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Research Article

H^∞ Control for a Networked Control Model of Systems with Two Additive Time-Varying Delays

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This paper is concerned with H^∞ control for a networked control model of systems with two additive time-varying delays. A new Lyapunov functional is constructed to make full use of the information of the delays, and for the derivative of the Lyapunov functional a novel technique is employed to compute a tighter upper bound, which is dependent on the two time-varying delays instead of the upper bounds of them. Then the convex polyhedron method is proposed to check the upper bound of the derivative of the Lyapunov functional. The resulting stability criteria have fewer matrix variables but less conservatism than some existing ones. The stability criteria are applied to designing a state feedback controller, which guarantees that the closed-loop system is asymptotically stable with a prescribed H^∞ disturbance attenuation level. Finally examples are given to show the advantages of the stability criteria and the effectiveness of the proposed control method.

1. Introduction

For years systems with time delays have received considerable attention since they are often encountered in various practical systems, such as engineering systems, biology, economics, neural networks, networked control systems, and other areas [1–6]. Since time-delay is frequently the main cause of oscillation, divergence, or instability, considerable effort has been devoted to stability for systems with time delays. According to whether stability criteria include the information of the delay, they are divided into two classes: delay-independent stability criteria and delay-dependent ones. It is well known that delay-independent stability criteria tend to be more conservative especially for small size delays. More attention has been paid to delay-dependent stability. For delay-dependent stability results, we refer readers to [7–14]. Among these papers, [11–13] were of systems with interval time-varying delay. Recently these delay-dependent stability results were extended to neutral systems with interval time-varying delay [14]. It should be pointed out that all the stability results mentioned are based on systems with one single delay in the state.

On the other hand, networked control systems have been receiving great attention these years due to their advantages in low cost, reduced weight and power requirements, simple installation and maintenance, and high reliability. It is well known that the transmission delay and the data packet dropout are two fundamental issues in networked control systems. The transmission delay generally includes the sensor-to-control delay and the control-to-actuator delay. In most of existing papers the sensor-to-control delay and the control-to-actuator delay were combined into one state delay, while the data packet dropouts were modeled as delays and absorbed by the state delay, thus formulating networked control systems as systems with one state delay [15]. Among recently reported results based on this modeling idea, to mention a few, event-triggered communication and H^∞ control codesign problems were addressed for networked control systems in [16], while exponential state estimation problems were considered for Markovian jumping neural networks in [17]. Note that the sensor-to-control delay and the control-to-actuator delay are different in nature because of the network transmission conditions. The transmission delay and the data packet dropout also have different properties. It is not rational to lump them into one state delay. In this paper, to study networked control systems we adopt the model of systems with multiadditive time-varying delay components. For simplicity, the system with two additive time-varying delay components will be employed to address H^∞ control problem for networked control systems. Now we write the system as follows:

$$\dot{x}(t) = Ax(t) + A_1 x(t - d_1(t) - d_2(t)) + Ew(t) + Bu(t), \quad (1)$$

$$y(t) = Cx(t) + C_1 x(t - d_1(t) - d_2(t)) + Fw(t) + Du(t), \quad (2)$$

$$x(t) = \phi(t), \quad t \in [-h, 0], \quad (3)$$

where $x(t) \in \mathbb{R}^n$ is the state; $y(t)$ is the measurement; $u(t)$ is the control; $w(t) \in L_2[0, \infty]$ is the disturbance; A, A_1, E, B, C, C_1, F, and D are known real constant matrices; $d_1(t)$ and $d_2(t)$ are two time-varying delays satisfying

$$0 \le d_1(t) \le h_1, \qquad 0 \le d_2(t) \le h_2, \quad (4)$$

$$\dot{d}_1(t) \le \mu_1, \qquad \dot{d}_2(t) \le \mu_2; \quad (5)$$

and $\phi(t)$ is a real-valued initial function on $[-h, 0]$ with

$$h = h_1 + h_2. \quad (6)$$

Stability analysis for this kind of system was conducted in [18], and a delay-dependent stability criterion was obtained. An improved stability criterion was derived in [19] by constructing a Lyapunov functional to employ the information of the marginally delayed state $x(t - h)$. However, another marginally delayed state $x(t - h_1)$ was not considered, which caused $-\int_{t-h_1}^{t-d_1(t)} \dot{x}(\alpha)^T Z_1 \dot{x}(\alpha) d\alpha$ to be discarded when bounding the derivative of the Lyapunov functional. On the other hand, in the process of the bounding, many free weighting matrices were introduced, making the stability result complicated.

In this paper we first revisit delay-dependent stability for system (1) and (2). We will construct a new Lyapunov functional to employ the information of the marginally delayed state $x(t - h_1)$ as well as $x(t - h)$. Motivated by [13], when bounding the derivative of the Lyapunov functional, we use a novel technique to avoid introducing too many matrix variables and compute a tighter upper bound. Considering that the upper bound depends on the two time-varying delays, we propose the so-called convex polyhedron method to check the negative definiteness for it. The resulting delay-dependent stability criteria turn out to be less conservative with fewer matrix variables. Then we take the advantages of the stability results to investigate the H^∞ state feedback control problem, which is to design a state feedback controller $u(t) = Kx(t)$ for the system such that the closed-loop system is asymptotically stable with an H^∞ disturbance attenuation level $\gamma > 0$ satisfying $\|y\|_2 < \gamma \|w\|_2$ for nonzero $w(t) \in L_2[0, \infty]$ under zero initial condition. A delay-dependent condition will be presented for the state feedback controller such that the closed-loop system is asymptotically stable with a prescribed H^∞ disturbance attenuation level. Formulated in LMIs the condition is readily verified, and when it is feasible the controller can be constructed.

Notation. Throughout this paper the superscript "T" stands for matrix transposition. I refers to an identity matrix with appropriate dimensions. For real symmetric matrices X and Y, the notation $X > Y$ means that the matrix $X - Y$ is positive definite. The $X \ge Y$ follows similarly. The symmetric term in a matrix is denoted by $*$. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

First we go about the stability analysis. To the end, a lemma is given, which will play an important role in deriving our criteria.

