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Research Article

The Larger Bound on the Domination Number of Fibonacci Cubes and Lucas Cubes

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Let Γ_n and Λ_n be the n-dimensional Fibonacci cube and Lucas cube, respectively. Denote by $\Gamma[u_{n,k,z}]$ the subgraph of Γ_n induced by the end-vertex $u_{n,k,z}$ that has no up-neighbor. In this paper, the number of end-vertices and domination number γ of Γ_n and Λ_n are studied. The formula of calculating the number of end-vertices is given and it is proved that $\gamma(\Gamma[u_{n,k,z}]) \leq 2^{k-1} + 1$. Using these results, the larger bound on the domination number γ of Γ_n and Λ_n is determined.

1. Introduction

The Fibonacci cube and Lucas cube were presented in [1, 2], respectively. Because their many properties (see [1–7]) such as domination number, 2-packing number, and observability can be applied to interconnection networks [1].

However, the number of vertices of Fibonacci cube Γ_n and Lucas cube Λ_n grows rapidly as n increases. So it is hard to calculate exactly the number of domination number of Fibonacci cubes and Lucas cubes. The lower bound on the domination number of Fibonacci cubes and Lucas cubes is determined in [3, 6], respectively. In this paper, we will give a larger bound on the domination number of Fibonacci cubes and Lucas cubes using construction method. We begin with some basic definitions.

Graphs considered in this paper are finite, simple, connected, and undirected. Let Q_n be the n-dimensional hypercube with $n > 0$. A Fibonacci string $b_1 b_2 \cdots b_n$ of order n is a binary string of length n without two consecutive ones. The Fibonacci cube Γ_n (see Figure 1) is the subgraph of Q_n induced by the Fibonacci strings of length n, whose vertices are the Fibonacci strings of length n, and two vertices are joined by an edge if their Hamming distance is exactly 1. A Fibonacci string $b_1 b_2 \cdots b_n$ is a Lucas string if $b_1 b_n = 0$. The Lucas cube Λ_n is the subgraph of Q_n induced by the Lucas strings of length n. It is well known that $|V(\Gamma_n)| = F_{n+2}$ and $|V(\Lambda_n)| = L_n$, where F_n and L_n are Fibonacci numbers and Lucas numbers, respectively. Recall that the Fibonacci numbers and Lucas numbers form a sequence of positive integers F_n and L_n, respectively, where $F_1 = 1, F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ and $L_1 = 1, L_2 = 3$, and $L_n = L_{n-1} + L_{n-2}$ for $n > 2$.

For a connected graph G, the distance $d_G(u, v)$ between vertices u and v is the usual shortest path distance. For $0 \leq k \leq n$, $n \geq 1$, let $\Gamma_{n,k}$ be the set of vertices of Γ_n that contain k ones. Hence $\Gamma_{n,k}$ is the set of vertices of Γ_n at distance k from $b_1 b_2 \cdots b_n, b_i = 0, 1 \leq i \leq n$. $\Lambda_{n,k}$ is defined analogously. If $uv \in \Gamma_n$, where $u \in \Gamma_{n,k}$ and $v \in \Gamma_{n,k-1}$ for $k \geq 1$, then we call v a down-neighbor of u and u an up-neighbor of v.

If a vertex u has no up-neighbor, we call it an end-vertex and denote by $\sigma(\Gamma_n)$ the number of end-vertices of Γ_n. Let $u_{n,k,z}$ $(0 \leq k \leq n/2, z \geq 0, n \geq 1)$ be an end-vertex with string length n, where k, z are the number of ones and consecutive 0^2, respectively. We denote by $\Gamma[u_{n,k,z}]$ the subgraph of Γ_n induced by the end-vertex $u_{n,k,z}$, whose strings of vertices were obtained from string of the vertex $u_{n,k,z}$ by changing i ones into i zeroes ($i = 0, 1, 2, \ldots, k$) and any two vertices have an edge if their Hamming distance is exactly 1 (see Figure 2).

Let G be a graph. Then $D \subseteq V(G)$ is a dominating set if every vertex from $V(G) \setminus D$ is adjacent to some vertex from D. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of G.

