

Research Article

The Complexity Uncertain Analysis about Three Differences Old and New Product Pricing Oligarch Retailers Closed-Loop Supply Chain

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This paper studied some realistic recycle fields at present in China, such as some daily machinery and equipment, electronic products, and plastic products. Under such circumstance, three oligarch retailers are a common situation. In this paper, we model and analyze the game of three oligarch retailers in a closed-loop supply chain. The stability of the Nash equilibrium, the bifurcation and chaos phenomenon of the recycling price and new product quantity when retailers change its adjustment speed, the profit trends of the three retailers, and the power spectrum are studied. At last, we used the parameter adjustment and the variable state feedback control strategy for chaos control, and the chaos of the system is delayed effectively. For closed-loop supply chain research, the conclusions of the numerical simulation in this paper not only have realistic guiding significance, but also have theoretical reference value.

1. Introduction

As an effective method for studying multistage decision problems in supply chain, game and complexity theory has been an enduring one. Zhang and Da [1] built a duopoly game model with different cost structures and studied the Nash equilibrium and the chaos of the model. Matsumoto and Nonaka [2] analyzed the complexity of a Cournot model with linear cost function. Lu et al. [3] studied a duopoly game model and its chaos control. Elabbasy et al. [4] discussed three different decision makers, bounded rationality, adaptive, and naïve, and studied the three-oligarch game under this situation. Gong and Li [5] analyzed a three-oligarch game based on two different structures which used differentiation competition and discrimination pricing strategy. Hu and Chen [6] considered a duopoly game with learning ability and its chaos control. Ma and Pu [7, 8] considered the nonlinear three-oligarch market and its complexity and discussed a Cournot-Bertrand model and its complexity characteristics. Zhang and Ma [9] studied the chaos and chaos control of a four-oligarch game. Ma and Sun [10] analyzed the complexity characteristics of a three-oligarch price game with

heterogeneous decision rules. Wang and Ma [11] studied a Cournot-Bertrand game model and its complexity with limited information.

Meanwhile, the game and complexity theory has been applied to different realistic fields. Ma and Bangura [12] applied complexity theory to financial and economic system. Junhai and Junling [13] used complexity theory to study insurance market and achieved some results. Ma and Tu [14] applied complexity theory to power market and discussed the chaos in duopoly game with bounded rationality, when delay was considered. Sun and Ma [15] researched Chinese cold rolled steel market and analyzed the complexity of a three-oligarch game based on Bertrand model. Ding et al. [16] studied an advertising game model and its chaos control using game and complexity theory. Yi [17] discussed the game model of a closed-loop supply chain with one manufacturer and two retailers. Guo et al. [18] considered a closed-loop supply chain with duopoly manufacturers and studied the chaos when the manufacturers recycled the waste products. Ma and Zhao [19] discussed the influence of the government replacement subsidy on four different types of closed-loop supply chain in three aspects.

Based on others' former research, for the realistic recycle fields at present in China, such as some daily machinery and equipment, electronic products, and plastic products, we build a closed-loop supply chain model with three oligarch retailers in this paper. According to this model, the retailers recycle the waste products, return them to the manufacturer for repair, and then sell the second-hand products after fix and new products together. After being repaired, the second-hand products are the same as the new products in performance and appearance, but they have different customer approval degree, so their prices are different.

To be more realistic, this paper builds a three oligarch retailers competing model based on price difference between new and second-hand products. The stability of the Nash equilibrium is discussed in this paper; through bifurcation diagram, the maximum Lyapunov index diagram, and attractor, the stability and chaos of the system are studied; the trends of the three oligarch retailers' profit and the system average profit are researched. At last, we successfully controlled the chaos of the system.

2. The Model

2.1. Assumptions and Symbols. At present, the research on closed-loop supply chain at home and abroad is mainly for game equilibrium with different market structures, in which role of the supply chain is to recycle the waste products and so on. Very few papers considered the new and second-hand product market together and rarely applied complexity theory to the combination market. This paper focuses on the new and second-hand combination market and considers the game and complexity of the three oligarch retailers in the closed-loop supply chain. In this model, the retailers recycle the waste products, return them to the manufacturer for repair, and pay some cost to the manufacturer for repair. After repair, the retailers sell the second-hand products and the new products together. The second-hand products have no difference in performance and appearance from the new products, but the difference in customer approval degree between them results in different prices. In this closed-loop supply chain, the manufacturer provides new products and fixes waste products; three retailers are three oligarchs; they recycle waste products and sell new and second-hand products.

Assumptions are as follows:

- (1) three retailers R_1 , R_2 , and R_3 are all independent decentralized decision makers, and they are all bounded rationally; at discrete time periods $t = 1, 2, \dots$, each retailer makes its decision to maximize its own profit;
- (2) the recycle quantity of each retailer is related to its own and competitors' recycle prices;
- (3) the manufacturer can repair all the waste products, with no waste in repair process;
- (4) for the difference in customer approval degree between new and second-hand products, their price is different;

- (5) demand is bigger than supply, so the new and second-hand products can both sell out, but price decreases when supply increases.

Symbols are as follows:

- (1) manufacturer M : new product wholesale price w ; waste product repair cost c ;
- (2) retailer R_i : waste product recycle price p_i ; waste product recycle quantity r_i ; second-hand product fixed cost of sale c_r ; second-hand product market price p_r ; new product supply quantity q_i ; new product fixed cost of sale c_n ; new product market price p_n .

2.2. Model Dynamical System. For the three oligarch retailers, the functions of each one's waste products recycle quantity are as follows:

$$\begin{aligned} r_1 &= g_1 + h_1 p_1 - j_1 p_2 - k_1 p_3, \\ r_2 &= g_2 + h_2 p_2 - j_2 p_1 - k_2 p_3, \\ r_3 &= g_3 + h_3 p_3 - j_3 p_1 - k_3 p_2. \end{aligned} \quad (1)$$

In the previous equation, $g_i \geq 0$ ($i = 1, 2, 3$) is the recycle quantity that the customers returned to retailer i for free, based on their environmental protection consciousness; $h_i \geq 0$ ($i = 1, 2, 3$) is the customers' recycle price sensitive coefficient; $j_i, k_i \geq 0$ ($i = 1, 2, 3$) is the retailers' competition coefficient, and $h_i > j_i, h_i > k_i$ ($i = 1, 2, 3$).

