

Research Article

Discrete-Time Event-Triggered Control of Nonlinear Wireless Networked Control Systems

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This paper investigates the problem of stabilization of nonlinear discrete-time networked control systems (NCSs) with event-triggering communication scheme in the presence of signal transmission delay. A Takagi-Sugeno (T-S) fuzzy model and parallel-distributed compensation (PDC) scheme are first employed to design a nonlinear fuzzy event-triggered controller for the stabilization of nonlinear discrete-time NCSs. The idea of the event-triggering communication scheme (i.e., a soft computation algorithm) under consideration is that the current sensor data is transmitted only when the current sensor data and the previously transmitted one satisfy a certain state-dependent trigger condition. By taking the signal transmission delay into consideration and using delay system approach, a T-S fuzzy delay system model is established to describe the nonlinear discrete-time NCSs with event-triggering communication scheme. Attention is focused on the design of fuzzy event-triggered controller which ensures asymptotic stability of the closed-loop fuzzy systems. Linear matrix inequality- (LMI-) based conditions are formulated for the existence of admissible fuzzy event-triggered controller. If these conditions are feasible, a desired fuzzy event-triggered controller can be readily constructed. A nonlinear mass-spring-damper mechanical system is presented to demonstrate the effectiveness of the proposed method.

1. Introduction

Recently, networked control systems (NCSs) have been drawing more and more attention from researchers working in the areas of system and control due to their low cost, high flexibility, and simple installation and maintenance [1], and a lot of important works have been reported; see, for example, [2–16]. These works have significant importance on both theoretical advancement and practical applications of NCSs. However, it should be pointed out that the time-triggered (or periodic-triggered) transmission scheme is adopted in the aforementioned works. Using the time-triggered transmission scheme implies that all the sampled data need to be transmitted through communication networks regardless of the state of the controlled plant. As is well known, the sampling period is determined according to the worst case operation conditions that rarely occur, and thus the periodic transmission scheme may result in conservative usage of the limited communication bandwidth in the context of

NCSs. On the other hand, with the development of network communication technology, the network bandwidth is significantly improved such as Ethernet (100 MB/s) and WiFi (11 MB/s), while there are also some types of network with low bandwidth for the purpose of control or power saving such as CAN (1 MB/s) and Zigbee (250 Kb/s). In these networks, if the number of sensors is large, network traffic may be very high. In this case, the reduction of data transmission rate is necessary and most feasible. Therefore, it is significant to investigate how to improve the bandwidth utilization in data transmission so that network bandwidth can be used for other traffic.

To save the limited bandwidth, one can make use of the so-called sporadic transmission scheme. As indicated in [17], event-based/triggered transmission scheme (EBTS/ETTS) just represents one way of generating such sporadic transmissions. EBTS has many potential advantages for NCSs, such as clock-free operation, less traffic requirement, and better resource utilization. Specifically, under the EBTS, the

sampled signals need to be transmitted only when some internal measure of the novelty in the sampled information exceeds a specified threshold, which implies that only part of the sampled signals is transmitted from the sensor to the controller, and the redundant signals are filtered in the sensor node. This, in turn, will generate a sporadic sequence of controller invocations. It is expected that the average rate of event-triggered task set will be much lower than the rate of a comparable time-triggered task set [17]. Hence, EBTS can be viewed as the possible and important alternative to time-triggered transmission scheme in terms of the network bandwidth utilization.

In the last five years, the research on EBTS has received considerable attention, and many interesting EBTSs have been developed in the literature to reduce the network bandwidth utilization; see, for example, [18–27] and the references therein. However, it is worth mentioning that the event-triggering conditions proposed in aforementioned publications need to be checked at every sampling instant, which leads to the higher computation cost of the smart sensor. Very recently, Peng and Yang [28] proposed a discrete event-triggered communication scheme, where the designed event-triggering condition only needs to be checked at every event-triggering instant. Notice that the event-triggering instant is a subset of the sampling instant, and thus the discrete event-triggered communication scheme not only reduces the network bandwidth utilization, but also saves the computation cost of the smart sensor in NCSs. Nevertheless, the above latest results still leave much room for improvement: (i) the main focus of related studies is on continuous-time linear NCSs (see [22, 23] and references therein), but little work has been conducted on that of event-triggering in discrete-time nonlinear NCSs setting; (ii) most of the existing results are based on the assumption that the lower bound of the network-induced delay is zero, which may bring some conservatism to some extent [29]. Until now, there is no work that discusses how to utilize the information of the lower bound of the network-induced delay in the event-triggered continuous-time/discrete-time nonlinear NCSs framework.

Motivated by the above observations, in this paper, we focus our attention on event-triggering in nonlinear discrete-time NCSs in the presence of signal transmission delay (lower bound is not equal to zero). Since fuzzy control is a simple and effective approach to study complex nonlinear systems under the framework of T-S fuzzy model and parallel-distributed compensation (PDC) technique [30], in this paper, we will use the T-S fuzzy model approach to approximate the nonlinear NCSs. However, due to the introduction of communication networks, the effect of the network-induced delay is considered and the premise variables in corresponding T-S fuzzy model and control are essentially different (time scales are different), which results in the fact that common PDC technique is not applicable. This point is ignored in some existing works; see, for example, [11, 14]. On the other hand, even though asynchronous property is utilized in [31, 32], it can only lead to linear controller rather than fuzzy controller. Just as pointed out in [33, 34], the results in [31, 32] are very conservative. In view of this, an interesting question

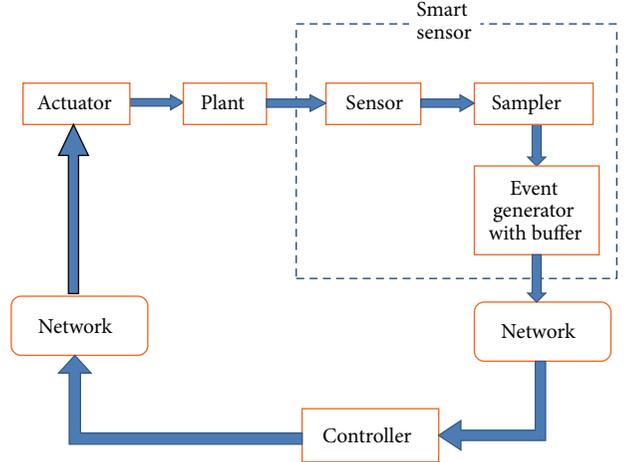


FIGURE 1: Schematic diagram of networked control systems with event-triggered communication scheme.

is how to tackle these issues effectively. In this paper, we will follow the work of Peng et al. [24] and give a feasible solution to overcome such a shortcoming. First of all, a discrete-time ETTS is proposed. The key idea of the ETTS is that if the current sensor value and the latest transmitted one satisfy a certain quantitative relation (its definition will be given in Section 2), then an event is happening and so as to trigger event generator (shown in Figure 1) to transmit (or release) the sampled signal; otherwise no event occurs. Then, to model the nonlinear NCSs effectively, synchronous premises are delicately constructed to solve the problem of nonuniform time scales in networked T-S fuzzy model and PDC fuzzy control rules. Considering the effects of signal transmission delay and event-triggering scheme, based on the well developed T-S fuzzy model approach together with delay system approach, a T-S fuzzy delay event-triggered closed system model is proposed. Based on this model, criteria for stability and fuzzy state-feedback controller design are derived, which are given in terms of LMIs. These LMIs conditions establish the relationship among the MATD (maximum allowable transmission delay) parameters of event-triggering condition and feedback gains of fuzzy controller. If these LMIs are feasible, a codesign for the parameters of event-triggering condition and fuzzy controller can be realized. Finally, a practical example is used to illustrate the applicability and effectiveness of the proposed method.

Notation. The notation used throughout the paper is standard. The superscripts “ T ” and “ -1 ” stand for matrix transposition and matrix inverse, respectively, \mathbb{R}^n denotes the n -dimensional Euclidean space, \mathbb{Z}_+ denotes the set of positive integers, and the notation $P > 0$ (≥ 0) means that P is real symmetric and positive definite (semidefinite). The symmetric term in a symmetric matrix is denoted by $*$; for example, $\begin{bmatrix} X & * \\ Y & Z \end{bmatrix}$.