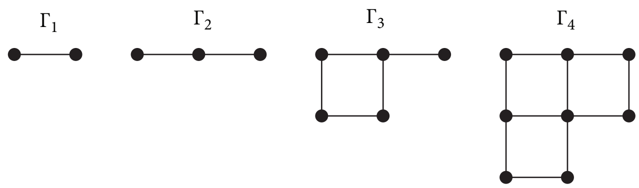

FIGURE 1: $\Gamma_1, \Gamma_2, \Gamma_3$, and Γ_4.

All graph-theoretical terms and concepts used but unexplained in this paper are standard, and that can be found in many textbooks such as [8].

2. The Larger Bound on the Domination Number of Fibonacci Cubes

In this section, we will determine the larger bound on the domination number of Fibonacci cubes. Firstly, we mention the following properties of Fibonacci cubes which will be used later.

Lemma 1 (see [3]). *Let $n \geq 3$ and $k \geq 2$. Then any different $u, v \in \Gamma_{n,k}$ have different sets of down-neighbors.*

Lemma 2 (see [6]). *For any $n \geq 4$, $\gamma(\Gamma_n) \geq \lceil (F_n - 3)/(n-2) \rceil$.*

Lemma 3. *Let $u_{n,k,z}$ be an end-vertex with string length n, where k and z are the number of ones and consecutive 0^2, respectively. Then $\gamma(\Gamma[u_{n,k,z}]) \leq 2^{k-1} + 1$ (see Figure 2).*

Proof. Let $C_n^m = (n\cdot(n-1)\cdots(n-m+1))/(m\cdot(m-1)\cdots 2\cdot 1)$, and it will be frequently used in latter. Let $u = u_{n,k,z}$, and v_h^j be a vertex with $d_{\Gamma[u_{n,k,z}]}(u, v_h^j) = j$ $(1 \leq h \leq C_k^j,\ j = 1, 2, \ldots, k)$, and

$$\begin{aligned} \Phi = \Big\{ & u, v_1^1, v_2^1, \ldots, v_{\lfloor C_k^1/2 \rfloor}^1, v_1^2, v_2^2, \ldots, \\ & v_{\lfloor C_k^2/2 \rfloor}^2, \ldots, v_1^{k-1}, v_2^{k-1}, \ldots, v_{\lfloor C_k^{k-1}/2 \rfloor}^{k-1}, v_1^k \Big\}, \end{aligned} \tag{1}$$

whose subset $\{v_1^i, v_2^i, \ldots, v_{\lfloor C_k^i/2 \rfloor}^i\}$ $(i = 2, 3, \ldots, k-1)$ contains all vertices $\{v_h^i \mid 1 \leq h \leq C_k^i\}$ such that satisfy the condition $d_{\Gamma[u_{n,k,z}]}(v_l^{i-1}, v_h^i) \neq 1$ for $l = 1, 2, \ldots, \lfloor C_k^{i-1}/2 \rfloor$. We will prove that Φ is a dominating set. In order to prove that Φ is a dominating set. It suffices to prove that any $\lfloor C_k^{i-1}/2 \rfloor$ vertices in $\{v_1^{i-1}, v_2^{i-1}, \ldots, v_{C_k^{i-1}}^{i-1}\}$ can dominate $\lfloor C_k^i/2 \rfloor$ vertices in $\{v_1^i, v_2^i, \ldots, v_{C_k^i}^i\}$. If $C_k^{i-1} \geq C_k^i$, the result is obviously correct. We assume that $C_k^{i-1} < C_k^i$. From Lemma 1, we know that any different $v_h^{i-1}, v_l^{i-1} \in V(\Gamma[u_{n,k,z}])$ $(1 \leq h, l \leq C_k^{i-1}, h \neq l)$ have different sets of down-neighbors and have at most one common down-neighbor vertex. Since each vertex v_h^{i-1} has exactly $k-i+1$ ones. If $k - i + 2 \geq \lfloor C_k^{i-1}/2 \rfloor$, then the number of vertices dominated by vertices set $\{v_1^{i-1}, v_2^{i-1}, \ldots, v_{\lfloor C_k^{i-1}/2 \rfloor}^{i-1}\}$ is at least as follows:

