Research Article H^{∞} Control for a Networked Control Model of Systems with
Two Additive Time-Varying Delays
Hanyong Shao,' Zhengqiang Zhang,' Xunlin Zhu,² and Guoying Miao³ **Two Additive Time-Varying Delays**

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Received 8 July 2014; Accepted 5 September 2014; Published 19 October 2014

Academic Editor: Valery Y. Glizer

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This paper is concerned with H^{∞} control for a networked control model of systems with two additive time-varying delays. A H^{∞} control for a networked control model of systems with two additive time-varying delays. An
structed to make full use of the information of the delays, and for the derivative of the Lyapunov
s employed to compute new Lyapunov functional is constructed to make full use of the information of the delays, and for the derivative of the Lyapunov functional a novel technique is employed to compute a tighter upper bound, which is dependent on the two time-varying delays instead of the upper bounds of them. Then the convex polyhedron method is proposed to check the upper bound of the derivative of the Lyapunov functional. The resulting stability criteria have fewer matrix variables but less conservatism than some existing ones. The stability criteria are applied to designing a state feedback controller, which guarantees that the closed-loop system is asymptotically stable with a prescribed H^{∞} disturbance attenuation level. Finally examples are given to show the advantages of the
stability criteria and the effectiveness of the proposed control method.
On the othe stability criteria and the effectiveness of the proposed control method.

1. Introduction

For years systems with time delays have received considerable attention since they are often encountered in various practical systems, such as engineering systems, biology, economics, neural networks, networked control systems, and other areas [1–6]. Since time-delay is frequently the main cause of oscillation, divergence, or instability, considerable effort has been devoted to stability for systems with time delays. According to whether stability criteria include the information of the delay, they are divided into two classes: delay-independent stability criteria and delay-dependent ones. It is well known that delay-independent stability criteria tend to be more conservative especially for small size delays. More attention has been paid to delay-dependent stability. For delay-dependent stability results, we refer readers to [7–14]. Among these papers, [11–13] were of systems with interval time-varying delay. Recently these delay-dependent stability results were extended to neutral systems with interval time-varying delay [14]. It should be pointed out that all the stability results mentioned are based on systems with one single delay in the state.

On the other hand, networked control systems have been receiving great attention these years due to their advantages in low cost, reduced weight and power requirements, simple installation and maintenance, and high reliability. It is well known that the transmission delay and the data packet dropout are two fundamental issues in networked control systems. The transmission delay generally includes the sensor-to-control delay and the control-to-actuator delay. In most of existing papers the sensor-to-control delay and the control-to-actuator delay were combined into one state delay, while the data packet dropouts were modeled as delays and absorbed by the state delay, thus formulating networked control systems as systems with one state delay [15]. Among recently reported results based on this modeling idea, to mention a few, event-triggered communication and H^{∞} con- H^∞ con-
ed control
problems
tworks in
ontrol-to-
enetwork trol codesign problems were addressed for networked control systems in [16], while exponential state estimation problems were considered for Markovian jumping neural networks in [17]. Note that the sensor-to-control delay and the control-toactuator delay are different in nature because of the network transmission conditions. The transmission delay and the data packet dropout also have different properties. It is not

rational to lump them into one state delay. In this paper, to study networked control systems we adopt the model of systems with multiadditive time-varying delay components. For simplicity, the system with two additive time-varying delay components will be employed to address H^{∞} control H^{∞} control
we write the
 $t) + Bu(t)$, (1) problem for networked control systems. Now we write the system as follows:

$$
\dot{x}(t) = Ax(t) + A_1x(t - d_1(t) - d_2(t)) + Ew(t) + Bu(t),
$$
\n(1)

$$
x(t) = Ax(t) + A_1x(t - a_1(t) - a_2(t)) + Ew(t) + Bu(t),
$$

(1)

$$
y(t) = Cx(t) + C_1x(t - d_1(t) - d_2(t)) + Fw(t) + Du(t),
$$

(2)

$$
x(t) = \phi(t), \quad t \in [-h, 0].
$$

$$
x(t) = \phi(t), \quad t \in [-h, 0], \tag{3}
$$

 $y(t) = Cx(t) + C_1x(t - u_1(t) - u_2(t)) + Fw(t) + Du(t),$

(2)
 $x(t) = \phi(t), \quad t \in [-h, 0],$ (3)

where $x(t) \in \mathbb{R}^n$ is the state; $y(t)$ is the measurement; $u(t)$

the control: $w(t) \in L_2[0, \infty)$ is the disturbance: A A, F, I $x(t) = \phi(t), \quad t \in [-h, 0],$ (3)

there $x(t) \in \mathbb{R}^n$ is the state; $y(t)$ is the measurement; $u(t)$ is

the control; $w(t) \in L_2[0, \infty]$ is the disturbance; A, A₁, E, B,
 x , C_1 , F, and D are known real constant matrices; $x(t) \in \mathbb{R}^n$ is the state; $y(t)$ is the measurement; $u(t)$ is
throl; $w(t) \in L_2[0, \infty]$ is the disturbance; A, A₁, E, B,
F, and D are known real constant matrices; $d_1(t)$ and
re two time-varying delays satisfying
 0 the control; $w(t) \in L_2[0, \infty]$ is the disturbance; *A*, *A*₁, *E*, *B*,
d *D* are known real constant matrices; *d*₁(*t*) and
o time-varying delays satisfying
 $0 \le d_1$ (*t*) ≤ *h*₁, 0 ≤ *d*₂ (*t*) ≤ *h*₂, (4)
 \dot{d} (*t*) C, C₁, F, and D are known real constant matrices; $d_1(t)$ and
 $d_2(t)$ are two time-varying delays satisfying
 $0 \le d_1(t) \le h_1, \qquad 0 \le d_2(t) \le h_2,$ (4)
 $\dot{d}_1(t) \le \mu_1, \qquad \dot{d}_2(t) \le \mu_2;$ (5) $d_2(t)$ are two time-varying delays satisfying

$$
0 \le d_1(t) \le h_1, \qquad 0 \le d_2(t) \le h_2,\tag{4}
$$

$$
0 \le d_1(t) \le h_1, \qquad 0 \le d_2(t) \le h_2,
$$
\n
$$
(4)
$$
\n
$$
\dot{d}_1(t) \le \mu_1, \qquad \dot{d}_2(t) \le \mu_2;
$$
\n
$$
(5)
$$
\nreal-valued initial function on $[-h, 0]$ with

\n
$$
h = h_1 + h_2.
$$
\n
$$
(6)
$$

̇ and

$$
h = h_1 + h_2. \tag{6}
$$

 $d_1(t) \le \mu_1, \qquad d_2(t) \le \mu_2;$ (5)

and $\phi(t)$ is a real-valued initial function on [-h, 0] with
 $h = h_1 + h_2.$ (6)

Stability analysis for this kind of system was conducted in $\phi(t)$ is a real-valued initial function on [−*h*, 0] with
 $h = h_1 + h_2$.

ility analysis for this kind of system was conduct

and a delay-dependent stability criterion was obtain

improved stability criterion was derived $h = h_1 + h_2.$ (6)
s kind of system was conducted in
lent stability criterion was obtained.
riterion was derived in [19] by con-
nctional to employ the information [18], and a delay-dependent stability criterion was obtained. An improved stability criterion was derived in [19] by constructing a Lyapunov functional to employ the information of the marginally delayed state $x(t - h)$. However, another $x(t - h)$. However, another
 $-(h_1)$ was not considered,
 $\dot{x}(\alpha)d\alpha$ to be discarded when
 α : Lyapunov functional. On

of the bounding, many free

luced, making the stability marginally delayed state $x(t - h_1)$ was not considered, $x(t - h_1)$ was not considered,
 $T Z_1 \dot{x}(\alpha) d\alpha$ to be discarded when

of the Lyapunov functional. On

cess of the bounding, many free

introduced, making the stability which caused – $\int_{t-h_1}^{t-d_1(t)}$
bounding the derivat
the other hand, in the
weighting matrices w
result complicated.
In this paper we $_{t-h_1}$ $\dot{x}(\alpha)^T Z_1 \dot{x}(\alpha) d\alpha$ to be discarded when
we of the Lyapunov functional. On
process of the bounding, many free
ere introduced, making the stability
first revisit delay-dependent stability bounding the derivative of the Lyapunov functional. On the other hand, in the process of the bounding, many free weighting matrices were introduced, making the stability result complicated.

In this paper we first revisit delay-dependent stability for system (1) and (2). We will construct a new Lyapunov functional to employ the information of the marginally delayed state $x(t - h_1)$ as well as $x(t - h)$. Motivated by [13], $x(t - h_1)$ as well as $x(t - h)$. Motivated by [13], ng the derivative of the Lyapunov functional, we chnique to avoid introducing too many matrix compute a tighter upper bound. Considering er bound depends on the two time-va when bounding the derivative of the Lyapunov functional, we use a novel technique to avoid introducing too many matrix variables and compute a tighter upper bound. Considering that the upper bound depends on the two time-varying delays, we propose the so-called convex polyhedron method to check the negative definiteness for it. The resulting delaydependent stability criteria turn out to be less conservative with fewer matrix variables. Then we take the advantages of the stability results to investigate the H^{∞} state feedback *H*[∞] state feedback
feedback controller
closed-loop system
urbance attenuation
or nonzero $w(t) \in$
A delay-dependent control problem, which is to design a state feedback controller $u(t) = Kx(t)$ for the system such that the closed-loop system
is asymptotically stable with an H^{∞} disturbance attenuation
level $\gamma > 0$ satisfying $||y||_2 < \gamma ||w||_2$ for nonzero $w(t) \in$
 $L_2[0, \infty]$ under zero initial con is asymptotically stable with an H^{∞} disturbance attenuation
level $\gamma > 0$ satisfying $||y||_2 < \gamma ||w||_2$ for nonzero $w(t) \in L_2[0, \infty]$ under zero initial condition. A delay-dependent
condition will be presented for the level $\gamma > 0$ satisfying $||y||_2 < y||w||_2$ for nonzero $w(t) \in$ $L_2[0,\infty]$ under zero initial condition. A delay-dependent $L_2[0, \infty]$ under zero initial condition. A delay-dependent condition will be presented for the state feedback controller such that the closed-loop system is asymptotically stable with condition will be presented for the state feedback controller such that the closed-loop system is asymptotically stable with

a prescribed H^{∞} disturbance attenuation level. Formulated in LMIs the condition is readily verified, and when it is feasible
the controller can be constructed.
Notation. Throughout this paper the superscript "T" sta LMIs the condition is readily verified, and when it is feasible the controller can be constructed.