The functions of the new and second-hand products' price are as follows:

$$\begin{aligned} p_n &= a_1 - b_1 (q_1 + q_2 + q_3) - d_1 (r_1 + r_2 + r_3), \\ p_r &= a_2 - b_2 (r_1 + r_2 + r_3) - d_2 (q_1 + q_2 + q_3). \end{aligned} \quad (2)$$

In the previous equation, $a_i > 0$ is the maximum demand of the market; $b_i > 0, d_i > 0$ is the replacement ratio between new and second-hand products. Then we get the profit of retailer i at period

$$\begin{aligned} \pi_i(t) &= (p_r(t) - p_i(t) - c - c_r) * r_i(t) \\ &+ (p_n(t) - w - c_n) * q_i(t). \end{aligned} \quad (3)$$

Each retailer has two decision variables: recycle price p_i and new product supply quantity q_i ; then the marginal profit functions of the three oligarch retailers at period t are according to the principle that maximizes its benefit, and in that time the marginal benefit is equal to 0:

$$\frac{\partial \pi_i(t)}{\partial p_i(t)}, \frac{\partial \pi_i(t)}{\partial q_i(t)} = 0 \quad (i = 1, 2, 3). \quad (4)$$

In real economic circumstance, the three oligarch retailers are all information limited; under the bounded rational decision rule, each retailer at period $t+1$ takes the adjustment strategy:

$$\begin{aligned} p_i(t+1) &= p_i(t) + v_i p_i(t) \frac{\partial \pi_i(t)}{\partial p_i(t)}, \\ q_i(t+1) &= q_i(t) + \alpha_i q_i(t) \frac{\partial \pi_i(t)}{\partial q_i(t)} \quad (i = 1, 2, 3). \end{aligned} \quad (5)$$

In the previous equation, $v_i, \alpha_i \geq 0$ ($i = 1, 2, 3$) are the adjustment speed of retailer i for its recycle price p_i and new product supply quantity q_i .

Integrate formulas (1)–(5); then we get the discrete dynamic system of the three oligarch retailers in this closed-loop supply chain:

$$\begin{aligned}
 p_1(t+1) &= p_1(t) + v_1 p_1(t) \\
 &\quad \times (A_1 p_1(t) + A_2 p_2(t) + A_3 p_3(t) \\
 &\quad + A_4 q_1(t) - d_2 h_1 q_2(t) \\
 &\quad - d_2 h_1 q_3(t) + A_5), \\
 p_2(t+1) &= p_2(t) + v_2 p_2(t) \\
 &\quad \times (B_1 p_1(t) + B_2 p_2(t) \\
 &\quad + B_3 p_3(t) - d_2 h_2 q_1(t) \\
 &\quad + B_4 q_2(t) - d_2 h_2 q_3(t) + B_5), \\
 p_3(t+1) &= p_3(t) + v_3 p_3(t) \\
 &\quad \times (C_1 p_1(t) + C_2 p_2(t) \\
 &\quad + C_3 p_3(t) - d_2 h_3 q_1(t) \\
 &\quad - d_2 h_3 q_2(t) + C_4 q_3(t) + C_5), \\
 q_1(t+1) &= q_1(t) + \alpha_1 q_1(t) \\
 &\quad \times (D_1 p_1(t) + D_2 p_2(t) \\
 &\quad + D_3 p_3(t) - 2b_1 q_1(t) \\
 &\quad - b_1 q_2(t) - b_1 q_3(t) + D_4), \\
 q_2(t+1) &= q_2(t) + \alpha_2 q_2(t) \\
 &\quad \times (E_1 p_1(t) + E_2 p_2(t) \\
 &\quad + E_3 p_3(t) - b_1 q_1(t) \\
 &\quad - 2b_1 q_2(t) - b_1 q_3(t) + E_4), \\
 q_3(t+1) &= q_3(t) + \alpha_3 q_3(t) \\
 &\quad \times (F_1 p_1(t) + F_2 p_2(t) \\
 &\quad + F_3 p_3(t) - b_1 q_1(t) \\
 &\quad - b_1 q_2(t) - 2b_1 q_3(t) + F_4).
 \end{aligned} \tag{6}$$

3. Numerical Simulation Analyses

3.1. The Equilibrium of System and Its Stability. In order to study the behavior of different retailers, we need to solve the equilibrium of discrete dynamic system (6) and then discuss its stability. Here we will study the connection between the equilibrium, its stability, and the decision behavior of each retailer. To be clear, first we assign a value for the market parameters, which are beyond the range of the retailers' decision.

At present in China, the recycling industry is still in its infancy; besides the customers' environmental protection consciousness is weak, so the customers' recycle price sensitivity would be strong. From the customer psychology, even though the new and second-hand products have no difference in their performance and appearance, the new product would have a higher price. Based on the actual investigation of Chinese retail market, we find that the situation is in this case, especially in the sales process of some high price products. This situation is more obvious; therefore, according to the actual statistical data, the values of the market parameters are assigned as follows:

$$\begin{aligned}
 a_1 &= 28, & a_2 &= 22, & b_1 &= 0.8, & b_2 &= 0.7, \\
 d_1 &= 0.3, & d_2 &= 0.5; & c &= 2, & w &= 10, \\
 c_{r1} &= 0.2, & c_{r2} &= 0.18, & c_{r3} &= 0.21, & c_{n1} &= 0.3, \\
 c_{n2} &= 0.32, & c_{n3} &= 0.27; & g_1 &= 0.3, \\
 g_2 &= 0.25, & g_3 &= 0.27, & k_1 &= 0.25, & k_2 &= 0.24, \\
 k_3 &= 0.26; & h_1 &= 1.2, & h_2 &= 0.95, & h_3 &= 1.1, \\
 j_1 &= 0.2, & j_2 &= 0.21, & j_3 &= 0.19.
 \end{aligned} \tag{7}$$

Discrete dynamic system (6) is a six-dimension space; the equilibriums of the system are the fixed points of the space, and there are 64 of them. As in realistic supply chain, any oligopoly will not make the price at 0 or have no production in the actual production process. Therefore, the quantity and price at equilibrium point need to be positive. From the calculated results, we find that 63 of the equilibriums are not totally positive, so they are meaningless in reality. Due to limited space, the 63 solutions are not listed. Here we will only solve the positive equilibrium E , and it is the only Nash equilibrium of the system; its value is as follows:

$$\begin{aligned}
 E &= [p_1^*, p_2^*, p_3^*, q_1^*, q_2^*, q_3^*] \\
 &= [4.4641, 4.9662, 4.7060, \\
 &\quad 4.2869, 4.4451, 4.3806].
 \end{aligned} \tag{8}$$

We can see that retailers' adjustment speed v_i, α_i has no influence on the value of the Nash equilibrium E . When the market parameters remain the same, the value of the equilibrium E would not change with the adjustment speed.

Next we will discuss the stability of the equilibrium; the Jacobian matrix of discrete dynamic system (6) is as follows:

$$J(p_1, p_2, p_3, q_1, q_2, q_3) = \begin{bmatrix} 1 + v_1 A & v_1 A_2 p_1 & v_1 A_3 p_1 & v_1 A_4 p_1 & -d_2 h_1 v_1 p_1 & -d_2 h_1 v_1 p_1 \\ v_2 B_1 p_2 & 1 + v_2 B & v_2 B_3 p_2 & -d_2 h_2 v_2 p_2 & v_2 B_4 p_2 & -d_2 h_2 v_2 p_2 \\ v_3 C_1 p_3 & v_3 C_2 p_3 & 1 + v_3 C & -d_2 h_3 v_3 p_3 & -d_2 h_3 v_3 p_3 & v_3 C_4 p_3 \\ \alpha_1 D_1 q_1 & \alpha_1 D_2 q_1 & \alpha_1 D_3 q_1 & 1 + \alpha_1 D & -b_1 \alpha_1 q_1 & -b_1 \alpha_1 q_1 \\ \alpha_2 E_1 q_2 & \alpha_2 E_2 q_2 & \alpha_2 E_3 q_2 & -b_1 \alpha_2 q_2 & 1 + \alpha_2 E & -b_1 \alpha_2 q_2 \\ \alpha_3 F_1 q_3 & \alpha_3 F_2 q_3 & \alpha_3 F_3 q_3 & -b_1 \alpha_3 q_3 & -b_1 \alpha_3 q_3 & 1 + \alpha_3 F \end{bmatrix} \quad (9)$$

In the previous matrix, $A = \partial\pi_1/\partial p_1 + A_1 p_1$, $B = \partial\pi_2/\partial p_2 + B_2 p_2$, $C = \partial\pi_3/\partial p_3 + C_3 p_3$, $D = \partial\pi_1/\partial q_1 - 2b_1 q_1$, $E = \partial\pi_2/\partial q_2 - 2b_1 q_2$, and $F = \partial\pi_3/\partial q_3 - 2b_1 q_3$.