$$\begin{aligned} \sum_{j=1}^{\lfloor C_k^{i-1}/2 \rfloor} (k - i - j + 2) &= \left\lfloor \frac{C_k^{i-1}\left(2k - 2i + 3 - \lfloor C_k^{i-1}/2 \rfloor\right)}{4} \right\rfloor \\ &\geq \left\lfloor \frac{C_k^{i-1}(k-i+1)}{4} \right\rfloor \\ &= \left\lfloor \frac{iC_k^i}{4} \right\rfloor \geq \left\lfloor \frac{C_k^i}{2} \right\rfloor \quad (\text{Since } i \geq 2). \end{aligned} \tag{2}$$

If $k - i + 2 < \lfloor C_k^{i-1}/2 \rfloor$, there must exist integer m such that $m < \lfloor C_k^{i-1}/2 \rfloor$ and $k - i - m + 1 = 0$, and then the number of vertices dominated by vertices set $\{v_1^{i-1}, v_2^{i-1}, \ldots, v_{\lfloor C_k^{i-1}/2 \rfloor}^{i-1}\}$ is at least as follows:

$$\sum_{j=1}^{m} (k - i - j + 2) = C_k^i > \left\lfloor \frac{C_k^i}{2} \right\rfloor. \tag{3}$$

Therefore the set Φ is a dominating set, and

$$|\Phi| = 2 + \sum_{i=1}^{k-1} \left\lfloor \frac{C_k^i}{2} \right\rfloor \leq 2^{k-1} + 1. \tag{4}$$

This completes our proof. □

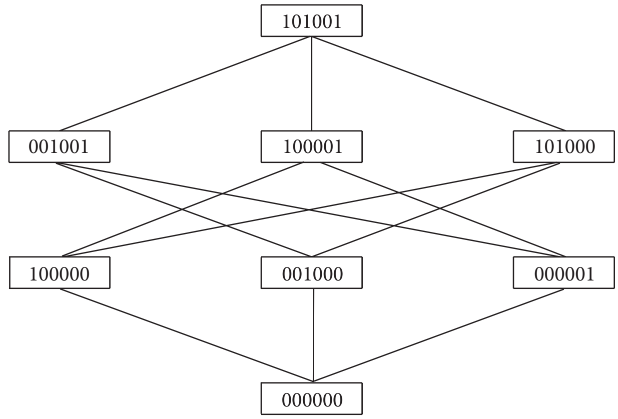

FIGURE 2: $\Gamma[u_{n,k,z}]$, $u_{n,k,z} = 101001$.

Theorem 4. *Let $\sigma(\Gamma_n)$ be the number of end-vertices in Γ_n. Then the followings hold.*

(i) *If n $(n = 2p + 1)$ is odd, then*

$$\sigma\left(\Gamma_{2p+1}\right) = \begin{cases} 1 + \sum_{k=1}^{(p-1)/3} \left(C_{p-k}^{2k-1} + C_{p-k}^{2k}\right) + \sum_{k=1}^{(p+2)/3} \left(C_{p-k}^{2k-2} + C_{p-k}^{2k-1}\right) & \text{if } n \equiv 0 \bmod 3; \\ 1 + \sum_{k=1}^{p/3} \left(C_{p-k}^{2k-1} + C_{p-k}^{2k}\right) + \sum_{k=1}^{p/3} \left(C_{p-k}^{2k-2} + C_{p-k}^{2k-1}\right) & \text{if } n \equiv 1 \bmod 3; \\ 1 + \sum_{k=1}^{(p+1)/3} \left(C_{p-k}^{2k-1} + C_{p-k}^{2k}\right) + \sum_{k=1}^{(p+1)/3} \left(C_{p-k}^{2k-2} + C_{p-k}^{2k-1}\right) & \text{if } n \equiv 2 \bmod 3. \end{cases} \tag{5}$$