Lemma 1 (see [20]). *For any symmetric positive definite matrix $M > 0$, scalar $\gamma > 0$, and vector function $\omega : [0, \gamma] \to R^n$ such that the integrations concerned are well defined, the following inequality holds:*

$$\left(\int_0^\gamma \omega(s) ds \right)^T M \left(\int_0^\gamma \omega(s) ds \right) \le \gamma \left(\int_0^\gamma \omega(s)^T M \omega(s) ds \right). \quad (7)$$

2. Stability Analysis

Consider system (1) with $w(t) = u(t) = 0$, namely,

$$\dot{x}(t) = Ax(t) + A_1 x(t - d_1(t) - d_2(t)). \quad (8)$$

Set

$$d(t) = d_1(t) + d_2(t), \quad (9)$$

$$\mu = \mu_1 + \mu_2. \quad (10)$$

Taking $d_1(t) + d_2(t)$ as one delay $d(t)$ we have the following system:

$$\dot{x}(t) = Ax(t) + A_1 x(t - d(t)), \quad (11)$$

with $0 \le d(t) \le h$, $\dot{d}(t) \le \mu$.

For this system there are many delay-dependent stability criteria available, but when used to check the stability for (8), they are more conservative [18]. In the following we present a new stability result for system (8) by considering the two delays separately.

Theorem 2. *The system* (8) *subject to* (4) *and* (5) *is asymptotically stable for given h_1, h_2, μ_1, and μ_2 if there exist matrices $P > 0$, $Q_i > 0$, $i = 1, 2, 3, 4$, and $Z_j > 0$, $j = 1, 2$, such that the following LMIs hold:*

$$\begin{gathered} \Phi - e_{13} h_1^{-1} Z_2 e_{13}^T - e_{23} h_2^{-1} Z_2 e_{23}^T < 0, \\ \Phi - e_{13} h_1^{-1} Z_2 e_{13}^T - e_{24} h_2 h^{-2} Z_2 e_{24}^T < 0, \\ \Phi - e_{24} h_1 h^{-2} Z_2 e_{24}^T - e_{23} h_2^{-1} Z_2 e_{23}^T < 0, \\ \Phi - e_{24} h^{-1} Z_2 e_{24}^T < 0, \end{gathered} \quad (12)$$

where $e_{13} = [I \;\; 0 \;\; -I \;\; 0 \;\; 0]^T$, e_{23}, e_{24}, *and* e_{35} *follow similarly, h is defined in* (6), *and*

$$\Phi = \begin{bmatrix} \varphi_1 & PA_1 & h_1^{-1}(Z_1+Z_2) & 0 & 0 \\ * & \varphi_2 & h_2^{-1}Z_2 & h^{-1}Z_2 & 0 \\ * & * & \varphi_3 & 0 & h_1^{-1}Z_1 \\ * & * & * & -Q_2 - h^{-1}Z_2 & 0 \\ * & * & * & * & -Q_4 - h_1^{-1}Z_1 \end{bmatrix} + \begin{bmatrix} A^T \\ A_1^T \\ 0 \\ 0 \\ 0 \end{bmatrix} [h_1 Z_1 + hZ_2] \begin{bmatrix} A^T \\ A_1^T \\ 0 \\ 0 \\ 0 \end{bmatrix}^T, \tag{13}$$

with μ *given in* (10) *and*

$$\begin{aligned} \varphi_1 &= PA + A^T P + \sum_{i=1}^{4} Q_i - h_1^{-1}(Z_1 + Z_2), \\ \varphi_2 &= -(1-\mu)Q_3 - (h_2^{-1} + h^{-1})Z_2, \\ \varphi_3 &= -(1-\mu_1)Q_1 - (h_2^{-1} + h_1^{-1})Z_2 - 2h_1^{-1}Z_1. \end{aligned} \tag{14}$$

Proof. Define a Lyapunov functional as follows:

$$\begin{aligned} V(t) &= x(t)^T P x(t) + \int_{t-d_1(t)}^{t} x(\alpha)^T Q_1 x(\alpha)\, d\alpha \\ &+ \int_{t-h}^{t} x(\alpha)^T Q_2 x(\alpha)\, d\alpha \\ &+ \int_{t-d(t)}^{t} x(\alpha)^T Q_3 x(\alpha)\, d\alpha \\ &+ \int_{t-h_1}^{t} x(\alpha)^T Q_4 x(\alpha)\, d\alpha \\ &+ \int_{t-h_1}^{t} \int_{s}^{t} \dot{x}(\alpha)^T Z_1 \dot{x}(\alpha)\, d\alpha\, ds \\ &+ \int_{t-h}^{t} \int_{s}^{t} \dot{x}(\alpha)^T Z_2 \dot{x}(\alpha)\, d\alpha\, ds, \end{aligned} \tag{15}$$

where $d(t)$ is defined in (9). Then calculating the time derivative of the Lyapunov functional along the trajectory of (8) yields

$$\begin{aligned} \dot{V}(x_t) &\le 2x(t)^T P (Ax(t) + A_1 x(t-d(t))) \\ &+ \sum_{i=1}^{4} x(t)^T Q_i x(t) - x(t-h_1)^T \\ &\times Q_4 x(t-h_1) - x(t-h)^T Q_2 x(t-h) \\ &- (1-\mu)\, x(t-d(t))^T Q_3 x(t-d(t)) \\ &- (1-\mu_1)\, x(t-d_1(t))^T Q_1 x(t-d_1(t)) \\ &+ (Ax(t) + A_1 x(t-d(t)))^T (h_1 Z_1 + hZ_2) \\ &\times [Ax(t) + A_1 x(t-d(t))] \\ &- \int_{t-h_1}^{t} \dot{x}(\alpha)^T Z_1 \dot{x}(\alpha)\, d\alpha - \int_{t-h}^{t} \dot{x}(\alpha)^T Z_2 \dot{x}(\alpha)\, d\alpha. \end{aligned} \tag{16}$$