For Nash equilibrium E , its Jacobian matrix is as follows:

$$J(E) = \begin{bmatrix} 1 - 16.7134v_1 & -0.4446v_1 & -0.5464v_1 & -2.4106v_1 & -1.3392v_1 & -1.3392v_1 \\ -1.2414v_2 & 1 - 12.6722v_2 & -0.4138v_2 & -1.1795v_2 & -1.9095v_2 & -1.1795v_2 \\ -1.6230v_3 & -0.0296v_3 & 1 - 14.774v_3 & -1.2941v_3 & -1.2941v_3 & -2.1553v_3 \\ -2.3149\alpha_1 & -0.4158\alpha_1 & -0.5166\alpha_1 & 1 - 6.8591\alpha_1 & -3.4295\alpha_1 & -3.4295\alpha_1 \\ -0.8335\alpha_2 & -1.7091\alpha_2 & -0.5467\alpha_2 & -3.5561\alpha_2 & 1 - 7.1121\alpha_2 & -3.5561\alpha_2 \\ -0.8433\alpha_3 & -0.3592\alpha_3 & -2.0063\alpha_3 & -3.5045\alpha_3 & -3.5045\alpha_3 & 1 - 7.009\alpha_3 \end{bmatrix} \quad (10)$$

From the Jacobian matrix of equilibrium E , we know that although the retailers' adjustment speed v_i , α_i has no influence on the value of equilibrium E , v_i , α_i affect the stability of E . So the Nash equilibrium is local stable, but its stability needs to meet certain conditions. The size of its stability range is decided by the retailers' adjustment speed to their recycle price and new product supply quantity.

3.2. Bifurcation and Chaos Numerical Simulation. From the Jacobian matrix of equilibrium E , we know that the retailers' adjustment speed v_i , α_i has influence on the stability of the Nash equilibrium E . As the three retailers are three oligarchs, theoretically they have a similar impact on the system, so we use v_1 , α_1 , for example, to study the influence of v_i , α_i on the system variables.

Take $v_2 = 0.05$, $v_3 = 0.1$, $\alpha_1 = 0.1$, $\alpha_2 = 0.15$, and $\alpha_3 = 0.05$; we get the track figures of each retailer's recycle price p_i and new product supply quantity q_i , when v_1 gradually increases from zero (Figure 1 is the track figure of p_i with v_1 , and Figure 2 is the track figure of q_i with v_1). Take $v_1 = 0.05$, $v_2 = 0.12$, $v_3 = 0.1$, $\alpha_2 = 0.1$, and $\alpha_3 = 0.15$; we get the track figures of each retailer's recycle price p_i and new product supply quantity q_i , when α_1 gradually increases from zero (Figure 3 is the track figure of p_i with α_1 , and Figure 4 is the track figure of q_i with α_1).

From Figures 1 to 4 we see that when the adjustment speed v_1 , α_1 of retailer 1 gradually increases, at first each retailer's recycle price p_i and new product supply quantity q_i all stay stable; then the first bifurcation came up, the second bifurcation came up, and at last it came to chaos. It means that if retailer 1 adjusts its recycle price p_1 or its new product

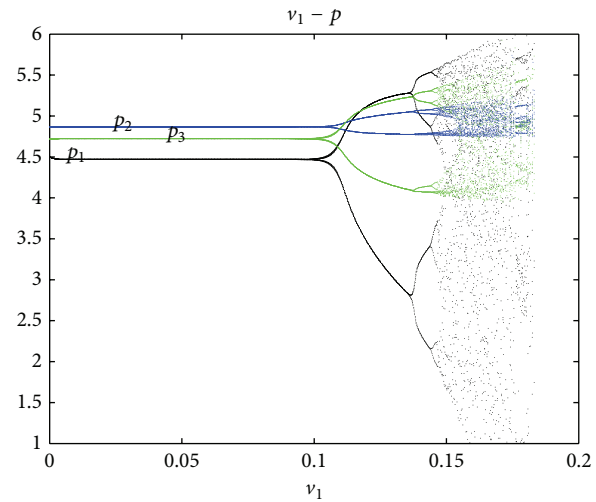


FIGURE 1: Price bifurcation with the change of v_1 .

supply quantity q_1 too fast, all the retailers' behavior will become unpredictable and complicated. Similarly, if retailer 2 and retailer 3 adjust their recycle price p_i or their new product supply quantity q_i too fast, it will also cause disorder of the market. What is more, from Figures 1-2, we can know that, with the increase of v_1 , the price of retailer 1 enters into the bifurcation condition first and then that of retailers 3 and 2. In quantity aspect, retailer 1 first entered the bifurcation condition and then retailers 2 and 3. This shows that, in this state, the different research objectives have different paths entering into chaos.

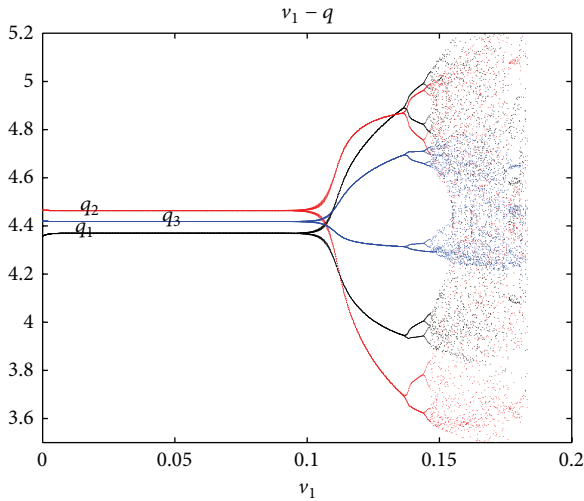


FIGURE 2: Quantity bifurcation with the change of v_1 .

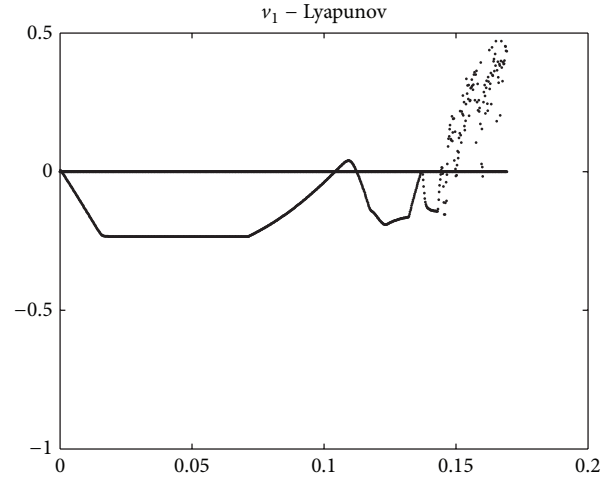


FIGURE 5: Lyapunov index with the change of v_1 .