(ii) *If n $(n = 2p)$ is even, then*

$$\sigma\left(\Gamma_{2p}\right) = \begin{cases} 1 + \sum_{k=1}^{p/3} \left(C_{p-k}^{2k-2} + C_{p-k}^{2k-1}\right) + \sum_{k=2}^{(p+3)/3} \left(C_{p-k}^{2k-3} + C_{p-k}^{2k-2}\right) & \text{if } n \equiv 0 \bmod 3; \\ 1 + \sum_{k=1}^{(p+1)/3} \left(C_{p-k}^{2k-2} + C_{p-k}^{2k-1}\right) + \sum_{k=2}^{(p+1)/3} \left(C_{p-k}^{2k-3} + C_{p-k}^{2k-2}\right) & \text{if } n \equiv 1 \bmod 3; \\ 1 + \sum_{k=1}^{(p+2)/3} \left(C_{p-k}^{2k-2} + C_{p-k}^{2k-1}\right) + \sum_{k=2}^{(p+2)/3} \left(C_{p-k}^{2k-3} + C_{p-k}^{2k-2}\right) & \text{if } n \equiv 2 \bmod 3. \end{cases} \tag{6}$$

Proof. We prove only that the theorem is correct if n $(n = 2p + 1)$ is odd and $n \equiv 0 \bmod 3$. And the proofs of the others cases are similar. End-vertices of Γ_n can be divided into two cases as follows.

Case 1. The end-vertices set is composed of end-vertices with strings form $1b_2b_3 \cdots b_n$.

Case 2. The end-vertices set is composed of end-vertices with strings form $0b_2b_3 \cdots b_n$.

In Case 1, since $n = 2p + 1$ and $n \equiv 0 \bmod 3$, then end-vertices $\{u_{n,k,z}\}$ are divided into $((4p-1)/3)+1$ cases with $z = 0, 1, 2, \ldots, (4p-1)/3$. Therefore the number of end-vertices with strings form $1b_2b_3 \cdots b_n$ is

$$1 + \sum_{k=1}^{(p-1)/3} \left(C_{p-k}^{2k-1} + C_{p-k}^{2k}\right). \tag{7}$$

In Case 2, as similar as Case 1, end-vertices $\{u_{n,k,z}\}$ are divided into $((4p + 2)/3) + 1$ cases with $z = 0, 1, 2, \ldots, (4p + 2)/3$. Then the number of end-vertices with strings form $0b_2b_3 \cdots b_n$ is

$$\sum_{k=1}^{(p+2)/3} \left(C_{p-k}^{2k-2} + C_{p-k}^{2k-1}\right). \tag{8}$$

Therefore

$$\sigma\left(\Gamma_{2p+1}\right) = 1 + \sum_{k=1}^{(p-1)/3} \left(C_{p-k}^{2k-1} + C_{p-k}^{2k}\right) + \sum_{k=1}^{(p+2)/3} \left(C_{p-k}^{2k-2} + C_{p-k}^{2k-1}\right) n \equiv 0 \bmod 3. \tag{9}$$

This completes our proof. □

Now we give the larger bound on the domination number of Fibonacci cubes as follows.

Theorem 5. *Let $n > 4$. Then for the Fibonacci cube Γ_n the followings hold.*

(i) *If n $(n = 2p + 1)$ is odd, then*

$$\left\lceil \frac{F_n - 3}{n-2} \right\rceil \leq \gamma\left(\Gamma_n\right) \leq \begin{cases} 2^p + 2 + \sum_{k=1}^{(p-1)/3} \left(C_{p-k}^{2k-1} + C_{p-k}^{2k}\right)\left(2^{p-k} - 1\right) + \sum_{k=1}^{(p+2)/3} \left(C_{p-k}^{2k-2} + C_{p-k}^{2k-1}\right)\left(2^{p-k} - 1\right) & \text{if } n \equiv 0 \bmod 3; \\ 2^p + 2 + \sum_{k=1}^{p/3} \left(C_{p-k}^{2k-1} + C_{p-k}^{2k}\right)\left(2^{p-k} - 1\right) + \sum_{k=1}^{p/3} \left(C_{p-k}^{2k-2} + C_{p-k}^{2k-1}\right)\left(2^{p-k} - 1\right) & \text{if } n \equiv 1 \bmod 3; \\ 2^p + 2 + \sum_{k=1}^{(p+1)/3} \left(C_{p-k}^{2k-1} + C_{p-k}^{2k}\right)\left(2^{p-k} - 1\right) + \sum_{k=1}^{(p+1)/3} \left(C_{p-k}^{2k-2} + C_{p-k}^{2k-1}\right)\left(2^{p-k} - 1\right) & \text{if } n \equiv 2 \bmod 3. \end{cases} \tag{10}$$