Notation. Throughout this paper the superscript "T" stands T" stands
atrix with
ces X and
is positive
ic term in
stated, are for matrix transposition. I refers to an identity matrix with *I* refers to an identity matrix with
For real symmetric matrices *X* and
ans that the matrix $X - Y$ is positive
vs similarly. The symmetric term in
Matrices, if not explicitly stated, are
ble dimensions. appropriate dimensions. For real symmetric matrices X and X and
ositive
erm in
ed, are
end, a *Y*, the notation *X* > *Y* means that the matrix *X* − *Y* is positive definite. The *X* ≥ *Y* follows similarly. The symmetric term in a matrix is denoted by $*$. Matrices, if not explicitly stated, are assumed to have definite. The $X \geq Y$ follows similarly. The symmetric term in enoted by $*$. Matrices, if not explicitly stated, are ave compatible dimensions.
go about the stability analysis. To the end, a gen, which will play an important role in d a matrix is denoted by *. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

∗. Matrices, if not explicitly stated, are atible dimensions.
the stability analysis. To the end, a will play an important role in deriving First we go about the stability analysis. To the end, a lemma is given, which will play an important role in deriving our criteria.

Lemma 1 (see [20]). *For any symmetric positive definite matrix* $M > 0$, *scalar* $\gamma > 0$, *and vector function* $\omega : [0, \gamma] \rightarrow$ *M* > 0*, scalar* γ > 0*, and vector function* ω : [0, γ] \rightarrow
n that the integrations concerned are well defined, the
ng inequality holds:
 $\nu(s)ds$ $\int^T M \left(\int_0^{\gamma} \omega(s) ds \right) \le \gamma \left(\int_0^{\gamma} \omega(s)^T M \omega(s) ds \right).$ *following inequality holds:*

$$
R^{n} such that the integrations concerned are well defined, the following inequality holds:\n
$$
\left(\int_{0}^{y} \omega(s)ds\right)^{T} M\left(\int_{0}^{y} \omega(s)ds\right) \leq \gamma \left(\int_{0}^{y} \omega(s)^{T} M \omega(s) ds\right).
$$
\n(7)\n2. Stability Analysis\nConsider system (1) with $w(t) = u(t) = 0$, namely,\n
$$
\dot{x}(t) = Ax(t) + A_{1}x(t - d_{1}(t) - d_{2}(t)).
$$
\n(8)\nSet\n
$$
d(t) = d_{1}(t) + d_{2}(t).
$$
\n(9)
$$

2. Stability Analysis

$$
\dot{x}(t) = Ax(t) + A_1 x(t - d_1(t) - d_2(t)).
$$
\n(8)
\n
$$
d(t) = d_1(t) + d_2(t),
$$
\n(9)
\n
$$
u = u_1 + u_2.
$$
\n(10)

Set

$$
d(t) = d_1(t) + d_2(t),
$$
\n(9)

$$
\mu = \mu_1 + \mu_2. \tag{10}
$$

d (*t*) = *d*₁ (*t*) + *d*₂ (*t*), (9)
 $\mu = \mu_1 + \mu_2.$ (10)

) as one delay *d*(*t*) we have the following $\mu = \mu_1 + \mu_2.$ (10)

one delay $d(t)$ we have the following
 $x(t) + A_1 x (t - d(t)),$ (11) Taking $d_1(t) + d_2(t)$ as one delay $d(t)$ we have the following
system:
 $\dot{x}(t) = Ax(t) + A_1x(t - d(t)),$ (11)
with $0 \le d(t) \le h, d(t) \le \mu.$ system:

$$
\dot{x}(t) = Ax(t) + A_1 x(t - d(t)),
$$
\n(11)

with $0 \leq d(t) \leq h, d(t) \leq \mu$.

 $\dot{x}(t) = Ax(t) + A_1x(t - d(t)),$ (11)
 $\le h, \dot{d}(t) \le \mu.$

stem there are many delay-dependent stability

le, but when used to check the stability for (8),

conservative [18]. In the following we present $0 \le d(t) \le h, d(t) \le \mu.$

So this system there are in available, but when

are more conservative
 w stability result for sy

So separately. For this system there are many delay-dependent stability criteria available, but when used to check the stability for (8), they are more conservative [18]. In the following we present a new stability result for system (8) by considering the two delays separately.

Theorem 2. *The system* (8) *subject to* (4) *and* (5) *is asymptotically stable for given* h_1 , h_2 , μ_1 , and μ_2 *if there exist matrices h*₁, *h*₂, *µ*₁, *and µ*₂ *if there exist matrices*
2, 3, 4, *and* $Z_j > 0$, *j* = 1, 2, *such that the*
 $I_1^{-1}Z_2e_{13}^T - e_{23}h_2^{-1}Z_2e_{23}^T < 0$,
 I_1Z^T *following LMIs hold:*

$$
P > 0, Q_i > 0, i = 1, 2, 3, 4, and Z_j > 0, j = 1, 2, such that the following LMIs hold:\n
$$
\Phi - e_{13}h_1^{-1}Z_2e_{13}^T - e_{23}h_2^{-1}Z_2e_{23}^T < 0,
$$
\n
$$
\Phi - e_{13}h_1^{-1}Z_2e_{13}^T - e_{24}h_2h^{-2}Z_2e_{24}^T < 0,
$$
\n
$$
\Phi - e_{24}h_1h^{-2}Z_2e_{24}^T - e_{23}h_2^{-1}Z_2e_{23}^T < 0,
$$
\n
$$
\Phi - e_{24}h^{-1}Z_2e_{24}^T < 0,
$$
\n(12)
$$

where
$$
e_{13} = [I \ 0 \ -I \ 0 \ 0]^T
$$
, e_{23} , e_{24} , and e_{35} follow similarly,
\nh is defined in (6), and
\n
$$
\Phi = \begin{bmatrix}\n\varphi_1 & PA_1 & h_1^{-1} (Z_1 + Z_2) & 0 & 0 \\
\ast & \varphi_2 & h_2^{-1} Z_2 & h^{-1} Z_2 & 0 \\
\ast & \ast & \varphi_3 & 0 & h_1^{-1} Z_1 \\
\ast & \ast & \ast & -Q_2 - h^{-1} Z_2 & 0 \\
\ast & \ast & \ast & \ast & -Q_4 - h_1^{-1} Z_1\n\end{bmatrix}
$$
\n
$$
+ \begin{bmatrix}\nA^T \\
A_1^T \\
0 \\
0 \\
0\n\end{bmatrix} [h_1 Z_1 + h Z_2] \begin{bmatrix}\nA^T \\
A_1^T \\
0 \\
0 \\
0\n\end{bmatrix}
$$
,
\nwith μ given in (10) and

with
$$
\mu
$$
 given in (10) and
\n
$$
\varphi_1 = PA + A^T P + \sum_{i=1}^4 Q_i - h_1^{-1} (Z_1 + Z_2),
$$
\n
$$
\varphi_2 = -(1 - \mu) Q_3 - (h_2^{-1} + h^{-1}) Z_2,
$$
\n
$$
\varphi_3 = -(1 - \mu_1) Q_1 - (h_2^{-1} + h_1^{-1}) Z_2 - 2h_1^{-1} Z_1.
$$
\nProof. Define a Lyapunov functional as follows:
\n
$$
V(t) = x(t)^T P x(t) + \int_0^t x(\alpha)^T O \cdot x(\alpha) d\alpha
$$

Proof. Define a Lyapunov functional as follows:

$$
- - (1 - \mu_1) Q_1 - (n_2 + n_1) Z_2 - 2n_1 Z_1.
$$

Define a Lyapunov functional as follows:

$$
V(t) = x(t)^T P x(t) + \int_{t-d_1(t)}^t x(\alpha)^T Q_1 x(\alpha) d\alpha
$$

$$
+ \int_{t-h}^t x(\alpha)^T Q_2 x(\alpha) d\alpha
$$

$$
+ \int_{t-d(t)}^t x(\alpha)^T Q_3 x(\alpha) d\alpha
$$

$$
+ \int_{t-h_1}^t x(\alpha)^T Q_4 x(\alpha) d\alpha
$$

$$
+ \int_{t-h_1}^t \int_s^t \dot{x}(\alpha)^T Z_1 \dot{x}(\alpha) d\alpha ds
$$

$$
+ \int_{t-h_1}^t \int_s^t \dot{x}(\alpha)^T Z_2 \dot{x}(\alpha) d\alpha ds,
$$

 $+ \int_{t-h} \int_s x(u) dz_2 x(u) du$ *us*,
defined in (9). Then calcule Lyapunov functional along J_s
d
ur ..
ir:
ווכ $\frac{1}{2}$ (9)
 $\frac{1}{2}$ fu where $d(t)$ is defined in (9). Then calculating the time
derivative of the Lyapunov functional along the trajectory of
(8) yields
 $\dot{V}(x_t) \le 2x(t)^T P(Ax(t) + A_1 x(t - d(t)))$ derivative of the Lyapunov functional along the trajectory of (8) yields

$$
\dot{V}(x_t) \le 2x(t)^T P\left(Ax(t) + A_1 x(t - d(t))\right) + \sum_{i=1}^4 x(t)^T Q_i x(t) - x(t - h_1)^T
$$

$$
\times Q_4 x (t - h_1) - x(t - h)^T Q_2 x (t - h)
$$

\n
$$
- (1 - \mu) x (t - d(t))^{T} Q_3 x (t - d(t))
$$

\n
$$
- (1 - \mu_1) x (t - d_1(t))^{T} Q_1 x (t - d_1(t))
$$

\n
$$
+ (Ax(t) + A_1 x (t - d(t)))^{T} (h_1 Z_1 + h Z_2)
$$

\n
$$
\times [Ax(t) + A_1 x (t - d(t))]
$$

\n
$$
- \int_{t - h_1}^{t} \dot{x} (\alpha)^T Z_1 \dot{x} (\alpha) d\alpha - \int_{t - h}^{t} \dot{x} (\alpha)^T Z_2 \dot{x} (\alpha) d\alpha.
$$

