}$$

Thus, $T(K) \subset K$.

Next, by similar arguments of Lemmas 5 and 6 [12] one can prove that $T : K \to K$ is completely continuous. So it is omitted, and the theorem is proved. □

To obtain positive solutions of problem (1), the following fixed point theorem in cones is fundamental, which can be found in [23, page 94].

**Lemma 7.** *Let $P$ be a cone in a real Banach space $E$. Assume $\Omega_1$, $\Omega_2$ are bounded open sets in $E$ with $0 \in \Omega_1$, $\overline{\Omega}_1 \subset \Omega_2$. If*

$$A : P \cap \left(\overline{\Omega}_2 \setminus \Omega_1\right) \longrightarrow P \tag{13}$$

*is completely continuous such that either*

(a) $\|Ax\| \le \|x\|$, $\forall x \in P \cap \partial\Omega_1$, *and* $\|Ax\| \ge \|x\|$, $\forall x \in P \cap \partial\Omega_2$, *or*

(b) $\|Ax\| \ge \|x\|$, $\forall x \in P \cap \partial\Omega_1$, *and* $\|Ax\| \le \|x\|$, $\forall x \in P \cap \partial\Omega_2$,

*then $A$ has at least one fixed point in $P \cap (\overline{\Omega}_2 \setminus \Omega_1)$.*

## 3. Proofs of the Main Results

For convenience we introduce some notations

$$\gamma = \int_0^1 \omega(s)\,ds, \qquad \rho = \max_{t\in J, 0\le y\le c} f(t,y), \qquad \rho_* = \max\{\rho_k,\ k = 1,2,\ldots,m\}, \tag{14}$$

where $\rho_k = \max_{t\in J, 0\le y\le c} I_k(t,y)$, $k = 1,2,\ldots,m$, and $c > 0$ is a constant.

*Proof of Theorem 1.* We need only prove this theorem under condition $(H_3)$ since the proof is similar when $(H_4)$ holds, provided the proper adjustments are made.

If $f^0 = 0$, $I^0(k) = 0$, then there exist $l > 0$ and $r > 0$ such that

$$f(t,y) < ly, \quad I_k(t,y) < ly, \qquad \forall t \in J, \quad 0 \le y \le r, \quad k = 1,2,\ldots,m, \tag{15}$$

where $l$ satisfies

$$\lambda \max\{\beta, 1\}\, l(\gamma + m) \le 1. \tag{16}$$

Then for $y \in K \cap \partial\Omega_r$ we have

$$\begin{aligned}(T_\lambda y)(t) &= \lambda \int_0^1 G(t,s)\,\omega(s)\, f(s,y(s))\,ds + \lambda \sum_{k=1}^{m} G(t,t_k)\, I_k(t_k, y(t_k)) \\ &\le \lambda\beta \int_0^1 \omega(s)\, l y(s)\,ds + \lambda\beta \sum_{k=1}^{m} l y(t_k) \\ &\le \lambda\beta l \|y\|_{\mathrm{PC}^1} \left(\int_0^1 \omega(s)\,ds + m\right) \\ &= \lambda\beta l \|y\|_{\mathrm{PC}^1}(\gamma + m) \le \|y\|_{\mathrm{PC}^1},\end{aligned}$$

$$\begin{aligned}\left|(T_\lambda y)'(t)\right| &\le \lambda \int_0^1 \left|G_t'(t,s)\right| \omega(s) f(s,y(s))\,ds \\ &\quad + \lambda \sum_{k=1}^m \left|G_t'(t,t_k)\right| I_k(t_k,y(t_k)) \\ &\le \lambda \int_0^1 \omega(s) f(s,y(s))\,ds \\ &\quad + \lambda \sum_{k=1}^m I_k(t_k,y(t_k)) \\ &\le \lambda l(\gamma+m)\|y\|_{PC^1} \\ &\le \|y\|_{PC^1},\end{aligned} \tag{17}$$

where

$$G_t'(t,s) = \begin{cases} \dfrac{-\sinh\left(\sqrt{\mathbf{M}}(1-t)\right)\cosh\left(\sqrt{\mathbf{M}}s\right)}{\sinh\left(\sqrt{\mathbf{M}}\right)}, & 0 \le s \le t \le 1, \\ \dfrac{\cosh\left(\sqrt{\mathbf{M}}t\right)\sinh\left(\sqrt{\mathbf{M}}(1-s)\right)}{\sinh\left(\sqrt{\mathbf{M}}\right)}, & 0 \le t \le s \le 1, \end{cases}$$
$$\max_{t,s\in J, t\ne s}\left|G_{1t}'(t,s)\right| = 1. \tag{18}$$