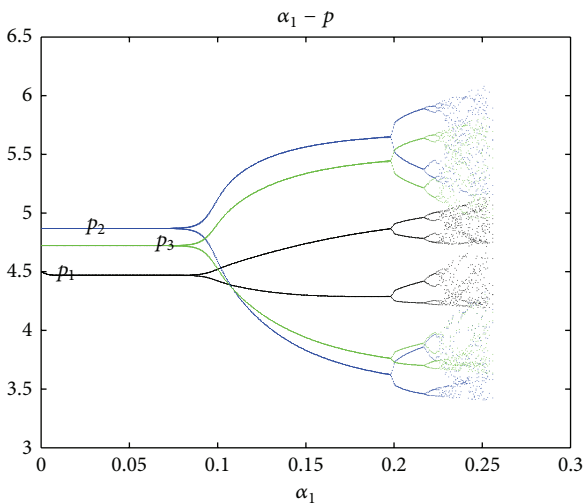


FIGURE 3: Price bifurcation with the change of α_1 .

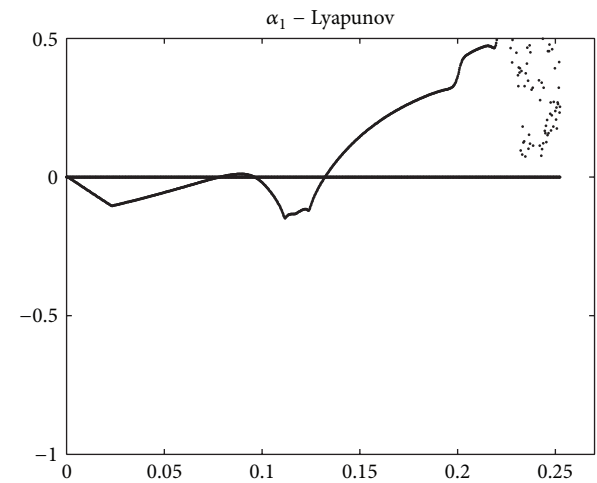


FIGURE 6: Lyapunov index with the change of α_1 .

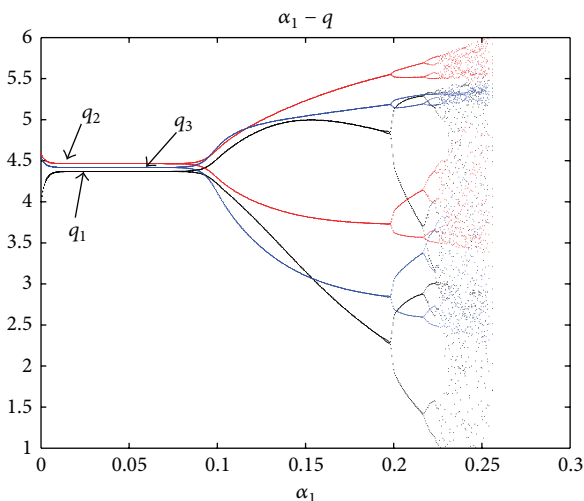


FIGURE 4: Quantity bifurcation with the change of α_1 .

Besides, we note that, in Figures 3 and 4, when α_1 gradually increases, after the first bifurcation of p_i, q_i occurred, in a wide range they remain in two-cycle state. In this situation, if we introduce chaos control to the system, the stability range will be remarkably expanded. Path entering into chaotic state is similar to the increase of v_1 .

Figure 5 is the maximum Lyapunov index variation figure of v_1 ; observing this figure we can also confirm the bifurcation and chaos of the retailers' decision variables p_i, q_i , when v_1 gradually increases. Figure 6 is the maximum Lyapunov index variation figure of α_1 . It also shows the process of the system changing from stability to chaos, when α_1 gradually increases.

Comparing Figure 5 to Figures 1 and 2, we see that when v_1 is small, p_i, q_i remain stable, and the maximum Lyapunov index of the system is less than zero; with the increase of v_1 , the first bifurcation of p_i, q_i occurred, and it is the point where the maximum Lyapunov index of the system is zero; when v_1 kept increasing, p_i, q_i came to chaos, and the maximum Lyapunov index of the system is bigger than

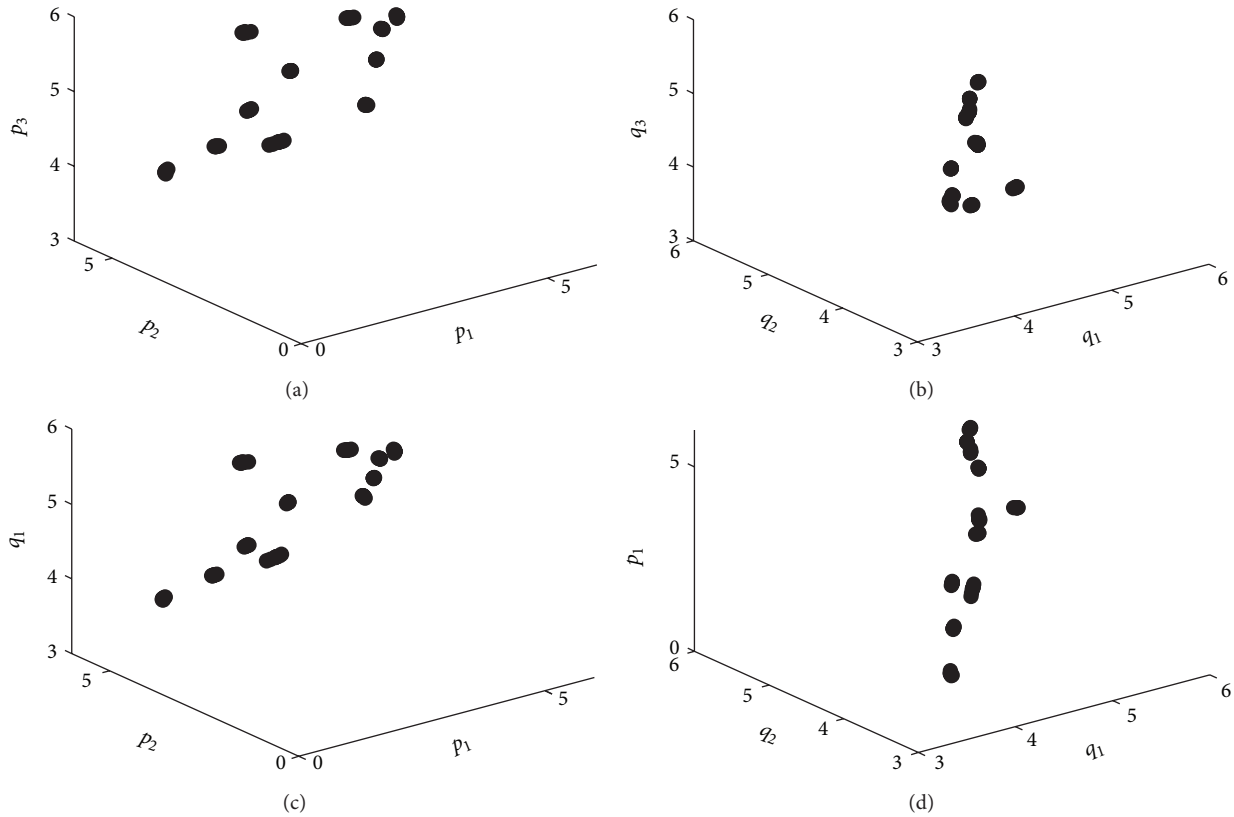


FIGURE 7: Chaos attractor of the system.

zero. The variation of the maximum Lyapunov index of α_1 in Figure 6 also confirmed the process in Figures 3 and 4: p_i, q_i change from stability to two-cycle state, then to four cycle state, and at last come to chaos.