Note that

$$\begin{aligned} &- \int_{t-h_1}^{t} \dot{x}(\alpha)^T Z_1 \dot{x}(\alpha)\, d\alpha - \int_{t-h}^{t} \dot{x}(\alpha)^T Z_2 \dot{x}(\alpha)\, d\alpha \\ &= - \int_{t-d_1(t)}^{t} \dot{x}(s)^T Z_1 \dot{x}(s)\, ds - \int_{t-h_1}^{t-d_1(t)} \dot{x}(s)^T Z_1 \dot{x}(s)\, ds \\ &\quad - \int_{t-d_1(t)}^{t} \dot{x}(s)^T Z_2 \dot{x}(s)\, ds - \int_{t-d(t)}^{t-d_1(t)} \dot{x}(s)^T Z_2 \dot{x}(s)\, ds \\ &\quad - \int_{t-h}^{t-d(t)} \dot{x}(s)^T Z_2 \dot{x}(s)\, ds. \end{aligned} \tag{17}$$

Write $\alpha = d_1(t)/h_1$ and $\beta = d_2(t)/h_2$. Then

$$\begin{aligned} &- \int_{t-d_1(t)}^{t} \dot{x}(s)^T Z_2 \dot{x}(s)\, d\alpha \\ &= -h_1^{-1} \int_{t-d_1(t)}^{t} h_1 \dot{x}(s)^T Z_2 \dot{x}(s)\, ds \\ &= -h_1^{-1} \int_{t-d_1(t)}^{t} d_1(t)\, \dot{x}(s)^T Z_2 \dot{x}(s)\, ds \\ &\quad - h_1^{-1} \int_{t-d_1(t)}^{t} [h_1 - d_1(t)]\, \dot{x}(s)^T Z_2 \dot{x}(s)\, ds. \end{aligned} \tag{18}$$

It follows from (18) that

$$\begin{aligned} &- \int_{t-d_1(t)}^{t} \dot{x}(s)^T Z_2 \dot{x}(s)\, ds \\ &\le -h_1^{-1} \int_{t-d_1(t)}^{t} d_1(t)\, \dot{x}(s)^T Z_2 \dot{x}(s)\, ds. \end{aligned} \tag{19}$$

Using (19) we have

$$\begin{aligned} -h_1^{-1}\int_{t-d_1(t)}^{t}&[h_1-d_1(t)]\,\dot{x}(s)^T Z_2\dot{x}(s)\,ds \\ &= -(1-\alpha)\int_{t-d_1(t)}^{t}\dot{x}(s)^T Z_2\dot{x}(s)\,ds \\ &\le -(1-\alpha)\,h_1^{-1}\int_{t-d_1(t)}^{t} d_1(t)\,\dot{x}(s)^T Z_2\dot{x}(s)\,ds. \end{aligned} \tag{20}$$

By Lemma 1, (18) and (20) imply

$$\begin{aligned} -\int_{t-d_1(t)}^{t}&\dot{x}(s)^T Z_2\dot{x}(s)\,ds \\ &\le -[x(t)-x(t-d_1(t))]^T h_1^{-1}Z_2 \\ &\quad\times[x(t)-x(t-d_1(t))] \\ &\quad-(1-\alpha)[x(t)-x(t-d_1(t))]^T h_1^{-1}Z_2 \\ &\quad\times[x(t)-x(t-d_1(t))]. \end{aligned} \tag{21}$$

Similarly it can be derived that

$$\begin{aligned} -\int_{t-d(t)}^{t-d_1(t)}&\dot{x}(s)^T Z_2\dot{x}(s)\,ds \\ &\le -[x(t-d(t))-x(t-d_1(t))]^T h_2^{-1}Z_2 \\ &\quad\times[x(t-d(t))-x(t-d_1(t))] \\ &\quad-(1-\beta)[x(t-d(t))-x(t-d_1(t))]^T h_2^{-1}Z_2 \\ &\quad\times[x(t-d(t))-x(t-d_1(t))], \\ -\int_{t-h}^{t-d(t)}&\dot{x}(s)^T Z_2\dot{x}(s)\,ds \\ &\le -[x(t-d(t))-x(t-h)]^T h^{-1}Z_2 \\ &\quad\times[x(t-d(t))-x(t-h)] \\ &\quad-\alpha[x(t-d(t))-x(t-h)]^T h_1h^{-2}Z_2 \\ &\quad\times[x(t-d(t))-x(t-h)] \\ &\quad-\beta[x(t-d(t))-x(t-h)]^T h_2h^{-2}Z_2 \\ &\quad\times[x(t-d(t))-x(t-h)]. \end{aligned} \tag{22}$$

Similar to [12] we have

$$\begin{aligned} -\int_{t-d_1(t)}^{t}&\dot{x}(s)^T Z_1\dot{x}(s)\,ds \\ &\le -[x(t)-x(t-d_1(t))]^T h_1^{-1}Z_1[x(t)-x(t-d_1(t))], \\ -\int_{t-h_1}^{t-d_1(t)}&\dot{x}(s)^T Z_1\dot{x}(s)\,ds \\ &\le -[x(t-d_1(t))-x(t-h_1)]^T h_1^{-1}Z_1 \\ &\quad\times[x(t-d_1(t))-x(t-h_1)]. \end{aligned} \tag{23}$$

Define

$$\zeta(t)=\begin{bmatrix}x(t)^T & x(t-d(t))^T & x(t-d_1(t))^T & x(t-h)^T & x(t-h_1)^T\end{bmatrix}^T. \tag{24}$$