2. Problem Formulation

2.1. Physical Plant. Consider a wireless NCS as depicted schematically in Figure 1. The wireless NCS consists of a nonlinear discrete-time plant:

$$x(k+1) = f(x(k)) + g(x(k))u(k), \quad (1)$$

where $k \in \mathbb{Z}_+$ is the time step, $x(k) \in \mathbb{R}^n$ is the state, $x(k+1) \in \mathbb{R}^n$ is the successor state, and $u(k) \in \mathbb{R}^m$ is the control input. The vector functions $f(x(k))$ and $g(x(k))$ are assumed to be continuous. Also assume without loss of generality that the origin is an equilibrium point for (1); that is, $f(0) = 0$. The initial state of system (1) is denoted by $x(0) \triangleq x_0$. Throughout the paper, we assume that system (1) is controlled via a shared network and that the system state is available for feedback. As shown in Figure 1, there are networks deployed between the smart sensor (including a sensor, a sampler, and an event generator) and the discrete-time static time-invariant controller (STIC) and between the STIC and the actuator. The sampler is assumed to be acting in a time-triggered fashion; the event generator, controller, and actuator are assumed to be acting in an event-triggered fashion; the data packets are assumed to be transmitted in a single packet at each time step and each data packet is time stamped [5, 6, 23]. The smart sensor of the plant transmits its measurement signal to the STIC, and the STIC transmits the control signal to the actuator over a shared and wireless network, which induces the so-called sensor-to-controller delay τ_{sc} and controller-to-actuator delay τ_{ca} . In fact, there exists computational delay τ_c in the controller. Since the computational delay τ_c is usually very small compared to τ_{sc} and τ_{ca} , it is omitted here. In the following discussions, the above network-induced delays are lumped together as a single delay τ_k [24]. On the other hand, it is well known that communication resources and/or energy sources, for example, the batteries for the wireless devices, are often limited. Given this, it is desirable to reduce the number of signal transmissions over the sensor-to-controller (S-C) and controller-to-actuator (C-A) channels as much as possible, while still guaranteeing the desirable closed-loop performance. This naturally leads us to the problem of designing smart sensor, controller, and actuator systems for the setup in Figure 1 such that this objective is achieved as expected.

In order to tackle this problem effectively, some interesting event-triggering techniques have been developed; see [35] and references therein. However, all of the above approaches have been performed in the continuous-time frame; little attention is paid to studying how the event-triggering scheme can be implemented in the discrete-time frame. A first attempt has been made for general nonlinear discrete-time system [36]; nevertheless, the common assumption used for the event-triggered strategy is the input-to-state property of the plant, which implies that the controller is given in advance. If the controller is not known a priori, the proposed method in the aforementioned work is no longer valid. In the sequel, we will start by proposing a discrete-time event-triggering transmission scheme for the configuration in Figure 1.

2.2. Event-Triggering Transmission Scheme. As indicated in the introduction, the traditional periodic transmission scheme may lead to many “unnecessary” signals being sent to controller through the network, which in turn increases the load of network transmission and wastes the limited network bandwidth. In order to reduce the load of network transmission and save the network bandwidth, it is necessary to introduce a signal scheduling strategy to effectively judge whether the current sampled signal should be sent out or not. In this sense, we will propose an event-triggering scheme in a discrete-time setting. Notice that the event-triggering condition is embedded in an event generator, whose structure is illustrated in Figure 1. In this setup, the sensor measurement $x(k)$ needs to be transmitted to the controller as long as the current sensor measurement $x(k)$ and the latest transmitted one $x(k_s)$ satisfy the following event-triggering condition:

$$[x(k) - x(k_s)]^T W_1 [x(k) - x(k_s)] > \sigma x^T(k_s) W_2 x(k_s), \quad (2)$$

where W_1 and W_2 are positive weighting matrices to be designed later for a given error tolerance $\sigma \geq 0$, $k_s \in \mathbb{Z}^+$, $s = 0, 1, 2, \dots, \infty$, and $k_0 = 0$. In this setup, it is not difficult to understand that any sensor data satisfying inequality (2) will be sent out to the controller. If we use k_s to represent the time instant when the latest event occurs (i.e., event-triggering instant), then

$$\begin{aligned} [x(k) - x(k_s)]^T W_1 [x(k) - x(k_s)] &\leq \sigma x^T(k_s) W_2 x(k_s), \\ k &\in [k_s, k_{s+1} - 1]. \end{aligned} \quad (3)$$

In other words, in the sensor node, only part of the sensor measurement will be transmitted to the controller for computation, and thus the burden of the network communication is reduced and the communication bandwidth in the network is saved and the computation burden of the controller is reduced as well. Especially in the wireless network, this method will save the transmission energy, increasing the lifespan of the battery of the nodes.

Remark 1. Different from the continuous event generator (CEG) with a continuous supervision of state [18], the event generator with (2) only supervises the difference between the current sensor value and the latest transmitted one in the discrete time instant. Moreover, the implementation of event-triggering scheme proposed in [18] needs some form of hardware event detector (e.g., analog integrated circuits or floating point gate array), while the ETTS (2) can be easily implemented by microprocessor control unit.

Remark 2. It can be seen from the discrete-time event-triggering condition (3) that the set of the event-triggering instants, that is, $\{k_1, k_2, \dots\}$, is a subset of the sampling instant

$\{1, 2, \dots\}$. Moreover, the next event-triggering instant k_{s+1} can be expressed as

$$k_{s+1} = k_s + \min_{l \geq 1} \left\{ l \mid [x(k_s + l) - x(k_s)]^T W_1 [x(k_s + l) - x(k_s)] > \sigma x^T(k_s) W_2 x(k_s) \right\}. \quad (4)$$

Clearly, the event-triggering instants depend not only on the latest transmitted state $x(k_s)$, but also on the difference between the current sensor value and the latest transmitted one. Particularly, when $\sigma = 0$, one has $k_{s+1} = k_s + 1$; the ETTS (2) reduces to the common periodic-triggering scheme (or time-triggering scheme). When $\sigma \neq 0$ and $W_1 = W_2 = W$, (2) becomes

$$[x(k) - x(k_s)]^T W [x(k) - x(k_s)] > \sigma x^T(k_s) W x(k_s). \quad (5)$$

Furthermore, when $\sigma \neq 0$ and $W_1 = W_2 = I$, (5) becomes

$$\|x(k) - x(k_s)\| > \sqrt{\sigma} \|x(k_s)\|. \quad (6)$$

It should be noted that the terms $x(k_s)$ in the right hand side of (5) and (6) denote the latest transmitted state not the current sensor measurement $x(k)$. If we use the current sensor measurement $x(k)$ instead of the latest transmitted state $x(k_s)$, then (5) and (6) become

$$[x(k) - x(k_s)]^T W [x(k) - x(k_s)] > \sigma x^T(k) W x(k), \quad (7)$$

$$\|x(k) - x(k_s)\| > \sqrt{\sigma} \|x(k)\|, \quad (8)$$

respectively, which has been used in [21, 25], respectively. So the proposed discrete-time ETTS (2) is essentially different from that in [21, 25], though they appear to be similar. Especially, in the process of event detection, for $k \in [k_s, k_{s+1} - 1]$, the proposed discrete-time ETTS (2) only needs to compute the relative threshold at every event-triggering instant ($\sigma x^T(k_s) W x(k_s)$ or $\sqrt{\sigma} \|x(k_s)\|$), while the event-triggering schemes proposed in [21, 25] have to compute the relative threshold at every sampling instant ($\sigma x^T(k) W x(k)$ or $\sqrt{\sigma} \|x(k)\|$). Hence, the proposed discrete-time ETTS (2) reduces the data transmission rate in communication network as well as the amount of computation of smart sensor.

2.3. Modeling of a Discrete-Time Networked T-S Fuzzy Model with ETTS. As is well known, the T-S fuzzy model [37] has been widely used to deal with the analysis and synthesis of nonlinear systems. In this subsection, a T-S fuzzy model will be presented to represent the nonlinear systems (1). Specifically, based on [37], the nonlinear system (1) can be represented by some local linear dynamic systems with their linguistic description. The i th rule of the T-S fuzzy model is of the following form:

Plant Rule i : IF $\theta_1(k)$ is F_1^i and, ..., and $\theta_g(k)$ is F_g^i , THEN

$$x(k+1) = A_i x(k) + B_i u(k), \quad (9)$$

where $i \in \mathcal{S} = \{1, 2, \dots, r\}$, r is the number of IF-THEN rules, $\theta_j(k)$ ($j = 1, 2, \dots, g$) represent premise variables, and F_g^i ($i \in \mathcal{S}$; $j = 1, 2, \dots, g$) are fuzzy sets. A_i and B_i are matrices with appropriate dimensions. Denoting $\theta(k) = [\theta_1(k), \dots, \theta_g(k)]^T$, we assume that $\theta(k)$ is either given or a function of $x(k)$, and it does not depend on $u(k)$.