\n(16)

Note that

$$
-\int_{t-h_1}^{t} \dot{x}(\alpha)^{T} Z_{1} \dot{x}(\alpha) d\alpha - \int_{t-h}^{t} \dot{x}(\alpha)^{T} Z_{2} \dot{x}(\alpha) d\alpha
$$

\n
$$
= -\int_{t-d_1(t)}^{t} \dot{x}(s)^{T} Z_{1} \dot{x}(s) ds - \int_{t-h_1}^{t-d_1(t)} \dot{x}(s)^{T} Z_{1} \dot{x}(s) ds
$$

\n
$$
-\int_{t-d_1(t)}^{t} \dot{x}(s)^{T} Z_{2} \dot{x}(s) ds - \int_{t-d(t)}^{t-d_1(t)} \dot{x}(s)^{T} Z_{2} \dot{x}(s) ds
$$

\n
$$
-\int_{t-h}^{t-d(t)} \dot{x}(s)^{T} Z_{2} \dot{x}(s) ds.
$$

\n
$$
(17)
$$

\n
$$
\text{it } \alpha = d_1(t)/h_1 \text{ and } \beta = d_2(t)/h_2. \text{ Then}
$$

$$
-\int_{t-h} x(s) Z_2 x(s) ds.
$$
\n(17)
\nWrite $\alpha = d_1(t)/h_1$ and $\beta = d_2(t)/h_2$. Then
\n
$$
-\int_{t-d_1(t)}^t \dot{x}(s)^T Z_2 \dot{x}(s) d\alpha
$$
\n
$$
= -h_1^{-1} \int_{t-d_1(t)}^t h_1 \dot{x}(s)^T Z_2 \dot{x}(s) ds
$$
\n
$$
= -h_1^{-1} \int_{t-d_1(t)}^t d_1(t) \dot{x}(s)^T Z_2 \dot{x}(s) ds
$$
\n
$$
-h_1^{-1} \int_{t-d_1(t)}^t [h_1 - d_1(t)] \dot{x}(s)^T Z_2 \dot{x}(s) ds.
$$
\nIt follows from (18) that

It follows from (18) that

$$
-\int_{t-d_1(t)}^{t} \dot{x}(s)^T Z_2 \dot{x}(s) ds
$$

\n
$$
\leq -h_1^{-1} \int_{t-d_1(t)}^{t} d_1(t) \dot{x}(s)^T Z_2 \dot{x}(s) ds.
$$
\n(19)

Using (19) we have

$$
- h_1^{-1} \int_{t-d_1(t)}^t [h_1 - d_1(t)] \dot{x}(s)^T Z_2 \dot{x}(s) ds
$$

\n
$$
= -(1 - \alpha) \int_{t-d_1(t)}^t \dot{x}(s)^T Z_2 \dot{x}(s) ds
$$
(20)
\n
$$
\leq -(1 - \alpha) h_1^{-1} \int_{t-d_1(t)}^t d_1(t) \dot{x}(s)^T Z_2 \dot{x}(s) ds.
$$

\n
$$
\text{erman 1, (18) and (20) imply}
$$

\n
$$
- \int_{t-d_1(t)}^t \dot{x}(s)^T Z_2 \dot{x}(s) ds
$$

By Lemma 1, (18) and (20) imply

$$
\begin{aligned}\n&= -(1-u) \, n_1 \int_{t-d_1(t)} u_1(t) \, x(s) \, Z_2 x \, (s) \, ds. \\
&\text{and (18) and (20) imply} \\
&= \int_{t-d_1(t)}^t \dot{x}(s)^T Z_2 \dot{x}(s) \, ds \\
&\leq -\left[x(t) - x(t - d_1(t))\right]^T h_1^{-1} Z_2 \\
&\times \left[x(t) - x(t - d_1(t))\right] \\
&= (1 - \alpha) \left[x(t) - x(t - d_1(t))\right]^T h_1^{-1} Z_2 \\
&\times \left[x(t) - x(t - d_1(t))\right].\n\end{aligned}\n\tag{21}
$$

Similarly it can be derived that

$$
\begin{aligned}\n&\times \left[x(t) - x(t - a_1(t))\right]. \\
\text{minally it can be derived that} \\
&-\int_{t-d(t)}^{t-d_1(t)} \dot{x}(s)^T Z_2 \dot{x}(s) \, ds \\
&\leq -\left[x(t - d(t)) - x(t - d_1(t))\right]^T h_2^{-1} Z_2 \\
&\times \left[x(t - d(t)) - x(t - d_1(t))\right] \\
&- (1 - \beta) \left[x(t - d(t)) - x(t - d_1(t))\right]^T h_2^{-1} Z_2 \\
&\times \left[x(t - d(t)) - x(t - d_1(t))\right], \\
&- \int_{t-h}^{t-d(t)} \dot{x}(s)^T Z_2 \dot{x}(s) \, ds \qquad (22) \\
&\leq -\left[x(t - d(t)) - x(t - h)\right]^T h_2^{-1} Z_2 \\
&\times \left[x(t - d(t)) - x(t - h)\right]^T h_1 h_2^{-1} Z_2 \\
&\times \left[x(t - d(t)) - x(t - h)\right]^T h_2 h_2^{-1} Z_2 \\
&\times \left[x(t - d(t)) - x(t - h)\right]^T h_2 h_2^{-1} Z_2 \\
&\times \left[x(t - d(t)) - x(t - h)\right].\n\end{aligned}
$$
\n
$$
\text{minar to } \left[12\right] \text{ we have}
$$

Similar to [12] we have

similar to [12] we have

\n
$$
-\int_{t-d_1(t)}^{t} \dot{x}(s)^T Z_1 \dot{x}(s) ds
$$
\n
$$
\leq -\left[x(t) - x(t - d_1(t))\right]^T h_1^{-1} Z_1 \left[x(t) - x(t - d_1(t))\right],
$$
\n
$$
-\int_{t-h_1}^{t-d_1(t)} \dot{x}(s)^T Z_1 \dot{x}(s) ds
$$

$$
\leq -[x(t - d_1(t)) - x(t - h_1)]^T h_1^{-1} Z_1
$$

$$
\times [x(t - d_1(t)) - x(t - h_1)].
$$

(23)
Since

Define

$$
\left(\lambda [t - d_1(t)] - x(t - h_1] \right).
$$
\n(23)
\nDefine
\n
$$
\zeta(t) = \left[x(t)^T x(t - d(t))^T x(t - d_1(t))^T x(t - h)^T x(t - h_1)^T \right]^T.
$$
\n(24)
\nCombining (16), (17), and (21)–(23) and using (13) yield
\n
$$
\dot{V}(t) \leq \zeta(t)^T \Phi \zeta(t) - (1 - \alpha)
$$
\n
$$
\times \left[x(t) - x(t - d_1(t)) \right]^T h_1^{-1} Z_2 \left[x(t) - x(t - d_1(t)) \right]
$$
\n
$$
- \alpha [x(t - d(t)) - x(t - h)]^T h_1 h^{-2} Z_2
$$
\n
$$
\times \left[x(t - d(t)) - x(t - h) \right]
$$
\n
$$
- \beta [x(t - d(t)) - x(t - h)]^T h_2 h^{-1} Z_2
$$
\n
$$
\times \left[x(t - d(t)) - x(t - h) \right]
$$
\n
$$
- (1 - \beta) \left[x(t - d(t)) - x(t - d_1(t)) \right]^T h_2^{-1} Z_2
$$
\n
$$
\times \left[x(t - d(t)) - x(t - d_1(t)) \right]
$$
\n
$$
= \zeta(t)^T M (\alpha, \beta) \zeta(t),
$$
\n(25)
\nwhere
\n
$$
M(\alpha, \beta) = \Phi \text{ and } h^{-2} Z^{-T}.
$$

where

(25)
\nwhere
\n
$$
M (α, β) = Φ – αe₂₄h₁h⁻²Z₂e₂₄^T
\n- (1 – α) e₁₃h₁⁻¹Z₂e₁₃^T – βe₂₄h₂h⁻²Z₂e₂₄^T
\n- (1 – β) e₂₃h₂⁻¹Z₂e₂₄^T
\n+ (1 – α) [Φ – e₁₃h₁⁻¹Z₂e₁₃^T] – βe₂₄h₂h⁻²Z₂e₂₄^T
\n- (1 – β) e₂₃h₂⁻¹Z₂e₂₃^T
\n= α [Φ – e₂₄h₁h⁻²Z₂e₂₄^T – βe₂₄h₂h⁻²Z₂e₂₄^T
\n- (1 – β) e₂₃h₂⁻¹Z₂e₂₃^T
\n= α [Φ – e₂₄h₁h⁻²Z₂e₂₄^T – βe₂₄h₂h⁻²Z₂e₂₄^T
\n- (1 – β) e₂₃h₂⁻¹Z₂e₂₃^T]
\n+ (1 – α) [Φ – e₁₃h₁⁻¹Z₂e₂₃^T]
\n= α [β (Φ – e₂₄h⁻¹
$$

By (12) it is derived that $M(\alpha, \beta) < 0$. Therefore system (8) is asymptotically stable. This ends the proof.

 $M(\alpha, \beta) < 0$. Therefore system (8) is
is ends the proof. \square
rovides a new delay-dependent sta-
n (8) with two additive time-varying
form of LMIs the criterion can be *Remark 3.* Theorem 2 provides a new delay-dependent stability criterion for system (8) with two additive time-varying delay components. In a form of LMIs the criterion can be checked easily.