Combining (16), (17), and (21)–(23) and using (13) yield

$$\begin{aligned} \dot{V}(t) &\le \zeta(t)^T\Phi\zeta(t)-(1-\alpha) \\ &\quad\times[x(t)-x(t-d_1(t))]^T h_1^{-1}Z_2[x(t)-x(t-d_1(t))] \\ &\quad-\alpha[x(t-d(t))-x(t-h)]^T h_1h^{-2}Z_2 \\ &\quad\times[x(t-d(t))-x(t-h)] \\ &\quad-\beta[x(t-d(t))-x(t-h)]^T h_2h^{-1}Z_2 \\ &\quad\times[x(t-d(t))-x(t-h)] \\ &\quad-(1-\beta)[x(t-d(t))-x(t-d_1(t))]^T h_2^{-1}Z_2 \\ &\quad\times[x(t-d(t))-x(t-d_1(t))] \\ &= \zeta(t)^T M(\alpha,\beta)\zeta(t), \end{aligned} \tag{25}$$

where

$$\begin{aligned} M(\alpha,\beta) &= \Phi-\alpha e_{24}h_1h^{-2}Z_2e_{24}^T \\ &\quad-(1-\alpha)e_{13}h_1^{-1}Z_2e_{13}^T-\beta e_{24}h_2h^{-2}Z_2e_{24}^T \\ &\quad-(1-\beta)e_{23}h_2^{-1}Z_2e_{23}^T \\ &= \alpha\left[\Phi-e_{24}h_1h^{-2}Z_2e_{24}^T\right] \\ &\quad+(1-\alpha)\left[\Phi-e_{13}h_1^{-1}Z_2e_{13}^T\right]-\beta e_{24}h_2h^{-2}Z_2e_{24}^T \\ &\quad-(1-\beta)e_{23}h_2^{-1}Z_2e_{23}^T \\ &= \alpha\left[\Phi-e_{24}h_1h^{-2}Z_2e_{24}^T-\beta e_{24}h_2h^{-2}Z_2e_{24}^T\right. \\ &\qquad\left.-(1-\beta)e_{23}h_2^{-1}Z_2e_{23}^T\right] \\ &\quad+(1-\alpha)\left[\Phi-e_{13}h_1^{-1}Z_2e_{13}^T-\beta e_{24}h_2h^{-2}Z_2e_{24}^T\right. \\ &\qquad\left.-(1-\beta)e_{23}h_2^{-1}Z_2e_{23}^T\right] \\ &= \alpha\left[\beta\left(\Phi-e_{24}h^{-1}Z_2e_{24}^T\right)\right. \\ &\qquad\left.+(1-\beta)\left(\Phi-e_{24}h_1h^{-2}Z_2e_{24}^T-e_{23}h_2^{-1}Z_2e_{23}^T\right)\right] \\ &\quad+(1-\alpha)\left[\beta\left(\Phi-e_{13}h_1^{-1}Z_2e_{13}^T-e_{24}h_2h^{-2}Z_2e_{24}^T\right)\right. \\ &\qquad+(1-\beta)\times\left(\Phi-e_{13}h_1^{-1}Z_2e_{13}^T\right. \\ &\qquad\left.\left.-e_{23}h_2^{-1}Z_2e_{23}^T\right)\right]. \end{aligned} \tag{26}$$

By (12) it is derived that $M(\alpha,\beta) < 0$. Therefore system (8) is asymptotically stable. This ends the proof. □

Remark 3. Theorem 2 provides a new delay-dependent stability criterion for system (8) with two additive time-varying delay components. In a form of LMIs the criterion can be checked easily.

Remark 4. Note that the corresponding matrix $M(\alpha,\beta)$ to the upper bound of $\dot{V}(x_t)$ is dependent on the two time-varying delays while those in [18, 19] are dependent on the upper bounds of the two time-varying delays. To check the negative definiteness of the function matrix $M(\alpha,\beta)$, one has to adopt a new method, which is motivated by [13]. The basic idea is that a function matrix is negative definite over a convex polyhedron only if the matrix is negative definite at the vertices. Note that

$$\begin{aligned} M(1,1) &= \Phi - e_{24}h^{-1}Z_2e_{24}^T, \\ M(1,0) &= \Phi + e_{24}h_1h^{-2}Z_2e_{24}^T - e_{23}h_2^{-1}Z_2e_{23}^T, \\ M(0,1) &= \Phi - e_{13}h_1^{-1}Z_2e_{13}^T - e_{24}h_2h^{-2}Z_2e_{24}^T, \\ M(0,0) &= \Phi - e_{13}h_1^{-1}Z_2e_{13}^T - e_{23}h_2^{-1}Z_2e_{23}^T. \end{aligned} \quad (27)$$

From this it can be seen the negative definiteness of $M(\alpha,\beta)$ over the rectangle: $0 \le \alpha \le 1$, $0 \le \beta \le 1$, is determined by that of $M(\alpha,\beta)$ at the vertices. One calls this approach to the negative definiteness of a function matrix a convex polyhedron method. Apparently the convex polyhedron method can be extended to more than two time-varying delays.

Remark 5. Gao et al. [19] took advantages of $x(t-h)$ to derive a stability criterion, which improved over that in [18], but another marginally delayed state $x(t-h_1)$ was not employed. In this paper one makes use of it to construct the Lyapunov functional $V(t)$ in (15), thus making $-\int_{t-h_1}^{t-d_1(t)} \dot{x}(\alpha)^T Z_1 \dot{x}(\alpha)d\alpha$ retained in the estimate of $\dot{V}(t)$. On the other hand, when estimating integrals in $\dot{V}(x_t)$ one does not introduce any free weighting matrix as [18, 19], but one uses new techniques reported recently in [12, 13]. Take $-\int_{t-h_1}^{t} \dot{x}(\alpha)^T Z_1 \dot{x}(\alpha)d\alpha$ as an example. One first divides it into two parts as (17) and then calculates them as (23). As for $-\int_{t-d_1(t)}^{t} \dot{x}(\alpha)^T Z_2 \dot{x}(\alpha)d\alpha$ and so forth, one deals with it in such a new way as (18)–(22). Thanks to the new techniques to calculate integrals in $\dot{V}(x_t)$ and the convex polyhedron method to check the negative definite for the upper bound of $\dot{V}(x_t)$, the resulting Theorem 2 is expected to be less conservative with fewer matrix variables, as shown in the following example.

When μ_1 and μ_2 are unknown, eliminating Q_1 and Q_3 one can obtain a delay-rate-independent stability criterion from Theorem 2 as follows.