It follows from (17) that

$$\|T_\lambda y\|_{PC^1} \le \|y\|_{PC^1}, \quad \forall y \in K \cap \partial\Omega_r. \tag{19}$$

If $f_\infty = \infty$, $I_\infty(k) = \infty$, then there exist $L > 0$ and $R > r > 0$ such that

$$\begin{gathered} f(t,y) > Ly, \quad I_k(t,y) > Ly, \\ \forall t \in J, \quad y \ge R, \quad k = 1,2,\ldots,m, \end{gathered} \tag{20}$$

where $L$ satisfies

$$\lambda\alpha L\delta(\gamma+m) \ge 1. \tag{21}$$

Let $\eta = R/\delta$. Thus, when $y \in K \cap \partial\Omega_\eta$ we have

$$y(t) \ge \delta\|y\|_{PC^1} = \delta\eta = R, \quad t \in J, \tag{22}$$

and then we get

$$\begin{aligned}(T_\lambda y)(t) &= \lambda \int_0^1 G(t,s)\omega(s) f(s,y(s))\,ds \\ &\quad + \lambda \sum_{k=1}^m G(t,t_k) I_k(t_k,y(t_k)) \\ &\ge \lambda\alpha \int_0^1 \omega(s) Ly(s)\,ds + \lambda\alpha \sum_{k=1}^m Ly(t_k) \\ &\ge \lambda\alpha L\delta\|y\|_{PC^1}\left(\int_0^1 \omega(s)\,ds + m\right) \\ &= \lambda\alpha L\delta\|y\|_{PC^1}(\gamma+m) \\ &\ge \|y\|_{PC^1}.\end{aligned} \tag{23}$$

This yields

$$\|T_\lambda y\|_{PC^1} \le \|y\|_{PC^1}, \quad \forall y \in K \cap \partial\Omega_\eta. \tag{24}$$

Hence, for given $\lambda > 0$ condition (a) of Lemma 7 is satisfied of operator $T_\lambda$, which implies that $T_\lambda$ has a fixed point $y_\lambda$ in $\overline{\Omega}_\eta \setminus \Omega_r$.

It remains to prove $\|y_\lambda\|_{PC^1} = +\infty$ as $\lambda \to 0^+$. In fact, if not, there would exist a number $c > 0$ and a sequence $\lambda_n \to 0^+$ such that

$$\left\|y_{\lambda_n}\right\|_{PC^1} \le c \quad (n = 1,2,3,\ldots). \tag{25}$$

Furthermore, the sequence $\{\|y_{\lambda_n}\|_{PC^1}\}$ contains a subsequence that converges into a number $d$, where $0 \le d \le c$. For simplicity, suppose that $\{\|y_{\lambda_n}\|_{PC^1}\}$ itself converges into $d$.

If $d > 0$, then $\|y_{\lambda_n}\|_{PC^1} > d/2$ for sufficiently large $n$ ($n > \mathbb{N}$), and therefore

$$\begin{aligned}\frac{1}{\lambda_n} &= \left(\left\|\int_0^1 G(t,s)\omega(s) f\left(s,y_{\lambda_n}(s)\right)ds \right.\right. \\ &\quad \left.\left. + \sum_{k=1}^m G(t,t_k) I_k\left(t_k,y_{\lambda_n}(t_k)\right)\right\|_{PC^1}\right) \\ &\quad \times \left(\left\|y_{\lambda_n}\right\|_{PC^1}\right)^{-1} \\ &\le \frac{\beta(\gamma\rho + m\rho^*)}{\left\|y_{\lambda_n}\right\|_{PC^1}} \\ &\le \frac{2\beta(\gamma\rho + m\rho^*)}{d} \quad (n > \mathbb{N}),\end{aligned} \tag{26}$$

which contradicts $\lambda_n \to 0^+$.