By using a center-average defuzzifier, a product inference, and a singleton fuzzifier, the global dynamics of the T-S fuzzy system (9) can be inferred as

$$x(k+1) = \sum_{i=1}^r \mu_i(\theta(k)) [A_i x(k) + B_i u(k)], \quad (10)$$

where

$$\mu_i(\theta(k)) = \frac{\omega_i(\theta(k))}{\sum_{i=1}^r \omega_i(\theta(k))}, \quad \omega_i(\theta(k)) = \prod_{j=1}^g F_j^i(\theta_j(k)), \quad (11)$$

and $F_j^i(\theta_j(k))$ is the grade membership of $\theta_j(k)$ in F_j^i , $\mu_i(\theta(k)) \geq 0$, and $\sum_{i=1}^r \mu_i(\theta(k)) = 1$. For notational simplicity, in the sequel, we use μ_i to represent $\mu_i(\theta(k))$.

In the following, in order to include the effect of the ETTS in the closed-loop NCSs model (given below), on one hand, we assume that the output of the event generator is denoted by $x(k_s)$ at the transmission times k_s ($s = 1, 2, \dots$). On the other hand, considering the effect of the network-induced delay, a natural assumption on the lumped network-induced delays τ_{k_s} can be made as $\tau_{k_s} \in [\tau_m, \tau_M]$, where τ_m and τ_M ($0 < \tau_m \leq \tau_M$) denote the minimum and the maximum delays, respectively. Based on the above assumptions, it can be seen that the transmitted states $x(k_s)$ will arrive at the controller side at the instants $k_s + \tau_{k_s}$ ($s = 1, 2, \dots$) and the premise variables for the control rules in the controller side should be $\theta(k_s)$, which implies that the available time-stamped packet to derive the premises in the system (10) and the controller should be asynchronous. That is to say, at the same time instant $k \in [k_s + \tau_{k_s}, k_{s+1} + \tau_{k_{s+1}} - 1]$, the premises variables $\theta_j(k)$ can be available in (10), while only $\theta_j(k_s)$ is available at the controller. To design the PDC control rules, it is assumed that the mechanical model of the considered nonlinear plant is known a priori. Then, the state of system (1) can be calculated for a known control signal as long as the initial condition is given [24]. Now, in the context of NCSs, since $\theta_j(k_s)$ is available at the controller, then $\theta_j(k)$ can be calculated for $k \in [k_s + \tau_{k_s}, k_{s+1} + \tau_{k_{s+1}} - 1]$. Therefore, one can derive the synchronous premise variable $\theta_j(k)$ in (10) and in the controller side. Based on the previous descriptions, the i th controller rule can be naturally expressed as follows:

Controller Rule i : IF $\theta_1(k)$ is F_1^i and, ..., and $\theta_g(k)$ is F_g^i , THEN

$$u(k) = K_i x(k_s), \quad k \in [k_s + \tau_{k_s}, k_{s+1} + \tau_{k_{s+1}} - 1], \quad (12)$$

where $\theta_g(k)$ and F_g^i have the same definition as in (9). K_j ($j = 1, 2, \dots, g$) are controller gains to be determined later.

Applying the PDC method, the inferred fuzzy controller is given by

$$u(k) = \sum_{j=1}^r \mu_j K_j x(k_s), \quad k \in [k_s + \tau_{k_s}, k_{s+1} + \tau_{k_{s+1}} - 1]. \quad (13)$$

Combining (10) and (13), the closed-loop nonlinear NCS is given by

$$x(k+1) = \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j [A_i x(k) + B_i K_j x(k_s)], \quad (14)$$

$$k \in [k_s + \tau_{k_s}, k_{s+1} + \tau_{k_{s+1}} - 1].$$

Next, for the convenience of analysis, we will convert system (14) to a delay system by using a novel interval analysis technique [23].

Case 1. If $k_s + \tau_M + 1 \geq k_{s+1} + \tau_{k_{s+1}} - 1$, where τ_M is the upper bound of τ_{k_s} , define a function d_k as

$$d_k = k - k_s - \tau_m, \quad k \in [k_s + \tau_{k_s}, k_{s+1} + \tau_{k_{s+1}} - 1]. \quad (15)$$

From (15), it is seen that

$$\tau_{k_s} - \tau_m \leq d_k \leq k_{s+1} - k_s + \tau_{k_{s+1}} - 1 \leq \tau_M - \tau_m + 1. \quad (16)$$

Case 2. If $k_s + \tau_M + 1 < k_{s+1} + \tau_{k_{s+1}} - 1$, consider the following intervals:

$$[k_s + \tau_{k_s}, k_s + \tau_M], \quad [k_s + \tau_M + l, k_s + \tau_M + l], \quad (17)$$

where $l \in \mathbb{Z}_+$ satisfies $l \geq 1$. Since $\tau_{k_s} \leq \tau_M$, it is easy to show that there exists a positive integer $d \geq 1$ such that

$$k_s + \tau_m \leq k_s + \tau_{k_s} < k_s + d + \tau_M < k_{s+1} + \tau_{k_{s+1}} - 1 \leq k_s + d + 1 + \tau_M; \quad (18)$$

moreover, $x(k_s)$ and $x(k_s + l)$ with $l = 1, 2, \dots, d$ satisfy

$$[x(k_s + l) - x(k_s)]^T W_1 [x(k_s + l) - x(k_s)] \leq \sigma x^T(k_s) W_2 x(k_s). \quad (19)$$

It can also be seen that

$$[k_s + \tau_{k_s}, k_{s+1} + \tau_{k_{s+1}} - 1] = [k_s + \tau_{k_s}, k_s + \tau_M] \cup \left\{ \bigcup_{l=1}^{d-1} [k_s + l + \tau_M, k_s + l + \tau_M] \right\} \cup [k_s + d + \tau_M, k_{s+1} + \tau_{k_{s+1}} - 1]. \quad (20)$$

Define

$$d_k = \begin{cases} k - k_s - \tau_m, & k \in \Omega_1, \\ k - k_s - l - \tau_m, & k \in \Omega_{2l}, \\ k - k_s - d - \tau_m, & k \in \Omega_3, \end{cases} \quad (21)$$

where

$$\begin{aligned} \Omega_1 &= [k_s + \tau_{k_s}, k_s + \tau_M], \\ \Omega_{2l} &= [k_s + l + \tau_M, k_s + l + \tau_M], \quad l = 1, 2, \dots, d-1, \\ \Omega_3 &= [k_s + d + \tau_M, k_{s+1} + \tau_{k_{s+1}} - 1]. \end{aligned} \quad (22)$$

From (21), we can obtain

$$\begin{aligned} 0 \leq \tau_{k_s} - \tau_m \leq d_k \leq \tau_M - \tau_m + 1, & \quad k \in \Omega_1, \\ 0 \leq \tau_{k_s} - \tau_m \leq \tau_M - \tau_m \leq d_k \leq \tau_M - \tau_m + 1, & \quad k \in \Omega_{2l}, \\ 0 \leq \tau_{k_s} - \tau_m \leq \tau_M - \tau_m \leq d_k \leq \tau_M - \tau_m + 1, & \quad k \in \Omega_3, \end{aligned} \quad (23)$$

where the third row in (23) is true since $[k_s + d + \tau_M, k_{s+1} + \tau_{k_{s+1}} - 1] \subset [k_s + d + \tau_M, k_{s+1} + d + 1 + \tau_M]$, and we can see from (23) that

$$0 \leq d_k \leq d_M \triangleq \tau_M - \tau_m + 1. \quad (24)$$

Under Case 1, for $k \in [k_s + \tau_{k_s}, k_{s+1} + \tau_{k_{s+1}} - 1]$, define $e(k) = 0$.

Under Case 2, define

$$e(k) = \begin{cases} x(k_s) - x(k_s), & k \in \Omega_1, \\ x(k_s) - x(k_s + l), & k \in \Omega_{2l}, \\ x(k_s) - x(k_s + d), & k \in \Omega_3. \end{cases} \quad (25)$$

Combining (15), (21), and (25), (14) can be rewritten as

$$x(k+1) = \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j [A_i x(k) + B_i K_j e(k) + B_i K_j x(k - d_k - \tau_m)], \quad (26)$$

$$k \in [k_s + \tau_{k_s}, k_{s+1} + \tau_{k_{s+1}} - 1],$$

where $d_k \in [0, \tau_M - \tau_m + 1]$, $x(k) = \phi(k)$ ($k \in [-\tau_m - d_M, 0]$) denotes the initial condition, and $e(k)$ satisfies

$$e^T(k) W_1 e(k) \leq \sigma [x(k - d_k - \tau_m) + e(k)]^T \times W_2 [x(k - d_k - \tau_m) + e(k)]. \quad (27)$$

Remark 3. It is worth pointing out that, in the above transformed system (26), τ_m is a known constant delay and d_k is a time-varying delay with bound τ_M . The aim of the above transformation is to represent system (14) as a discrete-time system with two successive delay components in the state, which makes us employ the well-developed delay system theory to the stability analysis and synthesis of the closed loop (26) later.