Corollary 6. *The system* (8) *subject to* (4) *is asymptotically stable for given h_1 and h_2 if there exist matrices $P > 0$, $Q_2 > 0$, $Q_4 > 0$, and $Z_j > 0$, $j = 1, 2$, such that the following LMIs hold:*

$$\begin{aligned} &\Phi_1 - e_{13}h_1^{-1}Z_2e_{13}^T - e_{23}h_2^{-1}Z_2e_{23}^T < 0, \\ &\Phi_1 - e_{13}h_1^{-1}Z_2e_{13}^T - e_{24}h_2h^{-2}Z_2e_{24}^T < 0, \\ &\Phi_1 - e_{24}h_1h^{-2}Z_2e_{24}^T - e_{23}h_2^{-1}Z_2e_{23}^T < 0, \\ &\Phi_1 - e_{24}h^{-1}Z_2e_{24}^T < 0, \end{aligned} \quad (28)$$

where

$$\Phi_1 = \begin{bmatrix} \tilde{\varphi}_1 & PA_1 & h_1^{-1}(Z_1+Z_2) & 0 & 0 \\ * & \tilde{\varphi}_2 & h_2^{-1}Z_2 & h^{-1}Z_2 & 0 \\ * & * & \tilde{\varphi}_3 & 0 & h_1^{-1}Z_1 \\ * & * & * & -Q_2 - h^{-1}Z_2 & 0 \\ * & * & * & * & -Q_4 - h_1^{-1}Z_1 \end{bmatrix} + \begin{bmatrix} A^T \\ A_1^T \\ 0 \\ 0 \\ 0 \end{bmatrix} [h_1Z_1 + hZ_2] \begin{bmatrix} A^T \\ A_1^T \\ 0 \\ 0 \\ 0 \end{bmatrix}^T, \quad (29)$$

with $\tilde{\varphi}_1 = PA + A^TP + Q_2 + Q_4 - h_1^{-1}(Z_1 + Z_2)$, $\tilde{\varphi}_2 = -(h_2^{-1} + h^{-1})Z_2$, *and* $\tilde{\varphi}_3 = -(h_2^{-1} + h_1^{-1})Z_2 - 2h_1^{-1}Z_1$.

When $d_1(t) \equiv h_1$, that is, $d_1(t)$ is a constant delay, Theorem 2 reduces to the following corollary, which was reported recently in [13].

Corollary 7. *The system* (8) *with $d_1(t) \equiv h_1$ and $d_2(t)$ satisfying $0 \le d_2(t) \le h_2$ and $\dot{d}(t) \le \mu_2$ is asymptotically stable for given $h_2 > 0$, $h_1 > 0$, and μ_2 if there exist $P > 0$, $Q_i > 0$, $i = 1, 2, 3$, and $Z_j > 0$, $j = 1, 2$, such that the following LMIs hold:*

$$\begin{aligned} &\Phi_2 - [0 \;\; I \;\; -I \;\; 0]^T Z_2 [0 \;\; I \;\; -I \;\; 0] < 0, \\ &\Phi_2 - [0 \;\; I \;\; 0 \;\; -I]^T Z_2 [0 \;\; I \;\; 0 \;\; -I] < 0, \end{aligned} \quad (30)$$

where

$$\Phi_2 = \begin{bmatrix} \hat{\varphi}_1 & PA_1 & 0 & h_1^{-1}Z_1 \\ * & \hat{\varphi}_2 & h_2^{-1}Z_2 & h_2^{-1}Z_2 \\ * & * & -Q_2 - h_2^{-1}Z_2 & 0 \\ * & * & * & -Q_4 - h_2^{-1}Z_2 - h_1^{-1}Z_1 \end{bmatrix} + \begin{bmatrix} A^T \\ A_1^T \\ 0 \\ 0 \end{bmatrix} (h_1Z_1 + h_2Z_2) \begin{bmatrix} A^T \\ A_1^T \\ 0 \\ 0 \end{bmatrix}^T, \quad (31)$$

with $\hat{\varphi}_1 = PA + A^TP + \sum_{i=1}^{3} Q_i - h_1^{-1}Z_1$ *and* $\hat{\varphi}_2 = -(1-\mu_2)Q_3 - 2h_2^{-1}Z_2$.

TABLE 1: Admissible upper bound h_2 for various h_1.

Method	h_1	1	1.2	1.5
[18]	h_2	0.415	0.376	0.248
[19]	h_2	0.512	0.406	0.283
Theorem 2	h_2	0.5955	0.4632	0.3129

Proof. Define the Lyapunov functional. Consider the following:

$$\begin{aligned} V(x_t) &= x(t)^T P x(t) + \int_{t-d(t)}^{t} x(\alpha)^T Q_3 x(\alpha)\, d\alpha \\ &+ \int_{t-h_1}^{t} x(\alpha)^T Q_1 x(\alpha)\, d\alpha + \int_{t-h}^{t} x(\alpha)^T Q_2 x(\alpha)\, d\alpha \\ &+ \int_{t-h_1}^{t}\int_{s}^{t} \dot{x}(\alpha)^T Z_1 \dot{x}(\alpha)\, d\alpha\, ds \\ &+ \int_{t-h}^{t-h_1}\int_{s}^{t} \dot{x}(\alpha)^T Z_2 \dot{x}(\alpha)\, d\alpha\, ds. \end{aligned} \tag{32}$$

Along a similar line as in the derivation of Theorem 2 the asymptotic stability can be established, and details are thus omitted. □

Remark 8. Note that when $d_1(t)$ is a constant delay h_1, system (8) can be regarded as a system in the form of (11) with interval time-varying delay: $h_1 \le d(t) \le h$, $0 \le \dot{d}(t) \le \mu_2$. The system can serve as a model for networked control systems with both network-induced delay and data dropout phenomenon [15, 16]. In the form of LMIs Corollary 7 can provide a delay-dependent stability criterion for the model. Derived by the convex polyhedron method Corollary 7 is less conservative than those recently reported in [11]; see [13].

In the following, we take the example in [19] to show that our stability criteria, though having much fewer matrix variables, are less conservative.

Example 9. Consider the system (8) with

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \qquad \dot{d}_1(t) \le 0.1, \qquad \dot{d}_2(t) \le 0.8. \tag{33}$$

For given upper bound h_1 of $d_1(t)$, we intend to find the admissible upper bound h_2 of $d_2(t)$, which guarantees the asymptotic stability of (8).

When h_2 is given, the admissible h_1 can be seen from Table 2.

As seen in Tables 1 and 2, Theorem 2 is less conservative than those in [15, 16]. It is worth noting that with fewer matrix variables involved, Theorem 2 needs less computational requirements.