If $d = 0$, then $\|y_{\lambda_n}\|_{PC^1} \to 0$ for sufficiently large $n$ ($n > \mathbb{N}$), and therefore it follows from ($H_3$) that for any $\varepsilon > 0$ there exists $b > 0$ such that

$$\begin{gathered} f\left(t,y_{\lambda_n}(t)\right) \le \varepsilon y_{\lambda_n}, \quad I_k\left(t,y_{\lambda_n}(t)\right) \le \varepsilon y_{\lambda_n}, \\ \forall y_{\lambda_n} : 0 \le y_{\lambda_n} \le b, \end{gathered} \tag{27}$$

and hence it follows from (10) that

$$\begin{aligned}\frac{1}{\lambda_n} &= \left(\left\|\int_0^1 G(t,s)\omega(s) f\left(s,y_{\lambda_n}(s)\right)ds \right.\right. \\ &\quad \left.\left. + \sum_{k=1}^m G(t,t_k) I_k\left(t_k,y_{\lambda_n}(t_k)\right)\right\|_{PC^1}\right) \\ &\quad \times \left(\left\|y_{\lambda_n}\right\|_{PC^1}\right)^{-1} \\ &\le \frac{\beta\left(\gamma\varepsilon\left\|y_{\lambda_n}\right\|_{PC^1} + m\varepsilon\left\|y_{\lambda_n}\right\|_{PC^1}\right)}{\left\|y_{\lambda_n}\right\|_{PC^1}} \\ &= \beta(\gamma+m)\varepsilon.\end{aligned} \tag{28}$$

Since $\varepsilon$ is arbitrary, we have $\lambda_n \to +\infty$ ($n \to +\infty$) in contradiction to $\lambda_n \to 0^+$. Therefore, $\|y_\lambda\| \to +\infty$ as $\lambda \to 0^+$ and the proof is complete. □

*Remark 8.* Comparing with Nieto and O'Regan [17], the main features of this paper are as follows.

(i) The parameter $\lambda > 0$ is considered.

(ii) The parameter dependence of the solution is available.

(iii) $\omega \in L^1_{\text{loc}}(0,1)$, not $\omega(t) \equiv 1$ for $t \in J$.

## 4. An Example

To illustrate how our main results can be used in practice we present an example.

*Example 9.* Consider the following boundary value problems

$$\begin{gathered} -y''(t) + y(t) = \lambda \frac{1}{\sqrt{t}} \sqrt[3]{1+t^2} y^2(t), \quad t \in J,\ t \neq \frac{1}{2}, \\ -\Delta y'\big|_{t_1=1/2} = \frac{1}{1+t} \frac{y^3}{1+y}, \quad k = 1, \\ y'(0) = y'(1) = 0. \end{gathered} \tag{29}$$

Evidently, $y(t) \equiv 0$ is the trivial solution of problem (29).

*Conclusion*. Problem (29) has at least one positive solution for any $\lambda > 0$.

*Proof.* Problem (29) can be regarded as a problem of the form (1), where

$$\begin{gathered} \mathbf{M} = 1, \qquad t_1 = \frac{1}{2}, \\ \omega(t) = \frac{1}{\sqrt{t}}, \qquad f(t,y) = \sqrt[3]{1+t^2} y^2(t), \\ I_1(t,y) = (1+t) \frac{y^3}{1+y}. \end{gathered} \tag{30}$$

It follows from the definition of $\omega$, $f$, and $I$ that $(H_1)$ and $(H_2)$ hold, and $\omega(t)$ is singular at $t = 0$ and $t = 1$. By calculating, we have

$$\begin{gathered} \delta = \frac{1}{\cosh^2(1)}, \qquad \gamma = \int_0^1 \frac{1}{\sqrt{t}} dt = 2, \\ \alpha = \frac{1}{\sinh(1)}, \qquad \beta = \frac{\cosh^2(1)}{\sinh(1)}, \\ f^0 = \limsup_{y \to 0} \max_{t \in J} \frac{\sqrt[3]{1+t^2} y^2(t)}{y} = 0, \\ I^0(k) = \limsup_{y \to 0} \max_{t \in J} \frac{(1+t)\left(y^3/(1+y)\right)}{y} = 0, \\ f_\infty = \liminf_{y \to \infty} \min_{t \in J} \frac{\sqrt[3]{1+t^2} y^2(t)}{y} = \infty, \\ I_\infty(k) = \limsup_{y \to \infty} \min_{t \in J} \frac{(1+t)\left(y^3/(1+y)\right)}{y} = \infty. \end{gathered} \tag{31}$$

Then, the condition $(H_3)$ of Theorem 1 holds. Hence, by Theorem 1, the conclusion follows, and the proof is complete. □

## Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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