Remark 4. Note that the corresponding matrix $M(\alpha, \beta)$ to $M(\alpha, \beta)$ to
two time-
lent on the
 ρ check the
 β), one has
 y [13]. The the upper bound of $V(x_t)$ is dependent on the two time-
varying delays while those in [18, 19] are dependent on the
upper bounds of the two time-varying delays. To check the
negative definiteness of the function matrix M varying delays while those in [18, 19] are dependent on the upper bounds of the two time-varying delays. To check the negative definiteness of the function matrix $M(\alpha, \beta)$, one has $M(\alpha, \beta)$, one has
ted by [13]. The
tive definite over
gative definite at to adopt a new method, which is motivated by [13]. The basic idea is that a function matrix is negative definite over a convex polyhedron only if the matrix is negative definite at the vertices. Note that

$$
M(1, 1) = \Phi - e_{24}h^{-1}Z_{2}e_{24}^{T},
$$

\n
$$
M(1, 0) = \Phi + e_{24}h_{1}h^{-2}Z_{2}e_{24}^{T} - e_{23}h_{2}^{-1}Z_{2}e_{23}^{T},
$$

\n
$$
M(0, 1) = \Phi - e_{13}h_{1}^{-1}Z_{2}e_{13}^{T} - e_{24}h_{2}h^{-2}Z_{2}e_{24}^{T},
$$

\n
$$
M(0, 0) = \Phi - e_{13}h_{1}^{-1}Z_{2}e_{13}^{T} - e_{23}h_{2}^{-1}Z_{2}e_{23}^{T}.
$$

\nthis it can be seen the negative definiteness of $M(\alpha, \beta)$
\nthe rectangle: $0 \le \alpha \le 1, 0 \le \beta \le 1$, is

M (0, 0) = Φ − $e_{13}h_1^{-1}Z_2e_{13}^T$ − $e_{23}h_2^{-1}Z_2e$
this it can be seen the negative definiter
the rectangle: 0 ≤ α ≤ 1, 0 ≤
mined by that of *M*(α, β) at the verti $\frac{1}{2}$. $\frac{1}{2}$. From this it can be seen the negative definiteness of $0 \leq \alpha \leq 1, 0 \leq \beta \leq 1$, is

of *M*(α, β) at the vertices. One calls

e negative definiteness of a function

hedron method. Apparently the convex

can be extended to more than two timeover the rectangle: $0 \le \alpha \le 1, 0 \le \beta \le 1$, is determined by that of $M(\alpha, \beta)$ at the vertices. One calls $M(\alpha, \beta)$ at the vertices. One calls
egative definiteness of a function
ron method. Apparently the convex
be extended to more than two timethis approach to the negative definiteness of a function matrix a convex polyhedron method. Apparently the convex polyhedron method can be extended to more than two timevarying delays.

Remark 5. Gao et al. [19] took advantages of $x(t-h)$ to derive $x(t-h)$ to derive
that in [18], but
as not employed.
ict the Lyapunov
 $\dot{x}(α)^T Z_1 \dot{x}(α) dα$ a stability criterion, which improved over that in [18], but another marginally delayed state $x(t - h_1)$ was not employed. $x(t - h_1)$ was not employed.

t to construct the Lyapunov
 $mg - \int_{t-h_1}^{t-d_1(t)} \dot{x}(\alpha)^T Z_1 \dot{x}(\alpha) d\alpha$

. On the other hand, when

does not introduce any free

t one uses new techniques In this paper one makes use of it to construct the Lyapunov functional $V(t)$ in (15), thus making $-\int_{t-h_1}^{t-d_1(t)}$
retained in the estimate of $\dot{V}(t)$. On the of
estimating integrals in $\dot{V}(x_t)$ one does not in
weighting matrix as [18, 19], but one uses
reported recently in [$t-h_1$ $\dot{x}(\alpha)^T Z_1 \dot{x}(\alpha) d\alpha$
her hand, when
troduce any free
new techniques
 $x \dot{y}^T Z_1 \dot{x}(\alpha) d\alpha$ as retained in the estimate of $V(t)$. On the other hand, when *V*(*t*). On the other hand, when
one does not introduce any free
], but one uses new techniques
. Take $-\int_{t-h_1}^{t} \dot{x}(\alpha)^T Z_1 \dot{x}(\alpha) d\alpha$ as
les it into two parts as (17) and
As for $-\int_{t}^{t} \dot{x}(\alpha)^T Z_1 \dot{x}(\alpha) d\alpha$ estimating integrals in $V(x_t)$ one does not introduce any free $V(x_t)$ one does not introduce any free

18, 19], but one uses new techniques

2, 13]. Take $-\int_{t-h_1}^{t} \dot{x}(\alpha)^T Z_1 \dot{x}(\alpha) d\alpha$ as

divides it into two parts as (17) and

s (23). As for $-\int_{t-d_1(t)}^{t} \dot{x}(\alpha)^T Z_2 \dot{x}(\alpha) d\alpha$ weighting matrix as [18, 19], but one uses new techniques reported recently in [12, 13]. Take $-\int_{t-h_1}^{t} \dot{x}(\alpha)^{T} Z_1 \dot{x}(\alpha) d\alpha$ as $-\int_{t}^{t}$
nto 1
∴ – \int sucl
ues $t-h_1$ $\dot{x}(\alpha)^T Z_1 \dot{x}(\alpha) d\alpha$ as

(*x*) parts as (17) and

(*x*) $\dot{x}(\alpha)^T Z_2 \dot{x}(\alpha) d\alpha$

(*x*) new way as (18)-

calculate integrals

extend to check the an example. One first divides it into two parts as (17) and then calculates them as (23). As for $-\int_t^t$
and so forth, one deals with it in such
(22). Thanks to the new techniques t
in $\dot{V}(x_t)$ and the convex polyhedron r
negative definite for the upper bound o
Theorem 2 is expe $t - d_1(t)$ $\dot{x}(\alpha)^T Z_2 \dot{x}(\alpha) d\alpha$
ew way as (18)-
culate integrals
od to check the
 \dot{x}_t), the resulting
tive with fewer and so forth, one deals with it in such a new way as (18)– (22). Thanks to the new techniques to calculate integrals in $V(x_t)$ and the convex polyhedron method to check the $V(x_t)$ and the convex polyhedron method to check the gative definite for the upper bound of $\dot{V}(x_t)$, the resulting eorem 2 is expected to be less conservative with fewer trix variables, as shown in the following exampl negative definite for the upper bound of Theorem 2 is expected to be less conservative with fewer matrix variables, as shown in the following example.

 $V(x_t)$, the resulting
ervative with fewer
ng example.
ating Q_1 and Q_3 one
sility criterion from When When μ_1 and μ_2 are unknown, eliminating Q_1 and Q_3 one
can obtain a delay-rate-independent stability criterion from
Theorem 2 as follows.
Corollary 6. The system (8) subject to (4) is asymptotically
stable can obtain a delay-rate-independent stability criterion from Theorem 2 as follows.

Corollary 6. *The system* (8) *subject to* (4) *is asymptotically*

 $Q_4 > 0$, and $Z_i > 0$, $j = 1, 2$, such that the following LMIs *hold:*

$$
\Phi_1 - e_{13}h_1^{-1}Z_2e_{13}^T - e_{23}h_2^{-1}Z_2e_{23}^T < 0,
$$

\n
$$
\Phi_1 - e_{13}h_1^{-1}Z_2e_{13}^T - e_{24}h_2h^{-2}Z_2e_{24}^T < 0,
$$

\n
$$
\Phi_1 - e_{24}h_1h^{-2}Z_2e_{24}^T - e_{23}h_2^{-1}Z_2e_{23}^T < 0,
$$

\n
$$
\Phi_1 - e_{24}h^{-1}Z_2e_{24}^T < 0,
$$

\n
$$
\Phi_1 - e_{24}h^{-1}Z_2e_{24}^T < 0,
$$

\n
$$
\Phi_1 h_1^{-1}(Z_1 + Z_2) = 0
$$

\n(28)

where

$$
\Phi_{1} - e_{24}h^{-1}Z_{2}e_{24}^{T} < 0,
$$
\nwhere\n
$$
\Phi_{1} = \begin{bmatrix}\n\tilde{\varphi}_{1} & PA_{1} & h_{1}^{-1}(Z_{1} + Z_{2}) & 0 & 0 \\
* & \tilde{\varphi}_{2} & h_{2}^{-1}Z_{2} & h^{-1}Z_{2} & 0 \\
* & * & \tilde{\varphi}_{3} & 0 & h_{1}^{-1}Z_{1} \\
* & * & * & -Q_{2} - h^{-1}Z_{2} & 0 \\
* & * & * & * & -Q_{4} - h_{1}^{-1}Z_{1}\n\end{bmatrix}
$$
\n
$$
+ \begin{bmatrix}\nA_{1}^{T} \\
A_{1}^{T} \\
0 \\
0 \\
0\n\end{bmatrix}\n\begin{bmatrix}\nh_{1}Z_{1} + hZ_{2}\n\end{bmatrix}\n\begin{bmatrix}\nA_{1}^{T} \\
A_{1}^{T} \\
0 \\
0 \\
0\n\end{bmatrix},
$$
\nwith $\tilde{\varphi}_{1} = PA + A^{T}P + Q_{2} + Q_{4} - h_{1}^{-1}(Z_{1} + Z_{2}), \tilde{\varphi}_{2} = -(h_{2}^{-1} + h^{-1})Z_{2}, and \tilde{\varphi}_{3} = -(h_{2}^{-1} + h_{1}^{-1})Z_{2} - 2h_{1}^{-1}Z_{1}.$ \nWhen $d_{1}(t) \equiv h_{1}$, that is, $d_{1}(t)$ is a constant delay, Theorem 2 reduces to the following corollary, which was reported recently in [13].

 $\frac{1}{n}$ -
|
! ! $\frac{1}{2}$ $\begin{bmatrix} 2 \ 1 \end{bmatrix}$ -
24
24 $\frac{1}{2}$

y,
as Z_1 .
is $\frac{1}{2}$. When $d_1(t) \equiv h_1$, that is, $d_1(t)$ is a constant delay, *d*₁(*t*) ≡ *h*₁, that is, *d*₁(*t*) is a constant delay,
reduces to the following corollary, which was
cently in [13].
7. *The system* (8) *with d*₁(*t*) ≡ *h*₁ and *d*₂(*t*)
≤ *d*₂(*t*) ≤ *h*₂ and *d* Theorem 2 reduces to the following corollary, which was reported recently in [13].