When $d_1(t)$ is a constant delay h_1, the system can be looked upon as those with interval time-varying delay. As indicated in Remark 8, the stability result in Corollary 7 as well as that in [11] can be turned to for computing the admissible upper bound h of $d(t)$, which are shown in Table 3.

TABLE 2: Admissible upper bound h_1 for various h_2.

Method	h_2	0.1	0.2	0.3
[18]	h_1	2.263	1.696	1.324
[19]	h_1	2.300	1.779	1.453
Theorem 2	h_1	2.3400	1.8337	1.5318

TABLE 3: Admissible upper bound h for various h_1.

Method	h_1	1	2	3	4
[11]	h	1.7423	2.4328	3.2234	4.0644
Corollary 7	h	1.8737	2.5049	3.2592	4.0745

Even as a delay-dependent criterion for systems with interval time-varying delay, Corollary 7 has advantages over [11] in the sense that the computed admissible upper bound of the time-varying delay is larger.

3. State Feedback Control

Without a free weighting matrix introduced, Theorem 2 only involves the matrices in the Lyapunov functional employed. It can be expected as a useful tool for the H^∞ state feedback control problem formulated above. We first present an H^∞ performance analysis result in the following.

Theorem 10. *System (1) and (2) with $u(t) = 0$ and delays subject to (4) and (5) is asymptotically stable with an H^∞ disturbance attenuation level γ for given h_1, h_2, μ_1, and μ_2, if there exist matrices $P > 0$, $Q_i > 0$, $i = 1,2,3,4$, and $Z_j > 0$, $j = 1,2$ such that the following LMIs hold:*

$$\begin{aligned} &\overline{\Phi} - \overline{e}_{13} h_1^{-1} Z_2 \overline{e}_{13}^T - \overline{e}_{23} h_2^{-1} Z_2 \overline{e}_{23}^T < 0, \\ &\overline{\Phi} - \overline{e}_{13} h_1^{-1} Z_2 \overline{e}_{13}^T - \overline{e}_{24} h_2 h^{-2} Z_2 \overline{e}_{24}^T < 0, \\ &\overline{\Phi} - \overline{e}_{24} h_1 h^{-2} Z_2 \overline{e}_{24}^T - \overline{e}_{23} h_2^{-1} Z_2 \overline{e}_{23}^T < 0, \\ &\overline{\Phi} - \overline{e}_{24} h^{-1} Z_2 \overline{e}_{24}^T < 0, \end{aligned} \tag{34}$$

where $\overline{e}_{13} = \begin{bmatrix} e_{13}^T & 0 \end{bmatrix}^T$, $\overline{e}_{23}$, $\overline{e}_{24}$, and $\overline{e}_{35}$ follow similarly and

$$\begin{aligned} \overline{\Phi} &= \begin{bmatrix} \varphi_1 & PA_1 & h_1^{-1}(Z_1 + Z_2) & 0 & 0 & PE \\ * & \varphi_2 & h_2^{-1} Z_2 & h^{-1} Z_2 & 0 & 0 \\ * & * & \varphi_3 & 0 & h_1^{-1} Z_1 & 0 \\ * & * & * & -Q_2 - h^{-1} Z_2 & 0 & 0 \\ * & * & * & * & -Q_4 - h_1^{-1} Z_1 & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix} \\ &+ \begin{bmatrix} A^T \\ A_1^T \\ 0 \\ 0 \\ 0 \\ E^T \end{bmatrix} [h_1 Z_1 + h Z_2] \begin{bmatrix} A^T \\ A_1^T \\ 0 \\ 0 \\ 0 \\ E^T \end{bmatrix}^T + \begin{bmatrix} C^T \\ C_1^T \\ 0 \\ 0 \\ 0 \\ F^T \end{bmatrix} \begin{bmatrix} C^T \\ C_1^T \\ 0 \\ 0 \\ 0 \\ F^T \end{bmatrix}^T, \end{aligned} \tag{35}$$

with φ_1, φ_2, φ_3, and h given in Theorem 2.

Proof. Comparing $\overline{\Phi}$ with Φ in (13), we can conclude that (34) implies (12). Therefore system (1) and (2) with $u(t) = 0$ is asymptotically stable. Now using the same Lyapunov functional as $V(t)$ in (15) and calculating $\dot{V}(t)$ similar to the derivation of Theorem 2 along the solution of system (1) and (2) with $u(t) = 0$, we have

$$y(t)^T y(t) - \gamma^2 w(t)^T w(t) + \dot{V}(t) \le \overline{\zeta}(t)^T \overline{M}(\alpha, \beta) \overline{\zeta}(t), \quad (36)$$

where

$$\overline{M}(\alpha, \beta) = \overline{\Phi} - \alpha \overline{e}_{24} h_1 h^{-2} Z_2 \overline{e}_{24}^T - (1-\alpha) \overline{e}_{13} h_1^{-1} Z_2 \overline{e}_{13}^T - \beta \overline{e}_{24} h_2 h^{-2} Z_2 \overline{e}_{24}^T - (1-\beta) \overline{e}_{23} h_2^{-1} Z_2 \overline{e}_{23}^T, \quad (37)$$

with α and β defined in the proof of Theorem 2 and $\overline{\zeta}(t) = \left[\zeta(t)^T \ \ w(t)^T\right]^T$ with $\zeta(t)$ in (24). On the one hand, using the convex polyhedron method we can prove that $\overline{M}(\alpha, \beta) < 0$ by (34). On the other hand, under the zero condition we have $V(0) = 0$ and $V(\infty) \ge 0$. Integrating both sides of (36) leads to $\|y\|_2 < \gamma \|w\|_2$ for all nonzero $w(t) \in L_2[0, \infty]$. This ends the proof. □

Now we are in a position to resolve the H^∞ state feedback control problem aforementioned.