The chaos state of the system also can be proved by its chaos attractor; when $v_1 = 0.15, v_2 = 0.05, v_3 = 0.1, \alpha_1 = 0.1, \alpha_2 = 0.15,$ and $\alpha_3 = 0.05$, the system is in chaos, and its chaos attractor is showed in Figure 7. From the figure, we see that the attractor of the system is divergent; it proves that the system is in chaos.

3.3. Profit Numerical Simulation. When each retailer changes its adjustment speed of its recycle price or its new product supply quantity, the decisions of them become complicated. In this situation, how would the profit of each retailer in the system become? Would it increase when the market uncertainty increases? Or would it decrease when the market became more complicated? Does every retailer's profit change in the same way? Here we still use the adjustment speed v_1, α_1 of retailer 1 as an example to study. When v_1 gradually increases, Figure 8 is the track figure of each retailer's profit, and Figure 9 is the average profit track figure of the three oligarchs. When α_1 gradually increases, the track of each retailer's profit is showed in Figure 10, and the average profit track of the three oligarchs is showed in Figure 11.

From Figures 8 to 11, we know that when the adjustment speed v_1, α_1 of retailer 1 about its recycler price p_1 and new product supply quantity q_1 gradually increases, at first, each

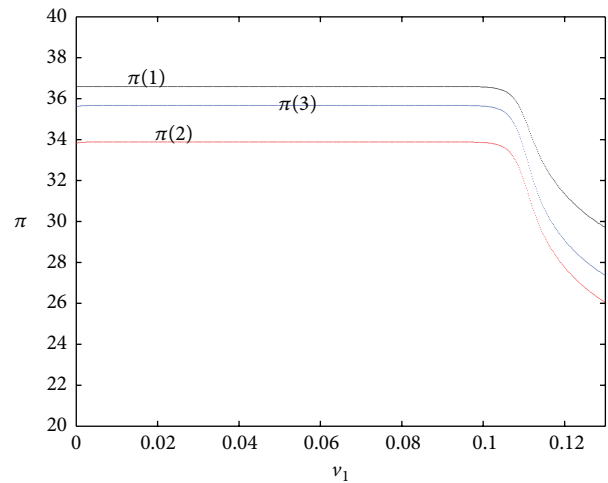


FIGURE 8: Track figure of profit with v_1 .

retailer's profit and the average profit of the three oligarchs all remained stable; then when v_1 and α_1 keep increasing and reach a critical value, all the profits fall fast. And the critical values of v_1, α_1 , where profits begin to fall, are the bifurcation points in Figures 1–4. Meanwhile, the maximum Lyapunov index equals zero.

So, if retailer 1 adjusts p_1, q_1 too fast, the system variables will become bifurcation and chaos. This means that all the decision makers' behavior in the system will be complicated,

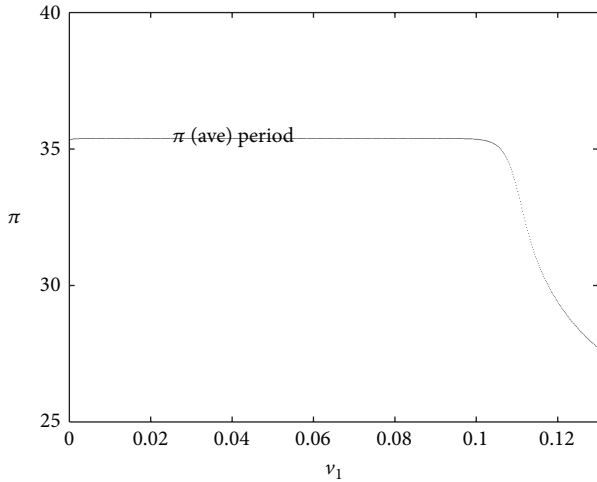


FIGURE 9: Average profit of the oligarchs with ν_1 .

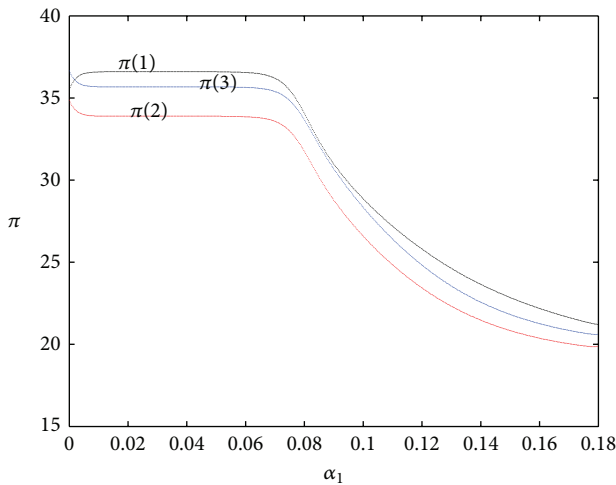


FIGURE 10: Track figure of profit with α_1 .

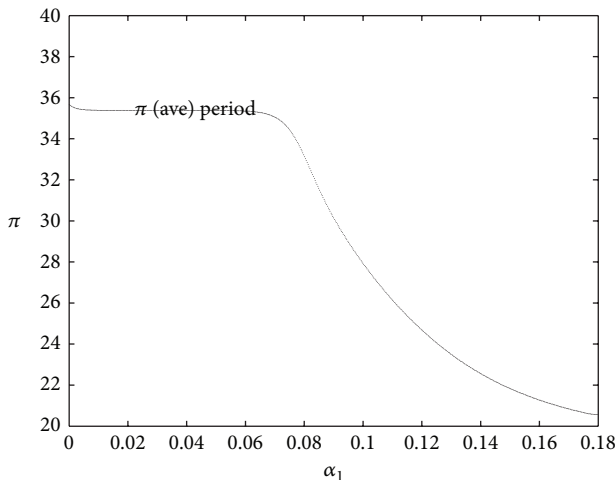


FIGURE 11: Average profit of the oligarchs with α_1 .

and it makes the profit of each retailer and the average profit of the system all reduce. As the same, if retailer 2 or retailer 3 adjusts its decision too fast, the profits of the three oligarchs would go the same way.