Corollary 7. *The system* (8) *with* $d_1(t) \equiv h_1$ *and* $d_2(t)$ $d_1(t) \equiv h_1$ and $d_2(t)$
 d_2 is asymptotically stable

nere exist $P > 0$, $Q_i > 0$,
 d_1 that the following LMIs *satisfying* $0 \le d_2(t) \le h_2$ *and* $d(t) \le \mu_2$ *is asymptotically stable* $0 ≤ d_2(t) ≤ h_2$ *and* $d(t) ≤ \mu_2$ *is asymptotically stable*
 $h_2 > 0$, $h_1 > 0$, *and* μ_2 *if there exist* $P > 0$, $Q_i > 0$,
 and $Z_j > 0$, $j = 1, 2$, *such that the following LMIs*
 $Φ_2 - [0 I - I 0]^T Z_2 [0 I - I 0] < 0$, *for given* $h_2 > 0$, $h_1 > 0$, and μ_2 *if there exist* $P > 0$, $Q_i > 0$, *h*₂ > 0*, h*₁ > 0*, and* μ_2 *if there exist P* > 0*, Q_i* > 0*,

3, and* Z_j > 0*, j* = 1*, 2, such that the following LMIs*
 $\Phi_2 - \begin{bmatrix} 0 & I & -I & 0 \end{bmatrix}^T Z_2 \begin{bmatrix} 0 & I & -I & 0 \end{bmatrix} < 0$,

(30) *hold:*

$$
i = 1, 2, 3, and Z_j > 0, j = 1, 2, such that the following LMIshold:
$$
\Phi_2 - \begin{bmatrix} 0 & I & -I & 0 \end{bmatrix}^T Z_2 \begin{bmatrix} 0 & I & -I & 0 \end{bmatrix} < 0,
$$

$$
\Phi_2 - \begin{bmatrix} 0 & I & 0 & -I \end{bmatrix}^T Z_2 \begin{bmatrix} 0 & I & 0 & -I \end{bmatrix} < 0,
$$
where

$$
\begin{bmatrix} \widehat{\omega}, PA, 0 & h^{-1}Z, 1 \end{bmatrix}
$$
$$

where

where
\n
$$
\Phi_2 = \begin{bmatrix}\n\hat{\varphi}_1 & PA_1 & 0 & h_1^{-1}Z_1 \\
* & \hat{\varphi}_2 & h_2^{-1}Z_2 & h_2^{-1}Z_2 \\
* & * & -Q_2 - h_2^{-1}Z_2 & 0 \\
* & * & * & -Q_4 - h_2^{-1}Z_2 - h_1^{-1}Z_1\n\end{bmatrix}
$$
\n
$$
+ \begin{bmatrix}\nA^T \\
A_1^T \\
0 \\
0\n\end{bmatrix}\n(h_1 Z_1 + h_2 Z_2) \begin{bmatrix}\nA^T \\
A_1^T \\
0 \\
0\n\end{bmatrix}^T,
$$
\nwith $\hat{\varphi}_1 = PA + A^T P + \sum_{i=1}^3 Q_i - h_1^{-1}Z_1$ and $\hat{\varphi}_2 = -(1 - \mu_2)Q_3 - 2h_2^{-1}Z_2.$ \n(31)

 $P/$ $\frac{1}{1}$ \overline{A}^2 $^{-1}$ 0
2
0
1 \overline{a} $2h_2^{-1}$ 2*.*

h_1		1.2	1.5
h ₂	0.415	0.376	0.248
h ₂	0.512	0.406	0.283
h,	0.5955	0.4632	0.3129

 h_2 0.5955 0.4632 0.3129

Elyapunov functional. Consider the follow *Proof.* Define the Lyapunov functional. Consider the following:

$$
V(x_t) = x(t)^T P x(t) + \int_{t-d(t)}^t x(\alpha)^T Q_3 x(\alpha) d\alpha
$$

+
$$
\int_{t-h_1}^t x(\alpha)^T Q_1 x(\alpha) d\alpha + \int_{t-h}^t x(\alpha)^T Q_2 x(\alpha) d\alpha
$$

+
$$
\int_{t-h_1}^t \int_s^t \dot{x}(\alpha)^T Z_1 \dot{x}(\alpha) d\alpha ds
$$

+
$$
\int_{t-h}^{t-h_1} \int_s^t \dot{x}(\alpha)^T Z_2 \dot{x}(\alpha) d\alpha ds.
$$
 (32)

+ \int_{t-h} \int_{s} $x(\alpha)$ \angle_{2} $x(\alpha)$ and as.

milar line as in the derivation

stability can be established, a J,
ne
y a
 a ()
s ir
n b Along a similar line as in the derivation of Theorem 2 the asymptotic stability can be established, and details are thus omitted. \Box

Remark 8. Note that when $d_1(t)$ is a constant delay h_1 , system *d*₁(*t*) is a constant delay *h*₁, system
system in the form of (11) with
y: *h*₁ ≤ *d*(*t*) ≤ *h*, 0 ≤ *d*(*t*) ≤
as a model for networked control
c-induced delay and data dropout
he form of LMIs Corollary 7 can (8) can be regarded as a system in the form of (11) with interval time-varying delay: $h_1 \le d(t) \le h$, $0 \le d(t) \le$ $h_1 \leq d(t) \leq h, 0 \leq d(t) \leq$
a model for networked control
nduced delay and data dropout
form of LMIs Corollary 7 can
ability criterion for the model.
dron method Corollary 7 is less μ_2 . The system can serve as a model for networked control
systems with both network-induced delay and data dropout
phenomenon [15, 16]. In the form of LMIs Corollary 7 can
provide a delay-dependent stability criterion systems with both network-induced delay and data dropout phenomenon [15, 16]. In the form of LMIs Corollary 7 can provide a delay-dependent stability criterion for the model. Derived by the convex polyhedron method Corollary 7 is less conservative than those recently reported in [11]; see [13].

In the following, we take the example in [19] to show that our stability criteria, though having much fewer matrix variables, are less conservative.

Example 9. Consider the system (8) with

$$
A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},
$$

\n
$$
\vec{d}_1(t) \le 0.1, \qquad \vec{d}_2(t) \le 0.8.
$$

\nupper bound h_1 of $d_1(t)$, we intend to find the
\nupper bound h_2 of $d_2(t)$, which guarantees the
\nstability of (8).

u
r
ili
g (*t*) \le 0.1, $u_2(t) \le$ 0.0.

bound h_1 of $d_1(t)$, we interval bound h_2 of $d_2(t)$, which y of (8).

ven, the admissible h_1 can For given upper bound h_1 of $d_1(t)$, we intend to find the h_2 of $d_2(t)$, which guarantees the
).

le admissible h_1 can be seen from

le admissible h_1 can be seen from admissible upper bound asymptotic stability of (8).

Table 2.

 h_2 of $d_2(t)$, which guarantees the

e admissible h_1 can be seen from

d 2, Theorem 2 is less conservative

is worth noting that with fewer When h_2 is given, the admissible h_1 can be seen from
le 2.
As seen in Tables 1 and 2, Theorem 2 is less conservative
1 those in [15, 16]. It is worth noting that with fewer
rix variables involved, Theorem 2 needs le As seen in Tables 1 and 2, Theorem 2 is less conservative than those in [15, 16]. It is worth noting that with fewer matrix variables involved, Theorem 2 needs less computational requirements.

When $d_1(t)$ is a constant delay h_1 , the system can be $d_1(t)$ is a constant delay h_1 , the system can be
on as those with interval time-varying delay. As looked upon as those with interval time-varying delay. As

TABLE 2: Admissible upper bound h_1 for various h_2 .						
Method	h,	0.1	0.2	0.3		
$\lceil 18 \rceil$	h,	2.263	1.696	1.324		
[19]	h_{1}	2.300	1.779	1.453		
Theorem 2	h,	2.3400	1.8337	1.5318		
TABLE 3: Admissible upper bound h for various h_1 .						
Method	n,					

h 1.8737 2.5049 3.2592 4.0745

demark 8, the stability result in Corollary

in [11] can be turned to for computing the repound *h* of *d*(*t*), which are shown in Table 3 indicated in Remark 8, the stability result in Corollary 7 as well as that in [11] can be turned to for computing the admissible upper bound h of $d(t)$, which are shown in Table 3.

h of $d(t)$, which are shown in Table 3.
pendent criterion for systems with
lay, Corollary 7 has advantages over
computed admissible upper bound
y is larger. Even as a delay-dependent criterion for systems with interval time-varying delay, Corollary 7 has advantages over [11] in the sense that the computed admissible upper bound of the time-varying delay is larger.

3. State Feedback Control

Without a free weighting matrix introduced, Theorem 2 only involves the matrices in the Lyapunov functional employed. It can be expected as a useful tool for the H^{∞} state feedback
control problem formulated above. We first present an H^{∞}
performance analysis result in the following.
Theorem 10. System (1) and (2) with $u(t) =$ performance analysis result in the following.

control problem formulated above. We first present an H^{∞}
performance analysis result in the following.
Theorem 10. System (1) and (2) with $u(t) = 0$ and delays
subject to (4) and (5) is asymptotically stable with a **Theorem 10.** System (1) and (2) with $u(t) = 0$ and delays $u(t) = 0$ *and delays*
y stable with an H^{∞}
n h_1 , h_2 , μ_1 , and μ_2 , *i*
1, 2, 3, 4, and $Z_j > 0$,
d: *subject to* (4) *and* (5) *is asymptotically stable with an* H^{∞} H^{∞}
 $u_2, \, \dot{u}_3$
> 0, *disturbance attenuation level* γ for given h_1 , h_2 , μ_1 , and μ_2 , if *f f for given* h_1 , h_2 , μ_1 , and μ_2 , i_j
 i > 0, *i* = 1, 2, 3, 4, and Z_j > 0,
 g LMIs hold:
 $-\bar{e}_{23}h_2^{-1}Z_2\bar{e}_{23}^T$ < 0,
 $\bar{e}_{23}h_1h_2^{-2}Z_3\bar{e}_{23}^T$ < 0, = 1, 2 *such that the following LMIs hold:*

there exist matrices
$$
P > 0
$$
, $Q_i > 0$, $i = 1, 2, 3, 4$, and $Z_j > 0$,
\n $j = 1, 2$ such that the following LMIs hold:
\n
$$
\overline{\Phi} - \overline{e}_{13} h_1^{-1} Z_2 \overline{e}_{13}^T - \overline{e}_{23} h_2^{-1} Z_2 \overline{e}_{23}^T < 0,
$$
\n
$$
\overline{\Phi} - \overline{e}_{13} h_1^{-1} Z_2 \overline{e}_{13}^T - \overline{e}_{24} h_2 h^{-2} Z_2 \overline{e}_{24}^T < 0,
$$
\n
$$
\overline{\Phi} - \overline{e}_{24} h_1 h^{-2} Z_2 \overline{e}_{24}^T - \overline{e}_{23} h_2^{-1} Z_2 \overline{e}_{23}^T < 0,
$$
\n(34)
\n
$$
\overline{\Phi} - \overline{e}_{24} h_1 h^{-2} Z_2 \overline{e}_{24}^T < 0,
$$
\nwhere $\overline{e}_{13} = \begin{bmatrix} e_{13}^T & 0 \end{bmatrix}^T$, \overline{e}_{23} , \overline{e}_{24} , and \overline{e}_{35} follow similarly and