Theorem 11. *Consider system* (1) *and* (2) *with delays subject to* (4) *and* (5). *Given* h_1, h_2, μ_1, *and* μ_2, *there exists a state feedback controller* $u(t) = Kx(t)$ *ensuring that the closed-loop system is asymptotically stable with an* H^∞ *disturbance attenuation level* γ, *if there exist matrices* $\overline{P} > 0$, $\overline{Q}_i > 0$, $i = 1, 2, 3, 4$, *and* $\overline{Z}_j > 0$, $j = 1, 2$, $\overline{K}$ *such that the following LMIs hold:*

$$\begin{bmatrix} \Omega_i & \Gamma \\ \Gamma^T & \Lambda \end{bmatrix} < 0, \quad i = 1, 2, 3, 4, \quad (38)$$

where

$$\Omega_1 = \Psi - \overline{e}_{13} h_1^{-1} \overline{Z}_2 \overline{e}_{13}^T - \overline{e}_{23} h_2^{-1} \overline{Z}_2 \overline{e}_{23}^T,$$
$$\Omega_2 = \Psi - \overline{e}_{13} h_1^{-1} \overline{Z}_2 \overline{e}_{13}^T - \overline{e}_{24} h_2 h^{-2} \overline{Z}_2 \overline{e}_{24}^T,$$
$$\Omega_3 = \Psi - \overline{e}_{24} h_1 h^{-2} \overline{Z}_2 \overline{e}_{24}^T - \overline{e}_{23} h_2^{-1} \overline{Z}_2 \overline{e}_{23}^T,$$
$$\Omega_4 = \Psi - \overline{e}_{24} h^{-1} \overline{Z}_2 \overline{e}_{24}^T,$$
$$\Lambda = \operatorname{diag}\left\{h_1^{-1}\left(\overline{Z}_1 - 2\overline{P}\right), h^{-1}\left(\overline{Z}_2 - 2\overline{P}\right), -I\right\},$$
$$\Gamma = \begin{bmatrix} \overline{P}A^T + \overline{K}^T B^T & \overline{P}A^T + \overline{K}^T B^T & \overline{P}C^T + \overline{K}^T D^T \\ \overline{P}A_1^T & \overline{P}A_1^T & \overline{P}C_1^T \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ E^T & E^T & F^T \end{bmatrix}, \quad (39)$$

with

$$\Psi = \begin{bmatrix} \overline{\varphi}_1 & A_1\overline{P} & h_1^{-1}\left(\overline{Z}_1 + \overline{Z}_2\right) & 0 & 0 & E \\ * & \overline{\varphi}_2 & h_2^{-1}\overline{Z}_2 & h^{-1}\overline{Z}_2 & 0 & 0 \\ * & * & \overline{\varphi}_3 & 0 & h_1^{-1}\overline{Z}_1 & 0 \\ * & * & * & -\overline{Q}_2 - h^{-1}\overline{Z}_2 & 0 & 0 \\ * & * & * & * & -\overline{Q}_4 - h_1^{-1}\overline{Z}_1 & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix},$$

$$\overline{\varphi}_1 = A\overline{P} + B\overline{K} + \left(A\overline{P} + B\overline{K}\right)^T + \sum_{i=1}^{4} \overline{Q}_i - h_1^{-1}\left(\overline{Z}_1 + \overline{Z}_2\right),$$
$$\overline{\varphi}_2 = -(1-\mu)\overline{Q}_3 - \left(h_2^{-1} + h^{-1}\right)\overline{Z}_2,$$
$$\overline{\varphi}_3 = -(1-\mu_1)\overline{Q}_1 - \left(h_2^{-1} + h_1^{-1}\right)\overline{Z}_2 - 2h_1^{-1}\overline{Z}_1, \quad (40)$$

and $\overline{e}_{13}$, $\overline{e}_{23}$, $\overline{e}_{24}$, $\overline{e}_{35}$, *h, and* μ *are given in Theorem 10. Moreover, if the foregoing condition holds, a desired controller gain matrix is given by*

$$K = \overline{K}\,\overline{P}^{-1}. \quad (41)$$

Proof. Apply the controller $u(t) = Kx(t)$ to system (1) and (2) and then the closed-loop system is formulated as follows:

$$\dot{x}(t) = (A + BK)x(t) + A_1 x\left(t - d_1(t) - d_2(t)\right) + Ew(t),$$
$$y(t) = (C + DK)x(t) + C_1 x\left(t - d_1(t) - d_2(t)\right) + Fw(t). \quad (42)$$

By Theorem 10 this system is asymptotically stable with an H^∞ disturbance attenuation level γ, if there exist matrices $P > 0$, $Q_i > 0$, $i = 1, 2, 3, 4$, and $Z_j > 0$, $j = 1, 2$, such that

$$\overline{\Phi}_c - \overline{e}_{13} h_1^{-1} Z_2 \overline{e}_{13}^T - \overline{e}_{23} h_2^{-1} Z_2 \overline{e}_{23}^T < 0,$$
$$\overline{\Phi}_c - \overline{e}_{13} h_1^{-1} Z_2 \overline{e}_{13}^T - \overline{e}_{24} h_2 h^{-2} Z_2 \overline{e}_{24}^T < 0,$$
$$\overline{\Phi}_c - \overline{e}_{24} h_1 h^{-2} Z_2 \overline{e}_{24}^T - \overline{e}_{23} h_2^{-1} Z_2 \overline{e}_{23}^T < 0,$$
$$\overline{\Phi}_c - \overline{e}_{24} h^{-1} Z_2 \overline{e}_{24}^T < 0, \quad (43)$$

where $\bar{e}_{13}, \bar{e}_{23}, \bar{e}_{24}, \bar{e}_{35}$, and h are the same as those in Theorem 10 and

$$\overline{\Phi}_c = \begin{bmatrix} \varphi_{c1} & PA_1 & h_1^{-1}(Z_1+Z_2) & 0 & 0 & PE \\ * & \varphi_2 & h_2^{-1}Z_2 & h^{-1}Z_2 & 0 & 0 \\ * & * & \varphi_3 & 0 & h_1^{-1}Z_1 & 0 \\ * & * & * & -Q_2-h^{-1}Z_2 & 0 & 0 \\ * & * & * & * & -Q_4-h_1^{-1}Z_1 & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} (A+BK)^T \\ A_1^T \\ 0 \\ 0 \\ 0 \\ E^T \end{bmatrix} [h_1Z_1+hZ_2] \begin{bmatrix} (A+BK)^T \\ A_1^T \\ 0 \\ 0 \\ 0 \\ E^T \end{bmatrix}^T + \begin{bmatrix} (C+DK)^T \\ C_1^T \\ 0 \\ 0 \\ 0 \\ F^T \end{bmatrix} \begin{bmatrix} (C+DK)^T \\ C_1^T \\ 0 \\ 0 \\ 0 \\ F^T \end{bmatrix}^T, \tag{44}$$

with φ_2 and φ_3 defined in Theorem 10 and $\varphi_{c1} = P(A+BK) + (A+BK)^T P + \sum_{i=1}^{4} Q_i - h_1^{-1}(Z_1+Z_2)$.