Then we have the study on the power spectrum of p_i, q_i when retailers adjust their recycle price p_i and new product supply quantity q_i in different speeds. From the result we are aware that when the system is in chaos state, the power spectrum of p_1, q_1 confirms that the power spectrum is continuous. Meanwhile, we know that no matter how fast retailer 1 adjusts p_1, q_1 , the values of recycle price p_1 and new product supply quantity q_1 would go through every value in the value area, and they will always be restricted within a certain value range. These are the chaos characteristics of the system: from inside, the chaotic motions of the system are irregular; from outside, the chaotic motions have a certain value space. It is the chaos attractor. So, the chaos is bounded; this is the foundation of the system chaos control.

4. System Chaos Control

When the system changes from stability to chaos, all the three-oligarch retailers' decisions become complicated, and the profit of each one declines fast, so no retailer expects the chaos. In this case, the three oligarch retailers can put some adjustment measures to the market, through the coordination and cooperation between them to control the system chaos, in order to delay or eliminate the chaos in the market and assure that their profits reach the maximum.

In this paper, we will use parameter adjustment and variable state feedback control strategy, to control the three retailers' recycle price p_i and new product supply quantity q_i , in order to delay the chaos of the system.

Note the original discrete dynamic system (6) as follows:

$$\begin{aligned}
 p_i(t+1) &= f_i(p_1(t), p_2(t), p_3(t), q_1(t), q_2(t), q_3(t)), \\
 & \quad i = 1, 2, 3, \\
 q_j(t+1) &= g_j(p_1(t), p_2(t), p_3(t), q_1(t), q_2(t), q_3(t)), \\
 & \quad j = 1, 2, 3.
 \end{aligned}
 \tag{11}$$

Put the parameter adjustment and variable state feedback control strategy to it:

$$\begin{aligned}
 p_i(t+m) &= (1-\mu) f_i^m(p_1(t), p_2(t), p_3(t), q_1(t), q_2(t), q_3(t)) \\
 & \quad + \mu p_i(t), \quad i = 1, 2, 3, \\
 q_j(t+m) &= (1-\mu) g_j^m(p_1(t), p_2(t), p_3(t), q_1(t), q_2(t), q_3(t)) \\
 & \quad + \mu q_j(t), \quad j = 1, 2, 3.
 \end{aligned}
 \tag{12}$$

In (12), $0 < \mu < 1$ is the adjustment parameter, and m is the iteration times in each control; here take $m = 1$. Then the controlled system is as follows:

$$\begin{aligned}
 p_i(t+1) &= (1-\mu) f_i(p_1(t), p_2(t), p_3(t), q_1(t), q_2(t), q_3(t)) \\
 &\quad + \mu p_i(t), \quad i = 1, 2, 3, \\
 q_j(t+1) &= (1-\mu) g_j(p_1(t), p_2(t), p_3(t), q_1(t), q_2(t), q_3(t)) \\
 &\quad + \mu q_j(t), \quad j = 1, 2, 3.
 \end{aligned}
 \tag{13}$$

Use the same market parameters as before, and solve the controlled system (13); then we know that the value of the only Nash equilibrium is the same as system (6). It is

$$\begin{aligned}
 E &= [p_1^*, p_2^*, p_3^*, q_1^*, q_2^*, q_3^*] \\
 &= [4.4641, 4.9662, 4.7060, \\
 &\quad 4.2869, 4.4451, 4.3806].
 \end{aligned}
 \tag{14}$$

The Jacobian matrix of the equilibrium E of controlled system (13) is

$$J(E) = \begin{bmatrix}
 1 - 16.7134v_1\mu & -0.4446v_1\mu & -0.5464v_1\mu & -2.4106v_1\mu & -1.3392v_1\mu & -1.3392v_1\mu \\
 -1.2414v_2\mu & 1 - 12.6722v_2\mu & -0.4138v_2\mu & -1.1795v_2\mu & -1.9095v_2\mu & -1.1795v_2\mu \\
 -1.6230v_3\mu & -0.0296v_3\mu & 1 - 14.774v_3\mu & -1.2941v_3\mu & -1.2941v_3\mu & -2.1553v_3\mu \\
 -2.3149\alpha_1\mu & -0.4158\alpha_1\mu & -0.5166\alpha_1\mu & 1 - 6.8591\alpha_1\mu & -3.4295\alpha_1\mu & -3.4295\alpha_1\mu \\
 -0.8335\alpha_2\mu & -1.7091\alpha_2\mu & -0.5467\alpha_2\mu & -3.5561\alpha_2\mu & 1 - 7.1121\alpha_2\mu & -3.5561\alpha_2\mu \\
 -0.8433\alpha_3\mu & -0.3592\alpha_3\mu & -2.0063\alpha_3\mu & -3.5045\alpha_3\mu & -3.5045\alpha_3\mu & 1 - 7.009\alpha_3\mu
 \end{bmatrix}.
 \tag{15}$$

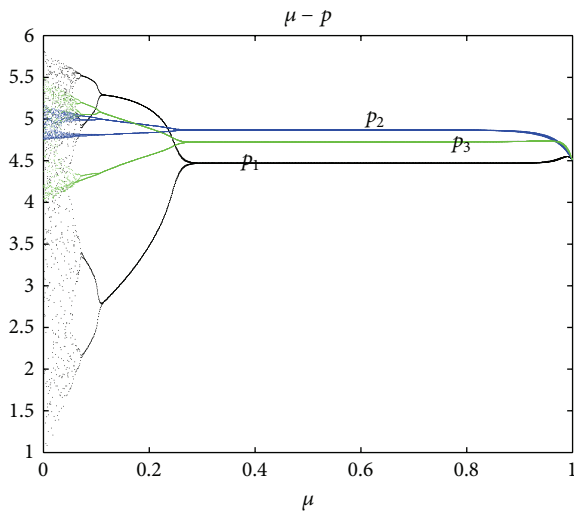


FIGURE 12: Chaos control of p_i with parameter μ .

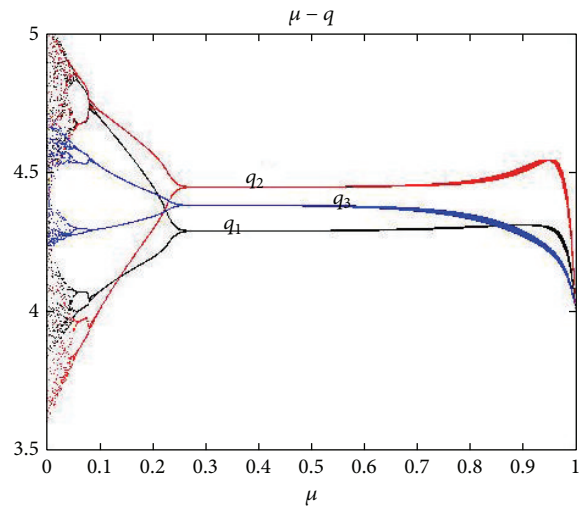


FIGURE 13: Chaos control of q_i with parameter μ .

Obviously, the value of μ has influence on the eigenvalues of the equilibrium's Jacobian matrix, so μ will affect the stable range of the Nash equilibrium.

Take $v_1 = 0.15, v_2 = 0.05, v_3 = 0.1, \alpha_1 = 0.1, \alpha_2 = 0.15,$ and $\alpha_3 = 0.05$; the system is in chaos, and the influence of the adjustment parameter μ on the stability of the recycle price p_i and new product supply quantity q_i is showed in Figures 12 and 13.