Remark 4. If the lower bound of the network-induced delays is assumed to be zero, that is, $\tau_m = 0$, the closed-loop NCS in (26) becomes

$$x(k+1) = \sum_{i,j=1}^r \mu_i \mu_j [A_i x(k) + B_i K_j x(k-d_k) + B_i K_j e(k)],$$

$$k \in [k_s + \tau_{k_s}, k_{s+1} + \tau_{k_{s+1}} - 1],$$
(28)

with $d_k \in [0, \tau_M + 1]$, and $e(k)$ satisfies (27) with $\tau_m = 0$. Compared with (26), the upper bound of d_k in (28) is increased by τ_m . In other words, without taking the lower bound of the transmission delays into consideration, τ_m will be treated as a time-varying delay instead of a constant one when it is nonzero. Therefore, the introduction of the lower bound τ_m will naturally reduce conservativeness, which will be shown via a numerical example later. However, existing results on event-triggered NCSs, such as [21, 23–25], did not offer to take the lower bound τ_m into consideration.

3. Main Results

3.1. Stability Analysis. In this subsection, we will concentrate on the problem of stability analysis of the closed-loop system (26) under the ETTS (27). To this end, we first introduce a finite sum inequality, which will play an important role in deriving our main results.

Lemma 5. For any constant matrix $R_l \in \mathbb{R}^{n \times n}$, $R_l = R_l^T \geq 0$, $l \in \mathbb{Z}_+$, scalar γ , a function h_k satisfying $1 \leq d_1 \leq h_k \leq d_3$, $d_1 \in \mathbb{Z}_+$, $d_3 \in \mathbb{Z}_+$, and a vector function $x(k) \in \mathbb{R}^n$, $\delta(k) = x(k+1) - x(k)$ such that the following sum is well defined, the following inequality is true:

$$-(d_3 - d_1) \sum_{\theta=k-d_3}^{k-d_1-1} \delta^T(\theta) R_l \delta(\theta)$$

$$\leq \xi_1^T(k) [1 + \mu_1] \mathcal{R}_l \xi_1(k) + \xi_3^T(k) [1 + \mu_2] \mathcal{R}_l \xi_3(k).$$
(29)

Further, the sufficient conditions that guarantee the following inequality is true

$$\gamma + \xi_1^T(k) [1 + \mu_1] \mathcal{R}_l \xi_1(k) + \xi_3^T(k) [1 + \mu_2] \mathcal{R}_l \xi_3(k) < 0$$
(30)

are given by

$$\gamma + \xi_1^T(k) 3\mathcal{R}_l \xi_1(k) + \xi_3^T(k) \mathcal{R}_l \xi_3(k) < 0,$$
(31)

$$\gamma + \xi_1^T(k) \mathcal{R}_l \xi_1(k) + \xi_3^T(k) 3\mathcal{R}_l \xi_3(k) < 0,$$

where $\mathcal{R}_l = \begin{bmatrix} -R_l & * \\ R_l & -R_l \end{bmatrix}$ and

$$\xi_1^T(k) = [x^T(k-d_1) \quad x^T(k-h_k)],$$

$$\xi_3^T(k) = [x^T(k-h_k) \quad x^T(k-d_3)],$$

$$\mu_1 = -1, \quad \mu_2 = 0, \quad \text{for } h_k = d_1$$
(32)

$$\mu_1 = \varepsilon, \quad \mu_2 = \frac{1}{\varepsilon}, \quad \text{for } d_1 < h_k < d_3, \quad \varepsilon = \frac{d_3 - h_k}{h_k - d_1}$$

$$\mu_1 = 0, \quad \mu_2 = -1, \quad \text{for } h_k = d_3.$$

Proof. Firstly, we prove that inequality (29) is true.

Case 1. When $h_k = d_1$ or $h_k = d_3$, one has $\xi_1^T(k) \mathcal{R}_l \xi_1(k) = 0$ or $\xi_3^T(k) \mathcal{R}_l \xi_3(k) = 0$, respectively. Then inequality (29) is reduced to a common discrete-time Jensen inequality (see Lemma 1 in [38]).

Case 2. When $d_1 < h_k < d_3$, applying discrete-time Jensen inequality [38], we have

$$-(d_3 - d_1) \sum_{\theta=k-d_3}^{k-d_1-1} \delta^T(\theta) R_l \delta(\theta)$$

$$= -(d_3 - d_1) \sum_{\theta=k-h_k}^{k-d_1-1} \delta^T(\theta) R_l \delta(\theta)$$

$$- (d_3 - d_1) \sum_{\theta=k-d_3}^{k-h_k-1} \delta^T(\theta) R_l \delta(\theta)$$

$$= -\frac{d_3 - d_1}{h_k - d_1} (h_k - d_1) \sum_{\theta=k-h_k}^{k-d_1-1} \delta^T(\theta) R_l \delta(\theta)$$

$$- \frac{d_3 - d_1}{d_3 - d_k} (d_3 - d_k) \sum_{\theta=k-d_3}^{k-h_k-1} \delta^T(\theta) R_l \delta(\theta)$$

$$- \left(1 + \frac{1}{\varepsilon}\right) \left(\sum_{\theta=k-d_3}^{k-h_k-1} \delta(\theta)\right)^T R_l \left(\sum_{\theta=k-d_3}^{k-h_k-1} \delta(\theta)\right)$$

$$= -(1 + \varepsilon) (h_k - d_1) \sum_{\theta=k-h_k}^{k-d_1-1} \delta^T(\theta) R_l \delta(\theta)$$

$$- \left(1 + \frac{1}{\varepsilon}\right) (d_3 - d_k) \sum_{\theta=k-d_3}^{k-h_k-1} \delta^T(\theta) R_l \delta(\theta)$$

$$\leq -(1 + \varepsilon) \left(\sum_{\theta=k-h_k}^{k-d_1-1} \delta(\theta)\right)^T R_l \left(\sum_{\theta=k-h_k}^{k-d_1-1} \delta(\theta)\right).$$
(33)

From (33), inequality (29) is true.

Secondly, we prove inequality (30) is true. Similarly, the proof is based on the above two cases. Under Case 1, it is easy

to see from (31) that (30) is true. Under Case 2, let $Y(k) = \xi_1^T(k)\mathcal{R}_1\xi_1(k) + \xi_3^T(k)\mathcal{R}_1\xi_3(k)$. If $Y(k) > 0$, multiplying the first and second rows of (31) by $\varepsilon/2$ and $1/2\varepsilon$, respectively, and summing them up, we have

$$\frac{Y(k)}{2}\varepsilon + \varepsilon\xi_1^T(k)\mathcal{R}_1\xi_1(k) + \frac{Y(k)}{2\varepsilon} + \frac{\xi_3^T(k)\mathcal{R}_1\xi_3(k)}{\varepsilon} < 0. \tag{34}$$

Then based on the basic inequality $(Y(k)/2)\varepsilon + Y(k)/2\varepsilon \geq Y(k)$, (30) is readily obtained. If $Y(k) \leq 0$, since $\mathcal{R}_l \leq 0$, it is clear that (30) is true. This completes the proof. \square

In the following discussion, in order to apply Lemma 5, define $h_k = d_k + \tau_m$; then from (23), one has $d_1 \triangleq \tau_m \leq h_k \leq d_3 \triangleq \tau_m + d_M$. Equations (27) and (26) can be rewritten as

$$e^T(k)W_1e(k) \leq \sigma[x(k-h_k) + e(k)]^T \times W_2[x(k-h_k) + e(k)], \tag{35}$$

$$x(k+1) = \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j [A_i x(k) + B_i K_j x(k-h_k) + B_i K_j e(k)], \tag{36}$$

$$k \in [k_s + \tau_{k_s}, k_{s+1} + \tau_{k_{s+1}} - 1],$$

respectively, where $x(k) = \phi(k)$ ($k \in [-d_3, 0]$) denotes the initial condition.