Write $J = \mathrm{diag}\{P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, I\}$, $\overline{P} = P^{-1}$, $\overline{Z}_i = P^{-1}Z_iP^{-1}$, $i = 1, 2$, $\overline{Q}_j = P^{-1}Q_jP^{-1}$, and $j = 1, 2, 3, 4$. Performing a congruence transformation to (43) by J yields

$$\begin{gathered} \Psi + \Gamma\widehat{\Lambda}^{-1}\Gamma^T - \bar{e}_{13}h_1^{-1}\overline{Z}_2\bar{e}_{13}^T - \bar{e}_{23}h_2^{-1}\overline{Z}_2\bar{e}_{23}^T < 0, \\ \Psi + \Gamma\widehat{\Lambda}^{-1}\Gamma^T - \bar{e}_{13}h_1^{-1}\overline{Z}_2\bar{e}_{13}^T - \bar{e}_{24}h_2h^{-2}\overline{Z}_2\bar{e}_{24}^T < 0, \\ \Psi + \Gamma\widehat{\Lambda}^{-1}\Gamma^T - \bar{e}_{24}h_1h^{-2}\overline{Z}_2\bar{e}_{24}^T - \bar{e}_{23}h_2^{-1}\overline{Z}_2\bar{e}_{23}^T < 0, \\ \Psi + \Gamma\widehat{\Lambda}^{-1}\Gamma^T - \bar{e}_{24}h^{-1}\overline{Z}_2\bar{e}_{24}^T < 0, \end{gathered} \tag{45}$$

where

$$\widehat{\Lambda} = \mathrm{diag}\left\{h_1^{-1}Z_1^{-1}, h^{-1}Z_2^{-1}, I\right\}. \tag{46}$$

By Schur complements we have

$$\begin{bmatrix} \Omega_i & \Gamma \\ \Gamma^T & -\widehat{\Lambda} \end{bmatrix} < 0, \quad i = 1, 2, 3, 4. \tag{47}$$

Note that (47) is not linear in $\overline{P}$, $\overline{K}$, $\overline{Q}_i$, and $\overline{Z}_j$ due to $Z_i^{-1} = \overline{P}\,\overline{Z}_i^{-1}\overline{P}$. However, noting that $(\overline{P} - \overline{Z}_i)\overline{Z}_i^{-1}(\overline{P} - \overline{Z}_i) \geq 0$, we have $\overline{P}\,\overline{Z}_i^{-1}\overline{P} \geq -\overline{Z}_i + 2\overline{P}$. Therefore, $Z_i^{-1} \geq -\overline{Z}_i + 2\overline{P}$. It follows immediately that $-\widehat{\Lambda} \leq \Lambda$, which means that (38) implies (47). This completes the proof. □

Due to the fact that $Z_i^{-1} \geq -\overline{Z}_i + 2\overline{P}$, condition (38) is more conservative than (47). However, based on (38) one can obtain an LMI approach to the H^∞ state feedback control problem for systems with two additive time-varying delays. The existence of the state feedback controller is guaranteed by the feasibility of LMIs (38). When LMIs (38) are solvable, the controller can be constructed according to (41). Based on (47), one can obtain a less conservative controller at the cost of more complexity by employing CCL method [21].

To illustrate the effectiveness of this method we provide an example.

Example 12. Consider system (1) and (2) with parameters given as follows:

$$\begin{gathered} A = \begin{bmatrix} 0.11 & 0 \\ 0 & -0.9 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} -2 & 0 \\ -1 & 1.1 \end{bmatrix}, \\ E = \begin{bmatrix} 0.56 \\ 0.61 \end{bmatrix}, \qquad B = \begin{bmatrix} 0.2 \\ -2.5 \end{bmatrix}, \\ C = \begin{bmatrix} 0.1 & 1.8 \end{bmatrix}, \qquad C_1 = \begin{bmatrix} 0.7 & -1 \end{bmatrix}, \\ F = 0.1, \qquad D = 0.4. \end{gathered} \tag{48}$$

Given $h_1 = 0.1$, $h_2 = 0.2$, $\mu_1 = 0.1$, $\mu_2 = 0.2$, and $\gamma = 0.6$ we can find that LMIs in (38) are feasible with

$$\overline{P} = \begin{bmatrix} 2.6423 & -0.2090 \\ -0.2090 & 0.7465 \end{bmatrix}, \qquad \overline{K} = \begin{bmatrix} -0.5241 & 0.8779 \end{bmatrix}. \tag{49}$$

By Theorem 11, there exists a state feedback controller

$$u(t) = \overline{K}\,\overline{P}^{-1}x(t) = \begin{bmatrix} -0.1078 & 1.1458 \end{bmatrix} x(t) \tag{50}$$

such that the closed-loop system is asymptotically stable for $0 \leq d_1(t) \leq 0.1$, $0 \leq d_2(t) \leq 0.2$ with an H^∞ disturbance attenuation level $\gamma = 0.6$.

4. Conclusion

This paper is concerned with H^∞ control for a networked control model of systems with two additive time-varying delays. For one thing a new delay-dependent stability criterion was derived, which improves over existing ones in that it has less conservatism with fewer matrix variables. A delay-rate-independent criterion was also obtained as a byproduct. When one of the delays is constant, a new stability criterion was given for systems with interval time-varying delay. Then examples were provided to illustrate the reduced conservatism of the criteria. Finally the H^{∞} state feedback control problem was solved via an LMI approach, which was demonstrated to be effective using another example.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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