From Figures 12 and 13 we see that when $\mu = 0$, before the adjustment, the original system is in chaos; with the increase of the adjustment parameter μ , the recycle price p_i and new product supply quantity q_i gradually change from the chaos to cycle phase; when μ reaches a critical value (about 0.3), the recycle price p_i and new product supply quantity q_i begin to keep stable, and the system chaos is well controlled.

Take $v_2 = 0.05, v_3 = 0.1, \alpha_1 = 0.1, \alpha_2 = 0.15, \alpha_3 = 0.05,$ and $\mu = 0.4$; the track figures of the recycle price p_i and new product supply quantity q_i when v_1 gradually increases are Figures 14 and 15. Take $v_1 = 0.05, v_2 = 0.12, v_3 = 0.1, \alpha_2 = 0.1, \alpha_3 = 0.15,$ and $\mu = 0.4$; the track figures of the recycle price p_i and new product supply quantity q_i when α_1 gradually increases are Figures 16 and 17.

Comparing Figures 14 and 15 with Figures 1 and 2, we noticed that the stable range of p_i, q_i when v_1 increases is much larger; comparing Figures 16 and 17 with Figures 3 and 4, we also noticed that the stable range of p_i, q_i when α_1 increases is much larger. This means that, through using the parameter adjustment and variable state feedback control strategy to control the chaos of the system, the stable range of the system is extremely expanded; in the condition that

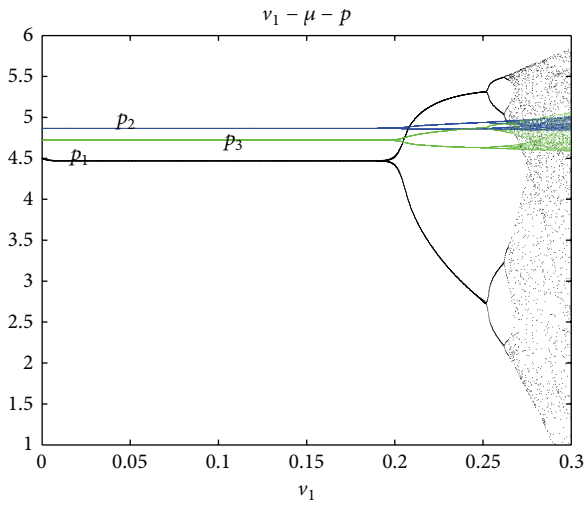


FIGURE 14: Track of p_i with v_1 after adjustment.

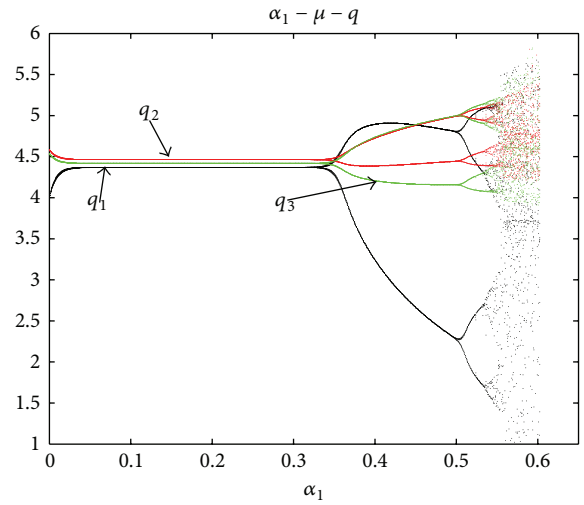


FIGURE 17: Track of the q_i with α_1 after adjustment.

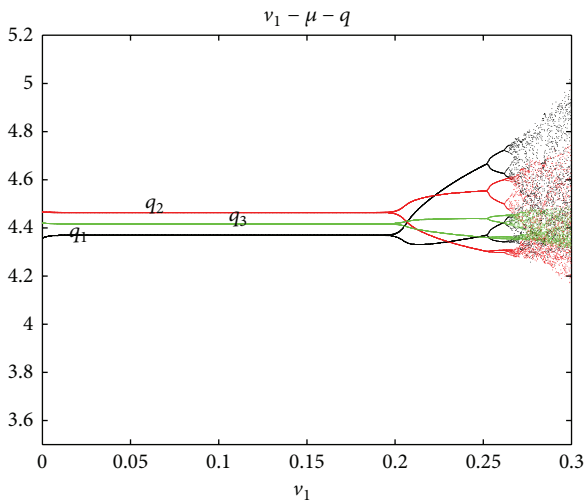


FIGURE 15: Track of the q_i with v_1 after adjustment.

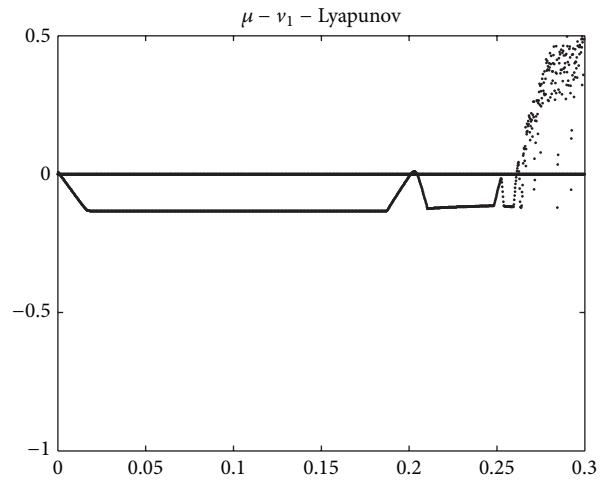


FIGURE 18: Lyapunov index variations of v_1 .

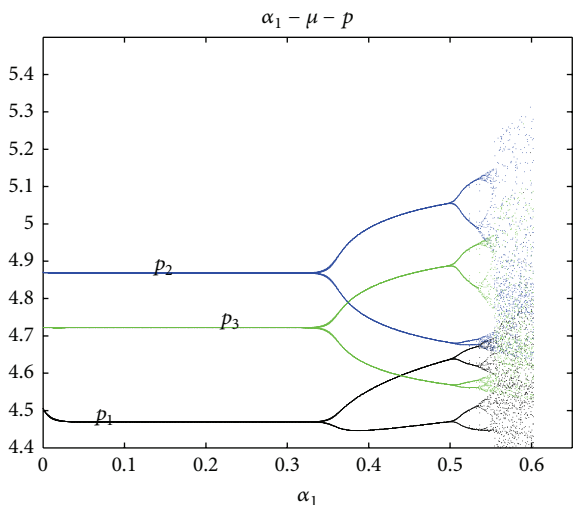


FIGURE 16: Track of p_i with α_1 after adjustment.

the market stays stable, the retailers' adjustment range can be much larger; it gives the retailers greater flexibility when they are making their decisions.

The larger stability range and the greater flexibility also can be reflected by the maximum Lyapunov index variation figure of retailer 1's adjustment speed v_1 , α_1 . In the case of system chaos control, the maximum Lyapunov index variation figures of v_1 , α_1 are Figures 18 and 19.