Theorem 6. For given positive integers d_1 and d_3 satisfying $1 \leq d_1 < d_3 < \infty$, a scalar $\sigma \geq 0$, and K_j ($j \in \mathcal{S}$), the closed-loop system (36) is asymptotically stable under the event-triggering scheme (27), if there exist matrices $P > 0$, $Q_l > 0$, $R_l > 0$ ($l = 1, 2, 3$), $W_1 > 0$, and $W_2 > 0$ with appropriate dimensions satisfying

$$\Xi^{ii}(m, n) = \begin{bmatrix} \Xi_{11}^{ii} + \Gamma_{11}^{ii}(m, n) & * \\ \Xi_{21}^{ii} & \Xi_{22}^{ii} \end{bmatrix} < 0, \quad i \in \mathcal{S}, \tag{37}$$

$$\Xi^{ij}(m, n) = \begin{bmatrix} \Xi_{11}^{ij} + \Xi_{11}^{ji} + 2\Gamma_{11}^{ij}(m, n) & * & * \\ \Xi_{21}^{ij} & \Xi_{22}^{ij} & * \\ \Xi_{21}^{ji} & 0 & \Xi_{22}^{ji} \end{bmatrix} < 0, \tag{38}$$

$$1 \leq i < j \leq r, \quad m, n = 1, 2,$$

where

$$\Xi_{11}^{ij} = \begin{bmatrix} \Gamma_{11}^i & * & * & * & * & * \\ R_1 & -Q_1 - R_1 & * & * & * & * \\ 0 & 0 & -Q_2 & * & * & * \\ K_j^T B_i^T P & 0 & 0 & \sigma W_2 & * & * \\ 0 & 0 & 0 & 0 & -Q_3 & * \\ K_j^T B_i^T P & 0 & 0 & \sigma W_2 & 0 & \sigma W_2 - W_1 \end{bmatrix},$$

$$\Gamma_{11}^i = P(A_i - I) + (A_i - I)^T P + \sum_{l=1}^3 Q_l - R_1,$$

$$\Xi_{21}^{ij} = [\mathcal{A}_{ij}^T P \quad d_1 \mathcal{A}_{ij}^T R_1 \quad (d_2 - d_1) \mathcal{A}_{ij}^T R_2 \quad (d_3 - d_2) \mathcal{A}_{ij}^T R_3]^T,$$

$$\Xi_{22} = \text{diag}\{-P, -R_1, -R_2, -R_3\},$$

$$\mathcal{A}_{ij} = [A_i - I \quad 0 \quad 0 \quad B_i K_j \quad 0 \quad B_i K_j],$$

$$\Gamma_{11}(1, 1) = \begin{bmatrix} 0 & * & * & * & * & * \\ 0 & -3R_2 & * & * & * & * \\ 0 & 0 & -R_2 - R_3 & * & * & * \\ 0 & 3R_2 & R_2 & -4R_2 & * & * \\ 0 & 0 & R_3 & 0 & -R_3 & * \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Gamma_{11}(1, 2) = \begin{bmatrix} 0 & * & * & * & * & * \\ 0 & -R_2 & * & * & * & * \\ 0 & 0 & -3R_2 - R_3 & * & * & * \\ 0 & R_2 & 3R_2 & -4R_2 & * & * \\ 0 & 0 & R_3 & 0 & -R_3 & * \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Gamma_{11}(2, 1) = \begin{bmatrix} 0 & * & * & * & * & * \\ 0 & -R_2 & * & * & * & * \\ 0 & R_2 & -R_2 - R_3 & * & * & * \\ 0 & 0 & R_3 & -4R_3 & * & * \\ 0 & 0 & 0 & 3R_3 & -3R_3 & * \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Gamma_{11}(2, 2) = \begin{bmatrix} 0 & * & * & * & * & * \\ 0 & -R_2 & * & * & * & * \\ 0 & R_2 & -R_2 - 3R_3 & * & * & * \\ 0 & 0 & 3R_3 & -4R_3 & * & * \\ 0 & 0 & 0 & R_3 & -R_3 & * \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(39)

Proof. Choose a Lyapunov functional candidate as

$$V(k) = V_1(k) + V_2(k) + V_3(k), \tag{40}$$

where

$$V_1(k) = x^T(k)Px(k),$$

$$V_2(k) = \sum_{l=1}^3 \sum_{s=k-d_l}^{k-1} x^T(s)Q_l x(s),$$

$$V_3(k) = \sum_{l=1}^3 \sum_{s=-d_l}^{-d_{l-1}-1} \sum_{\theta=k+s}^{k-1} (d_l - d_{l-1}) \delta^T(\theta) R_l \delta(\theta) \tag{41}$$

with $d_0 = 0$ and if $\text{mod}(d_3 - d_1, 2) = 0$, $d_2 = (d_1 + d_3)/2$, otherwise $d_2 = (d_1 + d_3 + 1)/2$.

Note that

$$\delta(k) = x(k+1) - x(k) = \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \mathcal{A}_{ij} \xi(k), \tag{42}$$

where \mathcal{A}_{ij} is defined in (37) and

$$\xi(k) = [x^T(k) \ x^T(k-d_1) \ x^T(k-d_2) \ x^T(k-h_k) \ x^T(k-d_3) \ e^T(k)]^T. \quad (43)$$

Taking the forward difference of (40) along the trajectory of system in (36) yields

$$\begin{aligned} \Delta V_1(k) &\leq \sum_{i,j=1}^r \mu_i \mu_j \left\{ x^T(k) [P(A_i - I) + (A_i - I)^T P] x(k) \right. \\ &\quad + 2x^T(k) P B_i K_j x(k - h_k) \\ &\quad \left. + 2x^T(k) P B_i K_j e(k) + \xi^T(k) \mathcal{A}_{ij}^T P \mathcal{A}_{ij} \xi(k) \right\}, \end{aligned} \quad (44)$$

$$\Delta V_2(k) = x^T(k) \sum_{l=1}^3 Q_l x(k) - \sum_{l=1}^3 x^T(k-d_l) Q_l x(k-d_l), \quad (45)$$

$$\begin{aligned} \Delta V_3(k) &= \sum_{l=1}^3 (d_l - d_{l-1})^2 \delta^T(k) R_l \delta(k) \\ &\quad - \sum_{l=1}^3 \sum_{\theta=k-d_l}^{k-d_{l-1}-1} (d_l - d_{l-1}) \delta^T(\theta) R_l \delta(\theta). \end{aligned} \quad (46)$$

In what follows, we will show that $\Delta V(k) < 0$ holds for both $1 \leq d_1 \leq h_k \leq d_2$ and $d_2 \leq h_k \leq d_3$.

Case 1. When $1 \leq d_1 \leq h_k \leq d_2$, from (42), we have

$$\begin{aligned} &\sum_{l=1}^3 (d_l - d_{l-1})^2 \delta^T(k) R_l \delta(k) \\ &\leq \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \xi^T(k) \mathcal{A}_{ij}^T \Theta \mathcal{A}_{ij} \xi(k), \end{aligned} \quad (47)$$

where $\Theta = d_1^2 R_1 + (d_2 - d_1)^2 R_2 + (d_3 - d_2)^2 R_3$. Using discrete Jessen inequality [39], it follows from the second term of (46) that

$$-d_1 \sum_{\theta=k-d_1}^{k-1} \delta^T(\theta) R_1 \delta(\theta) \leq \xi_4^T(k) \mathcal{R}_1 \xi_4(k) \quad (48)$$

$$-(d_3 - d_2) \sum_{\theta=k-d_3}^{k-d_2-1} \delta^T(\theta) R_3 \delta(\theta) \leq \xi_5^T(k) \mathcal{R}_3 \xi_5(k), \quad (49)$$

where $\xi_4^T(k) = [x^T(k) \ x^T(k-d_1)]$, $\xi_5^T(k) = [x^T(k-d_2) \ x^T(k-d_3)]$, and \mathcal{R}_1 and \mathcal{R}_3 are given in (29) with $l = 1$ and 3. Applying Lemma 5,

$$\begin{aligned} &-(d_2 - d_1) \sum_{\theta=k-d_2}^{k-d_1-1} \delta^T(\theta) R_2 \delta(\theta) \\ &\leq \xi_1^T(k) (1 + \lambda_1) \mathcal{R}_2 \xi_1(k) + \xi_2^T(k) (1 + \lambda_2) \mathcal{R}_2 \xi_2(k), \end{aligned} \quad (50)$$

where $\lambda_1 = (d_2 - h_k)/(h_k - d_1)$, $\lambda_2 = 1/\lambda_1$, for all $h_k \neq d_1$. From (44)–(50) and noting (35), we have

$$\begin{aligned} \Delta V(k) &\leq \Delta V_1(k) + \Delta V_2(k) + \Delta V_3(k) + \sigma [x(k-h_k) + e(k)]^T \\ &\quad \times W_2 [x(k-h_k) + e(k)] - e^T(k) W_1 e(k) \\ &\leq \sum_{i,j=1}^r \mu_i \mu_j \left\{ \xi^T(k) \left[\Xi_{11}^{ij} - (\Xi_{21}^{ij})^T \Xi_{22}^{-1} \Xi_{21}^{ij} \right] \xi(k) \right. \\ &\quad \left. + \xi_5^T(k) \mathcal{R}_3 \xi_5(k) \right\} \\ &\quad + \xi_1^T(k) (1 + \lambda_1) \mathcal{R}_2 \xi_1(k) + \xi_2^T(k) (1 + \lambda_2) \mathcal{R}_2 \xi_2(k). \end{aligned} \quad (51)$$