where $\overline{e}_{13} = \left[e_{13}^T \right]$

$$
\overline{\Phi} - \overline{e}_{24}h^{-1}Z_{2}\overline{e}_{24}^{T} < 0,
$$
\nwhere $\overline{e}_{13} = \begin{bmatrix} e_{13}^{T} & 0 \end{bmatrix}^{T}$, \overline{e}_{23} , \overline{e}_{24} , and \overline{e}_{35} follow similarly and\n
$$
\overline{\Phi} = \begin{bmatrix} \varphi_{1} & PA_{1} & h_{1}^{-1}(Z_{1} + Z_{2}) & 0 & 0 & PE \\ * & \varphi_{2} & h_{2}^{-1}Z_{2} & h_{2}^{-1}Z_{2} & 0 & 0 \\ * & * & \varphi_{3} & 0 & h_{1}^{-1}Z_{1} & 0 \\ * & * & * & -Q_{2} - h_{2}^{-1}Z_{2} & 0 & 0 \\ * & * & * & * & -Q_{4} - h_{1}^{-1}Z_{1} & 0 \\ * & * & * & * & * & -\gamma^{2}I \end{bmatrix}
$$
\n
$$
+ \begin{bmatrix} A^{T} \\ A_{1}^{T} \\ 0 \\ 0 \\ 0 \\ E^{T} \end{bmatrix} \begin{bmatrix} H_{1}Z_{1} + hZ_{2} \end{bmatrix} \begin{bmatrix} A_{1}^{T} \\ A_{1}^{T} \\ 0 \\ 0 \\ E^{T} \end{bmatrix}^{T} + \begin{bmatrix} C^{T} \\ C_{1}^{T} \\ 0 \\ 0 \\ E^{T} \end{bmatrix}^{T} + \begin{bmatrix} C^{T} \\ C_{1}^{T} \\ 0 \\ 0 \\ E^{T} \end{bmatrix}^{T} + \begin{bmatrix} C^{T} \\ C_{1}^{T} \\ 0 \\ 0 \\ E^{T} \end{bmatrix}^{T} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ E^{T} \end{bmatrix},
$$
\n(35)

(35)

with φ_1 , φ_2 , φ_3 , and *h* given in Theorem 2.

 φ_1 , φ_2 , φ_3 , and *h* given in Theorem 2.
 f. Comparing $\overline{\Phi}$ with Φ in (13), we implies (12). Therefore system (1) a

asymptotically stable. Now using t

ional as $V(t)$ in (15) and calculating *Proof.* Comparing Φ with Φ in (13), we can conclude that
Therefore system (1) and (2) with $u(t) = t$ stable. Now using the same Lyapunov
in (15) and calculating $\dot{V}(t)$ similar to the
em 2 along the solution of system (1) and
e have (34) implies (12). Therefore system (1) and (2) with $u(t)$ = () is asymptotically stable. Now using the same Lyapunov functional as $V(t)$ in (15) and calculating $\dot{V}(t)$ similar to the derivation of Theorem 2 along the solution of system (1) and (2) with $u(t) = 0$, we have $y(t)^T y(t$ functional as $V(t)$ in (15) and calculating $\dot{V}(t)$ similar to the *V*(*t*) in (15) and calculating *V*(*t*) similar to the
Theorem 2 along the solution of system (1) and
= 0, we have
 $-\gamma^2 w(t)^T w(t) + \dot{V}(t) \le \overline{\zeta}(t)^T \overline{M} (\alpha, \beta) \overline{\zeta}(t)$, (36) derivation of Theorem 2 along the solution of system (1) and

(2) with
$$
u(t) = 0
$$
, we have
\n
$$
y(t)^{T} y(t) - \gamma^{2} w(t)^{T} w(t) + \dot{V}(t) \leq \overline{\zeta}(t)^{T} \overline{M}(\alpha, \beta) \overline{\zeta}(t),
$$
\n(36)
\nwhere
\n
$$
\overline{M}(\alpha, \beta) = \overline{\Phi} - \alpha \overline{e}_{24} h_{1} h^{-2} Z_{2} \overline{e}_{24}^{T} - (1 - \alpha) \overline{e}_{13} h_{1}^{-1} Z_{2} \overline{e}_{13}^{T}
$$

where

$$
y(t) y(t) - \gamma \omega(t) \omega(t) + v(t) \le \zeta(t) \ln(\alpha, \beta) \zeta(t),
$$
\n(36)
\nwhere
\n
$$
\overline{M}(\alpha, \beta) = \overline{\Phi} - \alpha \overline{e}_{24} h_1 h^{-2} Z_2 \overline{e}_{24}^T - (1 - \alpha) \overline{e}_{13} h_1^{-1} Z_2 \overline{e}_{13}^T - \beta \overline{e}_{24} h_2 h^{-2} Z_2 \overline{e}_{24}^T - (1 - \beta) \overline{e}_{23} h_2^{-1} Z_2 \overline{e}_{23}^T,
$$
\n(37)
\nwith α and β defined in the proof of Theorem 2 and $\overline{\zeta}(t)$

 $-\beta \bar{e}_{24} h_2 h^{-2} Z_2 \bar{e}_{24}^T - (1 - \beta) \bar{e}_{23} h_2^{-1} Z_2 \bar{e}_1$

defined in the proof of Theorem 2 and

with $\zeta(t)$ in (24). On the one hand, u ,
ζ
1, α and β defined in the proof of Theorem 2 and $\zeta(t) =$
 $\left[\frac{w(t)^T}{T}\right]^T$ with $\zeta(t)$ in (24). On the one hand, using the

ex polyhedron method we can prove that $\overline{M}(\alpha, \beta) < 0$

4). On the other hand, under the $\left[\zeta(t)^T \ w(t)^T\right]^T$ with $\zeta(t)$ in (24). On the one hand, using the convex polyhedron method we can prove that $\overline{M}(\alpha, \beta) < 0$ by (34). On the other hand, under the zero condition we have $V(0) = 0$ and $V(\infty) \ge 0$. Int T with convex polyhedron method we can prove that $\overline{M}(\alpha, \beta) < 0$ $(.,)$
ition we have
of (36) leads
o]. This ends by (34). On the other hand, under the zero condition we have $V(0) = 0$ and $V(\infty) \ge 0$. Integrating both sides of (36) leads
to $||y||_2 < \gamma ||w||_2$ for all nonzero $w(t) \in L_2[0, \infty]$. This ends
the proof.
Now we are in a position to resolve the H^{∞} state feedback
control problem the proof.

to $||y||_2 < y ||w||_2$ for all nonzero $w(t) \in L_2[0, \infty]$. This ends
the proof.
Now we are in a position to resolve the H^{∞} state feedback
control problem aforementioned.

Theorem 11. Consider sutan (1) and (2) with dela Now we are in a position to resolve the H^{∞} state feedback control problem aforementioned.

Now we are in a position to resolve the H^{∞} state feedback

control problem aforementioned.
 Theorem 11. Consider system (1) and (2) with delays subject

to (4) and (5). Given h_1 , h_2 , μ_1 , and μ_2 , ther **Theorem 11.** *Consider system* (1) *and* (2) *with delays subject to* (4) and (5). Given h_1 , h_2 , μ_1 , and μ_2 , there exists a state h_1 , h_2 , μ_1 , and μ_2 , there exists a state
 $h(t) = Kx(t)$ ensuring that the closed-

bically stable with an H^{∞} disturbance

there exist matrices $\overline{P} > 0$, $\overline{Q}_i > 0$,
 $\overline{Q}_i > 0$, $\overline{Q}_i = 1, 2$, $\$ $u(t) = Kx(t)$ ensuring that the closed-
ptotically stable with an H^{∞} disturbance
if there exist matrices $\overline{P} > 0$, $\overline{Q}_i > 0$,
 $\overline{Q}_i > 0$, $j = 1, 2$, \overline{K} such that the following *loop system is asymptotically stable with an* H^{∞} *disturbance*
 $\overline{Q}_i > 0, \overline{Q}_i > 0,$

hat the following

(38) *attenuation level* γ , *if there exist matrices* $\overline{P} > 0$, $\overline{Q}_i > 0$, *y*, *if there exist matrices* $P > 0$ *,* $Q_i > 0$ *,
* $\overline{Z}_j > 0$ *,* $j = 1, 2$ *,* \overline{K} *such that the following
* $\Omega_i \Gamma$ *
* $\Gamma^T \Lambda$ *< 0,* $i = 1, 2, 3, 4$ *, (38)* $i = 1, 2, 3, 4,$ and $Z_j > 0$, $j = 1, 2, K$ such that the following
LMIs hold:
 $\begin{bmatrix} \Omega_i & \Gamma \\ \Gamma^T & \Lambda \end{bmatrix} < 0, \quad i = 1, 2, 3, 4,$ (38) *LMIs hold:*

$$
\begin{bmatrix} \Omega_i & \Gamma \\ \Gamma^T & \Lambda \end{bmatrix} < 0, \quad i = 1, 2, 3, 4,\tag{38}
$$
\n
$$
h_1^{-1} \overline{Z}_2 \overline{e}_{13}^T - \overline{e}_{23} h_2^{-1} \overline{Z}_2 \overline{e}_{23}^T,
$$

where

$$
\begin{bmatrix} r^T \\ \Gamma^T \end{bmatrix} < 0, \quad i = 1, 2, 3, 4,\tag{38}
$$
\nere