(ii) *If n $(n = 2p)$ is even, then*

$$\left\lceil \frac{F_n - 3}{n-2} \right\rceil \le \gamma(\Gamma_n) \le \begin{cases} 2^{p-1} + 2 + \sum_{k=1}^{p/3} \left(C_{p-k}^{2k-2} + C_{p-k}^{2k-1}\right)\left(2^{p-k} - 1\right) + \sum_{k=2}^{(p+3)/3} \left(C_{p-k}^{2k-3} + C_{p-k}^{2k-2}\right)\left(2^{p-k} - 1\right) & \text{if } n \equiv 0 \bmod 3; \\ 2^{p-1} + 2 + \sum_{k=1}^{(p+1)/3} \left(C_{p-k}^{2k-2} + C_{p-k}^{2k-1}\right)\left(2^{p-k} - 1\right) + \sum_{k=2}^{(p+1)/3} \left(C_{p-k}^{2k-3} + C_{p-k}^{2k-2}\right)\left(2^{p-k} - 1\right) & \text{if } n \equiv 1 \bmod 3; \\ 2^{p-1} + 2 + \sum_{k=1}^{(p+2)/3} \left(C_{p-k}^{2k-2} + C_{p-k}^{2k-1}\right)\left(2^{p-k} - 1\right) + \sum_{k=2}^{(p+2)/3} \left(C_{p-k}^{2k-3} + C_{p-k}^{2k-2}\right)\left(2^{p-k} - 1\right) & \text{if } n \equiv 2 \bmod 3. \end{cases} \quad (11)$$

Proof. According to the process of proof of Lemma 3, there must exist a dominating set of $\Gamma[u_{n,k,z}]$ with the string form $1b_2 \cdots b_n$ (or $0b_2 \cdots b_n$) such that it contains the vertex with the string form 10^{n-1} (or 010^{n-2}). So all dominating sets of $\Gamma[u_{n,k,z}]$ of Γ_n with strings form $1b_2 \cdots b_n$ (or $0b_2 \cdots b_n$) have at least two common vertices with strings $\{10^{n-1}, 0^n\}$ (or $\{010^{n-2}, 0^n\}$). Then the theorem follows directly from Lemmas 2 and 3 and Theorem 4. □

3. The Larger Bound on the Domination Number of Lucas Cubes

In this section, we will determine the larger bound on the domination number of Lucas cubes as follows.

Lemma 6 (see [3]). *For any $n \ge 7$, $\gamma(\Lambda_n) \ge \lceil (L_n - 2n)/(n-3) \rceil$.*

Theorem 7. *Let $\sigma(\Lambda_n)$ be the number of end-vertices in Λ_n. Then the followings hold.*

(i) *If n $(n = 2p + 1)$ is odd, then*

$$\sigma(\Lambda_{2p+1}) = \begin{cases} \sum_{k=1}^{(p-1)/3} C_{p-k}^{2k-1} + \sum_{k=1}^{(p+2)/3} \left(C_{p-k}^{2k-2} + C_{p-k}^{2k-1}\right) & \text{if } n \equiv 0 \bmod 3; \\ \sum_{k=1}^{p/3} C_{p-k}^{2k-1} + \sum_{k=1}^{p/3} \left(C_{p-k}^{2k-2} + C_{p-k}^{2k-1}\right) & \text{if } n \equiv 1 \bmod 3; \\ \sum_{k=1}^{(p+1)/3} C_{p-k}^{2k-1} + \sum_{k=1}^{(p+1)/3} \left(C_{p-k}^{2k-2} + C_{p-k}^{2k-1}\right) & \text{if } n \equiv 2 \bmod 3. \end{cases} \quad (12)$$