In order to apply Lemma 5, define

$$\begin{aligned} \lambda &= \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \left\{ \xi^T(k) \left[\Xi_{11}^{ij} - (\Xi_{21}^{ij})^T \Xi_{22}^{-1} \Xi_{21}^{ij} \right] \xi(k) \right. \\ &\quad \left. + \xi_5^T(k) \mathcal{R}_3 \xi_5(k) \right\}. \end{aligned} \quad (52)$$

Then, (51) can be rewritten as

$$\begin{aligned} \Delta V(k) &\leq \lambda + \xi_1^T(k) (1 + \lambda_1) \mathcal{R}_2 \xi_1(k) \\ &\quad + \xi_2^T(k) (1 + \lambda_2) \mathcal{R}_2 \xi_2(k). \end{aligned} \quad (53)$$

Using Lemma 5, it can be concluded that if we can prove

$$\begin{aligned} \lambda + \xi_1^T(k) 3 \mathcal{R}_2 \xi_1(k) + \xi_2^T(k) \mathcal{R}_2 \xi_2(k) &< 0, \\ \lambda + \xi_1^T(k) \mathcal{R}_2 \xi_1(k) + \xi_2^T(k) 3 \mathcal{R}_2 \xi_2(k) &< 0 \end{aligned} \quad (54)$$

then we have

$$\lambda + \xi_1^T(k) (1 + \lambda_1) \mathcal{R}_2 \xi_1(k) + \xi_2^T(k) (1 + \lambda_2) \mathcal{R}_2 \xi_2(k) < 0. \quad (55)$$

When $m = 1$ and $n = 1, 2$, by schur complement, the LMIs in (37)–(38) imply the inequalities in (54) hold. Therefore,

combining (53) and (55), we have $\Delta V(k) < 0$; that is, the closed-loop system (36) is asymptotically stable.

Case 2. When $d_2 \leq h_k \leq d_3$, inequality (48) remains unchanged. Using discrete Jessen inequality [39] and Lemma 5 to deal with the cross terms $-(d_2 - d_1) \sum_{\theta=k-d_2}^{k-d_1-1} \delta^T(\theta) R_2 \delta(\theta)$ and $-(d_3 - d_2) \sum_{\theta=k-d_3}^{k-d_2-1} \delta^T(\theta) R_3 \delta(\theta)$, respectively, then similar to the analysis method as in Case 1, we obtain

$$\begin{aligned} \Delta V(k) &\leq \bar{\lambda} + \xi_3^T(k) (1 + \bar{\lambda}_1) \mathcal{R}_3 \xi_3(k) \\ &\quad + \xi_2^T(k) (1 + \bar{\lambda}_2) \mathcal{R}_3 \xi_2(k), \end{aligned} \quad (56)$$

where $\xi_3(k)$ is defined in (31), $\bar{\lambda}_1 = (d_3 - h_k)/(h_k - d_2)$, $\bar{\lambda}_2 = 1/\bar{\lambda}_1$, for all $h_k \neq d_2, d_3$, and

$$\begin{aligned} \bar{\lambda} &= \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \left\{ \xi^T(k) \left[\Xi^{ij} - (\Xi_{21}^{ij})^T \Xi_{22}^{-1} \Xi_{21}^{ij} \right] \xi(k) \right. \\ &\quad \left. + \xi_6^T(k) \mathcal{R}_2 \xi_6(k) \right\} \end{aligned} \quad (57)$$

with $\xi_6^T(k) = [x^T(k - d_1) \quad x^T(k - d_2)]$. Applying Lemma 5 again, we have, when $m = 2$ and $n = 1, 2$, that the LMIs in (37)-(38) also guarantee the following inequality holds:

$$\bar{\lambda} + \xi_3^T(k) (1 + \bar{\lambda}_1) \mathcal{R}_3 \xi_3(k) + \xi_2^T(k) (1 + \bar{\lambda}_2) \mathcal{R}_2 \xi_2(k) < 0 \quad (58)$$

which implies $\Delta V(k) < 0$.

Based on above analysis, it can be concluded that if the LMIs in (37) and (38) hold, then system (36) is asymptotically stable for $1 \leq d_1 \leq h_k \leq d_3$. The proof is completed. \square

Remark 7. Since a new sum inequality (see Lemma 5) is used to deal with the difference terms of $\Delta V_3(k)$ in the proof of Theorem 6, neither model transformation nor bounding technique for inner product of cross terms is introduced. Although the same goal can be achieved by using the widely used free-weighting matrices method, it often leads to the case that more free matrices are introduced in the LMIs, which may increase the computational complexity. Observe that the number of LMIs in Theorem 6 is $2r(r + 1)$ and the number of scalar decision variables in Theorem 6 is $3.5n(n + 1)$. Obviously, if the free matrices are introduced, the involved variables must be larger than Theorem 6.

3.2. Controller Design. On the basis of Theorem 6, we can easily design the feedback gains K_j ($j \in \mathcal{S}$) such that closed-loop system (36) is asymptotically stable. The results are summarized as Theorem 8 below.

Theorem 8. For given scalars $d_1 \in \mathbb{Z}_+$, $d_3 \in \mathbb{Z}_+$, λ , and $\sigma \geq 0$, there exists a fuzzy state-feedback controller in the form of (13) such that the closed-loop system (36) is asymptotically stable if

there exist matrices X , $\widehat{Q}_l > 0$, $\widehat{R}_l > 0$ ($l = 1, 2, 3$), Y_j , $\widehat{W}_1 > 0$, and $\widehat{W}_2 > 0$ with appropriate dimensions satisfying

$$\begin{aligned} \widehat{\Xi}^{ii}(m, n) &= \begin{bmatrix} \widehat{\Xi}_{11}^{ii} + \widehat{\Gamma}_{11}^{ii}(m, n) & * \\ \widehat{\Xi}_{21}^{ii} & \widehat{\Xi}_{22}^{ii} \end{bmatrix} < 0, \quad i \in \mathcal{S}, \\ \widehat{\Xi}^{ij}(m, n) &= \begin{bmatrix} \widehat{\Xi}_{11}^{ij} + \Xi_{11}^{ij} + 2\widehat{\Gamma}_{11}^{ij}(m, n) & * & * \\ \widehat{\Xi}_{21}^{ij} & \widehat{\Xi}_{22}^{ij} & * \\ \widehat{\Xi}_{21}^{ji} & 0 & \widehat{\Xi}_{22}^{ij} \end{bmatrix} < 0, \quad (59) \\ &1 \leq i < j \leq r, \quad m, n = 1, 2, \end{aligned}$$

where

$$\widehat{\Xi}_{11}^{ij} = \begin{bmatrix} \widehat{\Gamma}_{11} & * & * & * & * & * \\ \widehat{R}_1 & -\widehat{Q}_1 - \widehat{R}_1 & * & * & * & * \\ 0 & 0 & -\widehat{Q}_2 & * & * & * \\ Y_j^T B_i^T & 0 & 0 & \sigma \widehat{W}_2 & * & * \\ 0 & 0 & 0 & 0 & -\widehat{Q}_3 & * \\ Y_j^T B_i^T & 0 & 0 & \sigma \widehat{W}_2 & 0 & \sigma \widehat{W}_2 - \widehat{W}_1 \end{bmatrix},$$

$$\widehat{\Gamma}_{11} = (A_i - I)X + X(A_i - I)^T + \sum_{l=1}^3 \widehat{Q}_l - \widehat{R}_1,$$

$$\widehat{\Xi}_{21}^{ij} = [\widehat{\mathcal{A}}_{ij}^T \quad d_1 \widehat{\mathcal{A}}_{ij}^T \quad (d_2 - d_1) \widehat{\mathcal{A}}_{ij}^T \quad (d_3 - d_2) \widehat{\mathcal{A}}_{ij}^T]^T,$$

$$\widehat{\mathcal{A}}_{ij} = [A_i X - X \quad 0 \quad 0 \quad B_i Y_j \quad 0 \quad B_i Y_j],$$

$$\widehat{\Xi}_{22} = \text{diag} \{-X, \lambda^2 \widehat{R}_1 - 2\lambda X, \lambda^2 \widehat{R}_2 - 2\lambda X, \lambda^2 \widehat{R}_3 - 2\lambda X\} \quad (60)$$

and $\widehat{\Gamma}_{11}(m, n)$ ($m, n = 1, 2$) are directly obtained from $\Gamma_{11}(m, n)$ by replacing the matrices R_2 and R_3 of $\Gamma_{11}(m, n)$ with \widehat{R}_2 and \widehat{R}_3 , respectively; other elements are zero. Moreover, if the above conditions are feasible, then under the event-triggering scheme (27) with $W_i = X^{-1} \widehat{W}_i X^{-1}$ ($i = 1, 2$), system (26) with feedback gains $K_j = Y_j X^{-1}$ ($j \in \mathcal{S}$) is asymptotically stable.