\n
$$
\Omega_1 = \Psi - \bar{e}_{13}h_1^{-1}\overline{Z}_2\bar{e}_{13}^T - \bar{e}_{23}h_2^{-1}\overline{Z}_2\bar{e}_{23}^T,
$$
\n
$$
\Omega_2 = \Psi - \bar{e}_{13}h_1^{-1}\overline{Z}_2\bar{e}_{13}^T - \bar{e}_{24}h_2h^{-2}\overline{Z}_2\bar{e}_{24}^T,
$$
\n
$$
\Omega_3 = \Psi - \bar{e}_{24}h_1h^{-2}\overline{Z}_2\bar{e}_{24}^T - \bar{e}_{23}h_2^{-1}\overline{Z}_2\bar{e}_{23}^T,
$$
\n
$$
\Omega_4 = \Psi - \bar{e}_{24}h^{-1}\overline{Z}_2\bar{e}_{24}^T,
$$
\n
$$
\Lambda = \text{diag}\left\{h_1^{-1}\left(\overline{Z}_1 - 2\overline{P}\right), h^{-1}\left(\overline{Z}_2 - 2\overline{P}\right), -I\right\},
$$
\n
$$
\Gamma = \begin{bmatrix} \overline{P}A^T + \overline{K}^T B^T & \overline{P}A^T + \overline{K}^T B^T & \overline{P}C^T + \overline{K}^T D^T \\ \overline{P}A_1^T & \overline{P}A_1^T & \overline{P}C_1^T \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ E^T & E^T \end{bmatrix},
$$
\n
$$
\text{(39)}
$$

with

$$
\Psi = \begin{bmatrix}\n\overline{\varphi}_{1} & A_{1}\overline{P} & h_{1}^{-1}(\overline{Z}_{1} + \overline{Z}_{2}) & 0 & 0 & E \\
* & \overline{\varphi}_{2} & h_{2}^{-1}\overline{Z}_{2} & h^{-1}\overline{Z}_{2} & 0 & 0 \\
* & * & \overline{\varphi}_{3} & 0 & h_{1}^{-1}\overline{Z}_{1} & 0 \\
* & * & * & -\overline{Q}_{2} - h^{-1}\overline{Z}_{2} & 0 & 0 \\
* & * & * & * & -\overline{Q}_{4} - h_{1}^{-1}\overline{Z}_{1} & 0 \\
* & * & * & * & * & -\overline{Q}_{4} - h_{1}^{-1}\overline{Z}_{1} & 0 \\
* & * & * & * & * & -\gamma^{2}I\n\end{bmatrix},
$$
\n
$$
\overline{\varphi}_{1} = A\overline{P} + B\overline{K} + (A\overline{P} + B\overline{K})^{T} + \sum_{i=1}^{4} \overline{Q}_{i} - h_{1}^{-1}(\overline{Z}_{1} + \overline{Z}_{2}),
$$
\n
$$
\overline{\varphi}_{2} = -(1 - \mu) \overline{Q}_{3} - (h_{2}^{-1} + h_{1}^{-1}) \overline{Z}_{2},
$$
\n
$$
\overline{\varphi}_{3} = -(1 - \mu_{1}) \overline{Q}_{1} - (h_{2}^{-1} + h_{1}^{-1}) \overline{Z}_{2} - 2h_{1}^{-1}\overline{Z}_{1},
$$
\n(40)\nand $\overline{e}_{13}, \overline{e}_{23}, \overline{e}_{24}, \overline{e}_{35}, h, and \mu \text{ are given in Theorem 10. Moreover, if the foregoing condition holds, a desired controller$

 $\begin{array}{c} \mathcal{L}_1, \ \mathcal{U}_2 \ \mathcal{L}_3, \end{array}$ *and* \bar{e}_{13} , \bar{e}_{23} , \bar{e}_{24} , \bar{e}_{35} , *h*, *and* μ *are given in Theorem 10.*
Moreover, if the foregoing condition holds, a desired controller gain matrix is given by
 $K - \overline{K} \overline{P}^{-1}$ (41) *Moreover, if the foregoing condition holds, a desired controller gain matrix is given by*

$$
K = \overline{K} \, \overline{P}^{-1}.
$$
 (41)

(42)

 $K = \overline{K} \overline{P}^{-1}$. (41)

ler $u(t) = Kx(t)$ to system (1) and (2)

system is formulated as follows: and then the closed-loop system is formulated as follows:

Proof. Apply the controller
$$
u(t) = Kx(t)
$$
 to system (1) and (2)
and then the closed-loop system is formulated as follows:

$$
\dot{x}(t) = (A + BK) x(t) + A_1 x(t - d_1(t) - d_2(t)) + Ew(t),
$$

$$
y(t) = (C + DK) x(t) + C_1 x(t - d_1(t) - d_2(t)) + Fw(t).
$$
(42)

 H^{∞} disturbance attenuation level γ , if there exist matrices By Theorem 10 this system is asymptotically stable with an H^{∞} disturbance attenuation level γ , if there exist matrices
 $P > 0$, $Q_i > 0$, $i = 1, 2, 3, 4$, and $Z_j > 0$, $j = 1, 2$, such

that
 $\overline{\Phi}_c - \overline{e}_{13} h_1^{-1} Z_2 \overline{e}_{13}^T - \overline{e}_{23} h_2^{-1} Z_2 \overline{e}_{23}^T < 0$, $P > 0$, $Q_i > 0$, $i = 1, 2, 3, 4$, and $Z_j > 0$, $j = 1, 2$, such
that
 $\overline{\Phi}_c - \overline{e}_{13} h_1^{-1} Z_2 \overline{e}_{13}^T - \overline{e}_{23} h_2^{-1} Z_2 \overline{e}_{23}^T < 0$, that

$$
\overline{\Phi}_c - \overline{e}_{13} h_1^{-1} Z_2 \overline{e}_{13}^T - \overline{e}_{23} h_2^{-1} Z_2 \overline{e}_{23}^T < 0,
$$
\n
$$
\overline{\Phi}_c - \overline{e}_{13} h_1^{-1} Z_2 \overline{e}_{13}^T - \overline{e}_{24} h_2 h^{-2} Z_2 \overline{e}_{24}^T < 0,
$$
\n
$$
\overline{\Phi}_c - \overline{e}_{24} h_1 h^{-2} Z_2 \overline{e}_{24}^T - \overline{e}_{23} h_2^{-1} Z_2 \overline{e}_{23}^T < 0,
$$
\n
$$
\overline{\Phi}_c - \overline{e}_{24} h^{-1} Z_2 \overline{e}_{24}^T < 0,
$$
\n(43)

(39)

Theorem 10 and

where
$$
\overline{e}_{13}, \overline{e}_{23}, \overline{e}_{24}, \overline{e}_{35}
$$
, and *h* are the same as those in Theorem 10 and
\n
$$
\overline{\Phi}_c = \begin{bmatrix}\n\varphi_{c1} & PA_1 & h_1^{-1}(Z_1 + Z_2) & 0 & 0 & PE \\
* & \varphi_2 & h_2^{-1}Z_2 & h^{-1}Z_2 & 0 & 0 \\
* & * & \varphi_3 & 0 & h_1^{-1}Z_1 & 0 \\
* & * & * & -Q_2 - h^{-1}Z_2 & 0 & 0 \\
* & * & * & * & -Q_4 - h_1^{-1}Z_1 & 0 \\
* & * & * & * & * & -\gamma^2I\n\end{bmatrix}
$$
\n
$$
+ \begin{bmatrix}\n(A + BK)^T \\
A_1^T \\
0 \\
0 \\
0 \\
0 \\
0\n\end{bmatrix}\n\begin{bmatrix}\nh_1Z_1 + hZ_2\n\end{bmatrix}\n\begin{bmatrix}\n(A + BK)^T \\
A_1^T \\
0 \\
0 \\
0 \\
E^T\n\end{bmatrix} + \begin{bmatrix}\n(C + DK)^T \\
C_1^T \\
0 \\
0 \\
0 \\
0 \\
0\n\end{bmatrix} + \begin{bmatrix}\n(C + DK)^T \\
C_1^T \\
0 \\
0 \\
0 \\
0 \\
0 \\
0\n\end{bmatrix},
$$
\n
$$
+ \begin{bmatrix}\n(A + BK)^T \\
A_1^T \\
0 \\
0 \\
E^T\n\end{bmatrix} = P(A + BK) + \begin{bmatrix}\n(2 + DK)^T \\
A_1^T \\
0 \\
0 \\
E^T\n\end{bmatrix} + \begin{bmatrix}\n(C + DK)^T \\
C_1^T \\
0 \\
0 \\
0 \\
0 \\
0 \\
0\n\end{bmatrix},
$$
\n
$$
+ \begin{bmatrix}\n(C + DK)^T \\
C_1^T \\
0 \\
0 \\
0 \\
0 \\
0\n\end{bmatrix} + \begin{bmatrix}\n(C + DK)^T \\
C_1^T \\
0 \\
0 \\
0 \\
0 \\
0 \\
0\n\end{bmatrix} + \begin{bmatrix}\n(C + DK)^T \\
C_1^T \\
0 \\
0 \\
0 \\
0 \\
0 \\
0\n\end{bmatrix} + \begin{bmatrix}\n(C + DK)^T \\
C_1^T \\
0 \\
0 \\
0 \\
0 \\
0 \\
0\n\end{bmatrix} + \begin{bmatrix}\n(C + DK)^T \\
C_1^T \\
0 \\
0 \\
0 \\
0 \\
0 \\
0\n\end{bmatrix} + \begin{bmatrix}\n(C + DK)^
$$

 $\frac{1}{2}$ E^T
n 1 $\frac{d}{dt}$ $\frac{1}{2}$ with φ_2 and φ_3 defined in Theorem 10 and $\varphi_{c1} = P(A + BK)$ + φ_2 and φ_3 defined in Theorem 10 and $\varphi_{c1} = P(A + BK) + BK)^T P + \sum_{i=1}^4 Q_i - h_1^{-1}(Z_1 + Z_2)$.
Vrite $J = \text{diag}\{P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, I\}$, $\overline{P} = P^{-1}, \overline{Z}_i = r_i P^{-1}$, $i = 1, 2, \overline{Q}_j = P^{-1}Q_j P^{-1}$, and $j = 1, 2, 3, 4$.
prmin $(A + BK)^T P + \sum_{i=1}^{4} Q_i - h_1^{-1} (Z_1 + Z_2).$

Write $J = \text{diag}\{P^{-1}, P^{-1}, P^{-1}, P^{-1}, \ P^{-1} Z_i P^{-1}, i = 1, 2, \overline{Q}_j = P^{-1} Q_j P^{-1}$