(ii) *If n $(n = 2p)$ is even, then*

$$\sigma(\Lambda_{2p}) = \begin{cases} 1 + \sum_{k=1}^{p/3} C_{p-k}^{2k-2} + \sum_{k=2}^{(p+3)/3} \left(C_{p-k}^{2k-3} + C_{p-k}^{2k-2}\right) & \text{if } n \equiv 0 \bmod 3; \\ 1 + \sum_{k=1}^{(p+1)/3} C_{p-k}^{2k-2} + \sum_{k=2}^{(p+1)/3} \left(C_{p-k}^{2k-3} + C_{p-k}^{2k-2}\right) & \text{if } n \equiv 1 \bmod 3; \\ 1 + \sum_{k=1}^{(p+2)/3} C_{p-k}^{2k-2} + \sum_{k=2}^{(p+2)/3} \left(C_{p-k}^{2k-3} + C_{p-k}^{2k-2}\right) & \text{if } n \equiv 2 \bmod 3. \end{cases} \quad (13)$$

Proof. The proof is similar to Theorem 4. □

Theorem 8. *Let $n > 7$. Then for the Lucas cube Λ_n the followings hold.*

(i) *If n $(n = 2p + 1)$ is odd, then*

$$\left\lceil \frac{L_n - 2n}{n-3} \right\rceil \le \gamma(\Lambda_n) \le \begin{cases} 2 + \sum_{k=1}^{(p-1)/3} C_{p-k}^{2k-1}\left(2^{p-k} - 1\right) + \sum_{k=1}^{(p+2)/3} \left(C_{p-k}^{2k-2} + C_{p-k}^{2k-1}\right)\left(2^{p-k} - 1\right) & \text{if } n \equiv 0 \bmod 3; \\ 2 + \sum_{k=1}^{p/3} C_{p-k}^{2k-1}\left(2^{p-k} - 1\right) + \sum_{k=1}^{p/3} \left(C_{p-k}^{2k-2} + C_{p-k}^{2k-1}\right)\left(2^{p-k} - 1\right) & \text{if } n \equiv 1 \bmod 3; \\ 2 + \sum_{k=1}^{(p+1)/3} C_{p-k}^{2k-1}\left(2^{p-k} - 1\right) + \sum_{k=1}^{(p+1)/3} \left(C_{p-k}^{2k-2} + C_{p-k}^{2k-1}\right)\left(2^{p-k} - 1\right) & \text{if } n \equiv 2 \bmod 3. \end{cases} \quad (14)$$

(ii) *If n ($n = 2p$) is even, then*

$$\left\lceil \frac{L_n - 2n}{n-3} \right\rceil \le \gamma(\Lambda_n) \le \begin{cases} 2^{p-1} + 2 + \sum_{k=1}^{p/3} C_{p-k}^{2k-2}\left(2^{p-k}-1\right) \\ \quad + \sum_{k=2}^{(p+3)/3} \left(C_{p-k}^{2k-3} + C_{p-k}^{2k-2}\right)\left(2^{p-k}-1\right) & \text{if } n \equiv 0 \bmod 3; \\ 2^{p-1} + 2 + \sum_{k=1}^{(p+1)/3} C_{p-k}^{2k-2}\left(2^{p-k}-1\right) \\ \quad + \sum_{k=2}^{(p+1)/3} \left(C_{p-k}^{2k-3} + C_{p-k}^{2k-2}\right)\left(2^{p-k}-1\right) & \text{if } n \equiv 1 \bmod 3; \\ 2^{p-1} + 2 + \sum_{k=1}^{(p+2)/3} C_{p-k}^{2k-2}\left(2^{p-k}-1\right) \\ \quad + \sum_{k=2}^{(p+2)/3} \left(C_{p-k}^{2k-3} + C_{p-k}^{2k-2}\right)\left(2^{p-k}-1\right) & \text{if } n \equiv 2 \bmod 3. \end{cases} \tag{15}$$

Proof. That follows directly from Lemmas 6 and Theorem 7. □

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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