Comparing Figures 18 and 19 with Figures 5 and 6, it is obvious that the maximum Lyapunov index of v_1 , α_1 remains less than zero in a much larger range. When the maximum Lyapunov index of the system is less than zero, the system stays stable, so now the system has a larger stable range.

From Figure 7 in Section 3.2, when $v_1 = 0.15$, $v_2 = 0.05$, $v_3 = 0.1$, $\alpha_1 = 0.1$, $\alpha_2 = 0.15$, and $\alpha_3 = 0.05$, the system is in chaos, and the attractor of the system is divergent. After the system chaos control, take $\mu = 0.4$; the new attractor of the system is showed in Figure 20. We see that the attractor

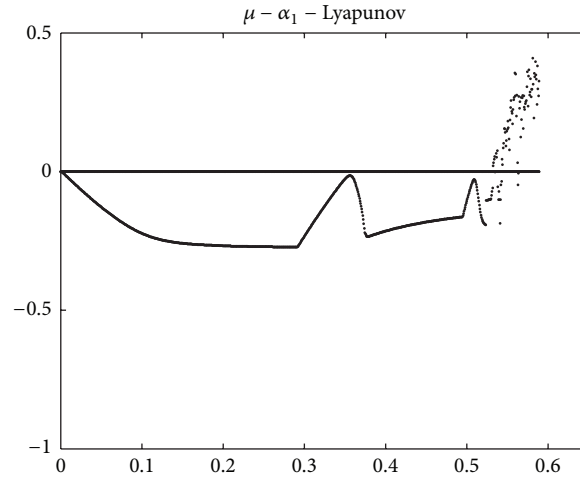


FIGURE 19: Lyapunov index variations of α_1 .

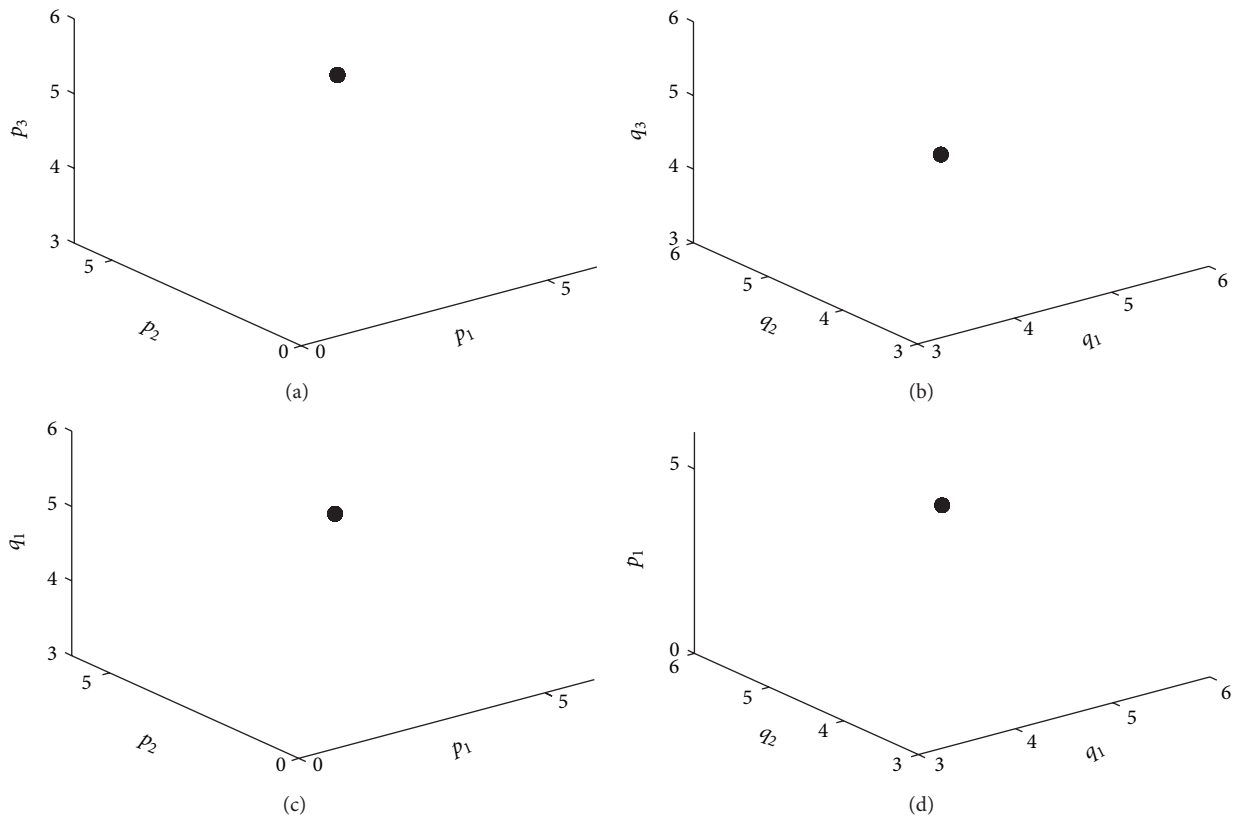


FIGURE 20: Chaos attractor of the system after adjustment.

in Figure 20 is a point; it means that the system now is stable, and the chaos of the system is controlled effectively.

According to the analyses of the three retailers' profit and their average profit in Section 3.3, when the system is stable, each of the three oligarch retailers' profit and their average profit are all stable and maximal; when the system is in chaos, each of the three oligarch retailers' profit and their average profit all fall fast. Through the system chaos control,

the chaos of the system is delayed, and the stable range of the system is expanded, so the stable range of the three oligarch retailers' profit becomes much larger, and it means a lot to the improvement of the retailers' profit.

In the reality of economic life, the value of the adjustment parameter μ is determined by the three oligarch retailers together, to make sure that they all get a stable and satisfied profit. Meanwhile, the three oligarch retailers also need to

control their adjustment speed of p_i , q_i , to make sure that the market remains stable and their own profit reaches the maximum.

5. Conclusions

In this paper, the model of a closed-loop supply chain based on price difference between new and second-hand products is established, and the game and complexity theory of the three oligarch retailers is discussed. Through numerical simulation, the conclusions are as follows.

(1) The retailers' adjustment speed about their recycle price and new product supply quantity has no influence on the value of the system equilibrium, but the adjustment speed would affect the stable range of the equilibrium.

(2) When the retailers adjust their recycle price and new product supply quantity too fast, the market behavior becomes complicated, and each retailer's decision variables go to chaos; at the same time, the three oligarch retailers' profit declines very fast.

(3) No matter what the inside of the system chaos is like, the value of the system variables would be restricted within a certain value range; the ergodicity and the boundedness are the foundation of chaos control.

(4) Chaos control can delay the system chaos state effectively and enlarge the system decision stable range.

Referencing to the model built in this paper, the participants in the supply chain can adjust their own decisions effectively in different time, and they can also work together to control the chaos in the supply chain. So the paper has a strong practical guiding significance to the market stability and the improvement of the retailers' profits. Besides, this paper studied the closed-loop supply chain with game and complexity theory and meanwhile combined the sale of the new product with the recycle and sale of the waste product, so it also has a certain reference value to the theory research of the closed-loop supply chain. The results of this paper provide the direction for further research on closed-loop supply chain with a time delay characteristics.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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