Proof. Defining $X = P^{-1}$, $\widehat{Q}_l = X Q_l X$, $\widehat{R}_l = X R_l X$ ($l = 1, 2, 3$), $\widehat{W}_i = X W_i X$ ($i = 1, 2$), $J_1 = \text{diag}\{X, R_1^{-1}, R_2^{-1}, R_3^{-1}\}$, and $Y_j = K_j X$ ($j \in \mathcal{S}$), then pre- and postmultiplying (37) with $\text{diag}\{\underbrace{X, \dots, X}_6, X, R_1^{-1}, R_2^{-1}, R_3^{-1}\}$ and its transpose, respectively, and pre- and postmultiplying (38) with $\text{diag}\{\underbrace{X, \dots, X}_6, J_1, J_1\}$ and its transpose, respectively, and using the inequality $-X \widehat{R}_l^{-1} X \leq \lambda^2 \widehat{R}_l - 2\lambda X$, the results (59) can be easily obtained. The proof is completed. \square

Remark 9. Note that the information of network-induced delay is involved in Theorem 8 (see the definition of parameter $d_3 \triangleq \tau_m + d_M$). Therefore, our method can be used to deal with the case that the network-induced delay exists. Moreover, due to the introduction of the event-triggering scheme (3), the release (or transmission) periods of the smart sensor data are no longer in periodic fashion. It can be seen from (3) that the parameters σ and W_i ($i = 1, 2$) can affect

the length of the release periods of the smart sensor data. In general, it is not easy to determine the scalar σ and the matrix W_i ($i = 1, 2$) in advance before solving the LMIs in Theorem 8 under some given conditions. Here, it should be pointed out that when the parameters d_1 , d_3 , and σ in Theorem 8 are known, the conditions in Theorem 8 are LMIs over the matrix variables X , $\widehat{Q}_l > 0$, $\widehat{R}_l > 0$ ($l = 1, 2, 3$), Y_j ($j = 1, 2, \dots, r$), and $\widehat{W}_i > 0$ ($i = 1, 2$). However, when we apply Theorem 8 to compute the maximum trigger parameter σ_{\max} and the corresponding maximum allowable bound of d_3 , (59) are not strict matrix inequalities. In this case, one way to find optimal σ and d_3 is to use a numerical software like MATLAB with optimization toolbox fminsearch. Another way is to use the algorithm below.

Algorithm 10. We have the following.

Step 1. Specify the bounds of $d_3 \in [d_{3\min}, d_{3\max}]$ and $\sigma \in [\sigma_{\min}, \sigma_{\max}]$ and select suitable steps Δd for d_3 and $\Delta\sigma$ for σ .

Step 2. Solve the LMIs in (59) with specified d_3 and σ .

Step 3. If there exists a feasible solution to (59), output the feedback gains K_j , $j = 1, 2, \dots, r$, and W_i ($i = 1, 2$). Otherwise, go to Step 4.

Step 4. Change $d_3 = d_3 + \Delta d$. If $d_3 > d_{3\max}$, set $d_3 = d_{3\min}$, $\sigma = \sigma + \Delta\sigma$. If $\sigma > \sigma_{\max}$, then there is no feasible solution. Otherwise, go to Step 2.

Both that the ranges and iteration steps of d_3 and σ are important values to search the optimal values of d_3 and σ . A computation burden increases as the ranges get wider and iteration steps get smaller.

Remark 11. Note that the main results obtained in this paper are based on the quadratic Lyapunov function approach, which has been widely used in existing literatures; see, for example, [40–43]. Generally speaking, basis-dependent Lyapunov function approach can also be used to solve the problem proposed in this paper. It is seen from [40] that with the use of basis-dependent Lyapunov function, the obtained results are less conservative than with the use of quadratic Lyapunov function, while the design procedures become more complex, and the computational requirement is usually demanding. Maybe there is a tradeoff between the computational complexity and the conservatism of the obtained results. For brevity of analysis process, in this paper, we just employ the quadratic Lyapunov function approach. How to further reduce the conservatism of the results will be left for our future study.

4. Application to Mass-Spring-Damper Mechanical System

Consider a nonlinear mass-spring-damper mechanical system [38, 44]:

$$M\ddot{y}(t) + g(y(t), \dot{y}(t)) + f(y(t)) = \phi(\dot{y}(t))u(t), \quad (61)$$

where M is the mass, $u(t)$ is the force, and $f(y(t))$, $g(y(t), \dot{y}(t))$, and $\phi(\dot{y}(t))$ are the nonlinear or uncertain

terms with respect to the spring, the damper, and the input, respectively. Assume that $g(y(t), \dot{y}(t)) = c_1 y(t) + c_2 \dot{y}(t)$, $f(y(t)) = c(t)y(t)$, and $\phi(\dot{y}(t)) = 1 + c_5 \dot{y}^3(t)$, where $c(t)$ is the uncertain term within the sector $[c_3, c_4]$ and $M = 1.0$, $c_1 = 0$, $c_2 = 1$, $c_3 = 0.5$, $c_4 = 1.81$, and $c_5 = 0.13$. Similar to [38, 44], choose the state variable $x(t) = [\dot{y}(t) \ y(t)]^T$; the uncertain nonlinear system (61) can be represented by the following fuzzy model:

$$\dot{x}(t) = A(h)x(t) + B(h)u(t), \quad (62)$$

where

$$A(h) = \sum_{i=1}^2 h_i A_i, \quad B(h) = \sum_{i=1}^2 h_i B_i \quad (63)$$

with

$$\begin{aligned} A_1 &= \begin{bmatrix} -1 & -1.155 \\ 1 & 0 \end{bmatrix}, & B_1 &= \begin{bmatrix} 1.4387 \\ 0 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -1 & -1.155 \\ 1 & 0 \end{bmatrix}, & B_2 &= \begin{bmatrix} 0.5613 \\ 0 \end{bmatrix}. \end{aligned} \quad (64)$$

For simulation, choose a sampling period $T = 0.2$, and then the corresponding discrete-time model (10) can be obtained with A_1, A_2, B_1 , and B_2 given by

$$\begin{aligned} A_1 &= \begin{bmatrix} 0.7986 & -0.2078 \\ 0.1799 & 0.9784 \end{bmatrix}, & B_1 &= \begin{bmatrix} 0.3119 \\ 0.0023 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 0.7986 & -0.2078 \\ 0.1799 & 0.9784 \end{bmatrix}, & B_2 &= \begin{bmatrix} 0.1217 \\ 0.0023 \end{bmatrix}. \end{aligned} \quad (65)$$

The membership functions are chosen as $\mu_1(x_1(k)) = \sin^2(x_1(k))$ and $\mu_2(x_1(k)) = 1 - \mu_1(x_1(k))$. The purpose of this example is to design the fuzzy controller (13) with $r = 2$ such that the closed-loop system (26) with event-triggering scheme (2) is asymptotically stable.

Firstly, we show that the introduction of the parameter λ will affect the maximum allowable value of d_3 for given σ . Given $d_1 = 1, \sigma = 0.5$, using Theorem 8, the obtained maximum d_3 for different λ is shown in Table 1 (note that some other parameters are also listed in Table 1). From Table 1, it can be found that the larger the λ , the bigger the d_3 . Secondly, it should be pointed out that, according to Algorithm 10, we know the maximum of σ and the maximum of d_3 may not exist simultaneously, so the tradeoff between them needs to be considered.

Next, we will show the advantage of the utilization of the event-triggering scheme. To this end, we consider the following two cases.

Case A. When $\sigma = 0$ (i.e., under the periodic transmission scheme). Applying Theorem 8 with $\lambda = 40, \sigma = 0, d_1 = 1$,

TABLE 1: The maximum d_3 obtained for different λ with $\sigma = 0.5$, $d_1 = 1$.