Performing a congruence transformati
 $\Psi + \Gamma \widehat{\Lambda}^{-1} \Gamma^T - \overline{e}_{13} h_1^{-1} \overline{Z}_2 \overline{e}_{13}^T - \overline{e}_{23} h_1$ Write $J = \text{diag}\{P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, I\}, \overline{P} = P^{-1}, \overline{Z}_i = i = 1, 2, \overline{Q}_j = P^{-1}Q_jP^{-1}$, and $j = 1, 2, 3, 4$.

g a congruence transformation to (43) by *J* yields
 $\Gamma \widehat{\Lambda}^{-1} \Gamma^T - \overline{e}_{13} h_1^{-1} \overline{Z}_2 \overline{e}_{13}^T - \overline{e}_{23} h_2^{-1}$

$$
P^{-1}Z_iP^{-1}, i = 1, 2, \overline{Q}_j = P^{-1}Q_jP^{-1}, \text{ and } j = 1, 2, 3, 4.
$$

Performing a congruence transformation to (43) by *J* yields

$$
\Psi + \Gamma \widehat{\Lambda}^{-1} \Gamma^T - \overline{e}_{13} h_1^{-1} \overline{Z}_2 \overline{e}_{13}^T - \overline{e}_{23} h_2^{-1} \overline{Z}_2 \overline{e}_{23}^T < 0,
$$

$$
\Psi + \Gamma \widehat{\Lambda}^{-1} \Gamma^T - \overline{e}_{13} h_1^{-1} \overline{Z}_2 \overline{e}_{13}^T - \overline{e}_{24} h_2 h^{-2} \overline{Z}_2 \overline{e}_{24}^T < 0,
$$

$$
\Psi + \Gamma \widehat{\Lambda}^{-1} \Gamma^T - \overline{e}_{24} h_1 h^{-2} \overline{Z}_2 \overline{e}_{24}^T - \overline{e}_{23} h_2^{-1} \overline{Z}_2 \overline{e}_{23}^T < 0,
$$

$$
\Psi + \Gamma \widehat{\Lambda}^{-1} \Gamma^T - \overline{e}_{24} h^{-1} \overline{Z}_2 \overline{e}_{24}^T < 0,
$$
where

$$
\widehat{\Lambda} - \text{diag} \{ h^{-1} Z^{-1} h^{-1} Z^{-1} I \}
$$
 (46)

where

$$
\Psi + \Gamma \widehat{\Lambda}^{-1} \Gamma^T - \overline{e}_{24} h^{-1} \overline{Z}_2 \overline{e}_{24}^T < 0,
$$
\n
$$
\widehat{\Lambda} = \text{diag} \left\{ h_1^{-1} Z_1^{-1}, h^{-1} Z_2^{-1}, I \right\}.
$$
\n(46)

By Schur complements we have

$$
\widehat{\Lambda} = \text{diag}\left\{ h_1^{-1} Z_1^{-1}, h^{-1} Z_2^{-1}, I \right\}.
$$
\n
\n146)
\n160
\n160
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\n19

 $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ $\left[\Gamma^T \right] - \widehat{\Lambda}$ $\left[\begin{array}{cc} 0, & i = 1, 2, 3, 4. \end{array}\right]$ (47)

not linear in \overline{P} , \overline{K} , \overline{Q}_i , and \overline{Z}_j due to $Z_i^{-1} =$

r, noting that $(\overline{P} - \overline{Z}_i) \overline{Z}_i^{-1} (\overline{P} - \overline{Z}_i) \ge 0$, we
 $\overline{Z}_i + 2\overline{P$ Note that (47) is not linear in P, K, Q_i , and Z_j due to Z_i \overline{P} , \overline{K} , \overline{Q}_i , and \overline{Z}_j due to Z_i^{-1}
 $(\overline{P} - \overline{Z}_i) \overline{Z}_i^{-1} (\overline{P} - \overline{Z}_i) \ge 0$,

fore, $Z_i^{-1} \ge -\overline{Z}_i + 2\overline{P}$. It follo

ch means that (38) implies (4 $\frac{1}{\sqrt{2}}$
 $\frac{1}{\sqrt{2}}$ $\overline{P} \, \overline{Z}_i^{-1}$
have \overline{l}
imme
This c
D i \overline{P} . However, noting that $(\overline{P} - \overline{Z}_i) \overline{Z}_i^{-1}$
 $\overline{P} \overline{Z}_i^{-1} \overline{P} \ge -\overline{Z}_i + 2\overline{P}$. Therefore, $Z_i^{-1} \ge -$

diately that $-\widehat{\Lambda} \le \Lambda$, which means that

completes the proof.

ue to the fact that Z_i^{-1 i $(P - Z_i) \ge 0$, we
 $\overline{Z}_i + 2\overline{P}$. It follows

(38) implies (47). □

condition (38) is have $\overline{P} \, \overline{Z}_i^{-1}$
immediate
This comp
Due to
more conse $i^1 \leq i^1 \mathbb{Z}$. Increase, \mathbb{Z}_i $\overline{P} \ge -\overline{Z}_i + 2\overline{P}$. Therefore, $Z_i^{-1} \ge -\overline{Z}_i + 2\overline{P}$. It follows
ly that $-\widehat{\Lambda} \le \Lambda$, which means that (38) implies (47).
letes the proof.
the fact that $Z_i^{-1} \ge -\overline{Z}_i + 2\overline{P}$, condition (38) is
ryati immediately that $-\widehat{\Lambda} \leq \Lambda$, which means that (38) implies (47). This completes the proof.

 $-\Lambda \le \Lambda$, which means that (38) implies (47).

le proof. \square

let that $Z_i^{-1} \ge -\overline{Z}_i + 2\overline{P}$, condition (38) is

e than (47). However, based on (38) one can

pproach to the H^{∞} state feedback control Due to the fact that $Z_i^{-1} \ge -\overline{Z}_i + 2\overline{P}$, condition (38) is
re conservative than (47). However, based on (38) one can
ain an LMI approach to the H^{∞} state feedback control
blem for systems with two additive t more conservative than (47). However, based on (38) one can obtain an LMI approach to the H^{∞} state feedback control
problem for systems with two additive time-varying delays.
The existence of the state feedback controller is guaranteed
by the feasibility of LMIs (38). When L problem for systems with two additive time-varying delays. The existence of the state feedback controller is guaranteed by the feasibility of LMIs (38). When LMIs (38) are solvable, the controller can be constructed according to (41). Based on

),
),
T $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ F^T
bta
exi $\frac{1}{\sqrt{\epsilon}}$ |
|
|
|
|
|
| |
|-
|
|Ct E_1
 $S6$ F^T

use:
 $\frac{1}{2}$
 $\frac{1}{2}$ $\frac{1}{2}$ |
|
|
|
|
| h (47), one can obtain a less conservative controller at the cost of more complexity by employing CCL method [21].

To illustrate the effectiveness of this method we provide an example.

Example 12. Consider system (1) and (2) with parameters given as follows:

$$
A = \begin{bmatrix} 0.11 & 0 \\ 0 & -0.9 \end{bmatrix}, \t A_1 = \begin{bmatrix} -2 & 0 \\ -1 & 1.1 \end{bmatrix},
$$

\n
$$
E = \begin{bmatrix} 0.56 \\ 0.61 \end{bmatrix}, \t B = \begin{bmatrix} 0.2 \\ -2.5 \end{bmatrix}, \t (48)
$$

\n
$$
C = [0.1 \quad 1.8], \t C_1 = [0.7 \quad -1],
$$

\n
$$
F = 0.1, \t D = 0.4.
$$

\n
$$
= 0.1, h_2 = 0.2, \mu_1 = 0.1, \mu_2 = 0.2, \text{ and } \gamma = 0.6 \text{ we that LMIs in (38) are feasible with}
$$

can find that LMIs in (38) are feasible with

Given
$$
h_1 = 0.1
$$
, $h_2 = 0.2$, $\mu_1 = 0.1$, $\mu_2 = 0.2$, and $\gamma = 0.6$ we
can find that LMIs in (38) are feasible with

$$
\overline{P} = \begin{bmatrix} 2.6423 & -0.2090 \\ -0.2090 & 0.7465 \end{bmatrix}, \qquad \overline{K} = \begin{bmatrix} -0.5241 & 0.8779 \end{bmatrix}.
$$
(49)
By Theorem 11, there exists a state feedback controller

$$
u(t) = \overline{K} \overline{P}^{-1} x(t) = \begin{bmatrix} -0.1078 & 1.1458 \end{bmatrix} x(t)
$$
(50)

By Theorem 11, there exists a state feedback controller

$$
u(t) = \overline{K} \, \overline{P}^{-1} x(t) = [-0.1078 \, 1.1458] \, x(t) \qquad (50)
$$

 $u(t) = \overline{K} \overline{P}^{-1} x(t) = [-0.1078 \text{ } 1.1458] x(t)$ (50)
t the closed-loop system is asymptotically stable for
 $t > 0.1$, $0 \le d_2(t) \le 0.2$ with an H^{∞} disturbance
on level $\gamma = 0.6$. such that the closed-loop system is asymptotically stable for $0 \le d_1(t) \le 0.1$, $0 \le d_2(t) \le 0.2$ with an H^{∞} disturbance
attenuation level $\gamma = 0.6$.
4. Conclusion
This paper is concerned with H^{∞} control for a networked attenuation level $\gamma = 0.6$.

4. Conclusion

 $\gamma = 0.6.$
 a meerned f system This paper is concerned with H^{∞} control for a networked control model of systems with two additive time-varying delays. For one thing a new delay-dependent stability criterion was derived, which improves over existi control model of systems with two additive time-varying delays. For one thing a new delay-dependent stability criterion was derived, which improves over existing ones in

that it has less conservatism with fewer matrix variables. A delay-rate-independent criterion was also obtained as a byproduct. When one of the delays is constant, a new stability criterion was given for systems with interval time-varying delay. Then examples were provided to illustrate the reduced conservatism of the criteria. Finally the H^{∞} state feedback H^{∞} state feedback
pproach, which was
er example. control problem was solved via an LMI approach, which was demonstrated to be effective using another example.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This work was partially supported by the National Natural Science Foundation of China under Grants 61374090, 61174085, and 61104007, the Program for Scientific Research Innovation Team in Colleges and Universities of Shandong Province, and the Taishan Scholarship Project of Shandong Province, China.

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