λ	20	30	40	50
d_3	5	7	9	9
K_1	$\begin{bmatrix} -0.0572 & -0.0103 \end{bmatrix}$	$\begin{bmatrix} -0.0015 & -0.0004 \end{bmatrix}$	$\begin{bmatrix} -0.0012 & -0.0004 \end{bmatrix}$	$\begin{bmatrix} -0.0078 & -0.0015 \end{bmatrix}$
K_2	$\begin{bmatrix} -0.1267 & -0.0002 \end{bmatrix}$	$\begin{bmatrix} -0.0037 & -0.0004 \end{bmatrix}$	$\begin{bmatrix} -0.0169 & -0.0010 \end{bmatrix}$	$\begin{bmatrix} -1.4388 & -4.3494 \end{bmatrix}$
W	$\begin{bmatrix} 36.8200 & 12.3900 \\ 12.3900 & 26.1333 \end{bmatrix}$	$\begin{bmatrix} 40.5271 & 15.2914 \\ 15.2914 & 39.4541 \end{bmatrix}$	$10^4 \times \begin{bmatrix} 7.7602 & 3.6213 \\ 3.6213 & 8.1029 \end{bmatrix}$	$\begin{bmatrix} 453.9902 & 176.6285 \\ 176.6285 & 421.6631 \end{bmatrix}$

and $d_3 = 9$, the corresponding feedback gains, K_1, K_2 , and weighting matrices W_1 and W_2 are given by

$$\begin{aligned}
 K_1 &= \begin{bmatrix} -0.0023 & -0.0007 \end{bmatrix}, \\
 K_2 &= \begin{bmatrix} -0.0052 & -0.0008 \end{bmatrix}, \\
 W_1 &= 10^6 \times \begin{bmatrix} 6.2327 & 5.3622 \\ 5.3622 & 7.8832 \end{bmatrix}, \\
 W_2 &= 10^7 \times \begin{bmatrix} 0.8150 & 0.7012 \\ 0.7012 & 1.0309 \end{bmatrix}.
 \end{aligned} \tag{66}$$

On the other hand, if we use the event-triggering condition (7), applying Theorem 8 with small modifications under the conditions of $\lambda = 40$, $\sigma = 0$, $d_1 = 1$, and $d_3 = 9$, the corresponding feedback gains, K_1, K_2 , and weighting matrix W are given by

$$\begin{aligned}
 K_1 &= \begin{bmatrix} -0.0023 & -0.0008 \end{bmatrix}, \\
 K_2 &= \begin{bmatrix} -0.0054 & -0.0009 \end{bmatrix}, \\
 W &= 10^9 \times \begin{bmatrix} 2.0590 & 1.7728 \\ 1.7728 & 2.6052 \end{bmatrix}.
 \end{aligned} \tag{67}$$

With the initial conditions $x(0) = [1 \ -1]^T$, the state responses of the closed-loop system (26) with (65) and (66) are plotted in Figure 2. The state responses of the closed-loop system (26) with (65) and (67) are plotted in Figure 3. From Figures 2 and 3, it can be seen that the closed-loop system is asymptotically stable.

Case B. When $\sigma \neq 0$ (under the ETTS). Applying Theorem 8 with $\lambda = 40$, $\sigma = 0.5$, and $d_1 = 1$, we can obtain the maximum value of $d_3 = 9$, and the corresponding feedback gains, K_1, K_2 , and weighting matrices W_1 and W_2 are given by

$$\begin{aligned}
 K_1 &= \begin{bmatrix} -0.0024 & -0.0070 \end{bmatrix}, \\
 K_2 &= \begin{bmatrix} -0.0053 & -0.0009 \end{bmatrix}, \\
 W_1 &= 10^6 \times \begin{bmatrix} 5.1745 & 4.4536 \\ 4.4536 & 6.5479 \end{bmatrix}, \\
 W_2 &= \begin{bmatrix} 54.8856 & 26.3025 \\ 26.3025 & 57.1294 \end{bmatrix}.
 \end{aligned} \tag{68}$$

However, similar to Case A, if we use the event-triggering condition (7), applying Theorem 8 with small modifications

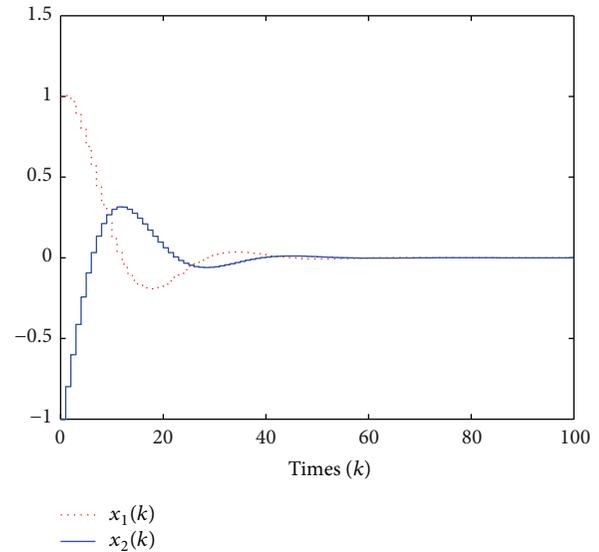


FIGURE 2: State responses for Case A under (66).

under the conditions of $\lambda = 40$, $\sigma = 0.5$, and $d_1 = 1$, one has the maximum value of $d_3 = 5$, and the corresponding feedback gains, K_1, K_2 , and weighting matrix W are given by

$$\begin{aligned}
 K_1 &= \begin{bmatrix} -0.0394 & -0.0070 \end{bmatrix}, \\
 K_2 &= \begin{bmatrix} -0.0808 & -0.0125 \end{bmatrix}, \\
 W &= \begin{bmatrix} 12.5702 & 5.0030 \\ 5.0030 & 11.8096 \end{bmatrix}.
 \end{aligned} \tag{69}$$

For this example, under the condition that $\sigma = 0.5$ and with the obtained weighting matrix in (68) and (69), the sensor data $x(k)$ is transmitted whenever the current sensor data $x(k)$ and the previously transmitted one $x(k_s)$ satisfy (2) and (7), respectively. Choose the same initial conditions as in Case A, then under the above two event-triggering schemes, the state is released (or transmitted) at a rate of approximately 4.2 and 3.2 samples per second, respectively. Figure 4 shows the state responses of the closed-loop system (26) with (65) when using the triggering scheme (2) with (68). Figure 5 shows the state responses of the closed-loop system (26) with (65) when using the triggering scheme (7) with (69). From Figures 4 and 5, the system performance degradation shown in Figure 4 is relatively small. Figure 6 shows the release instants and release interval due to the event-triggering condition

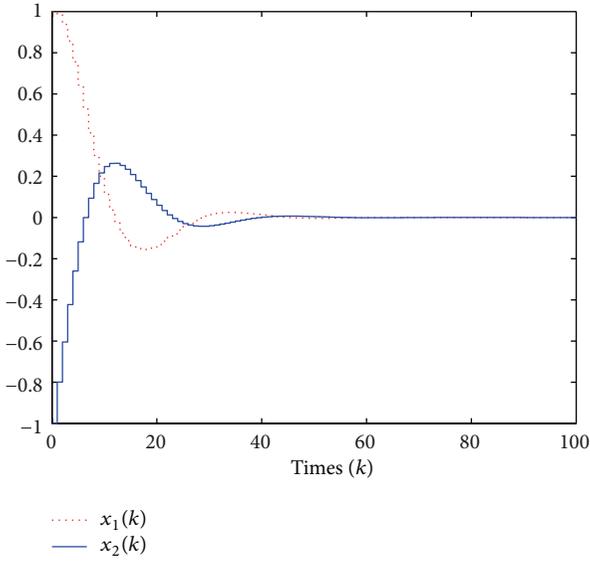


FIGURE 3: State responses for Case A under (67).

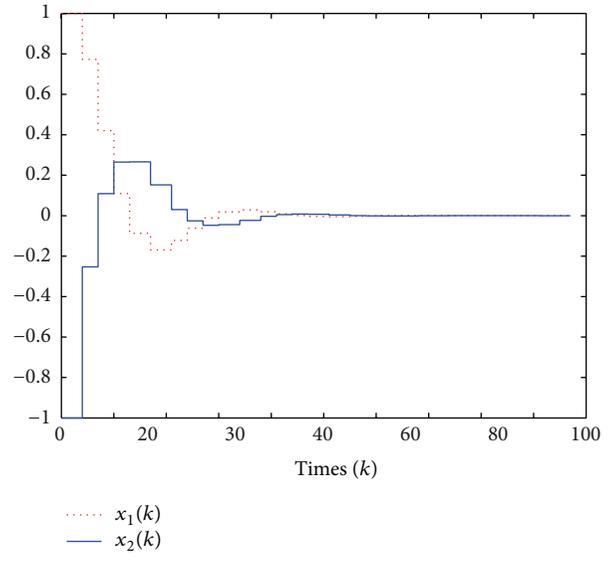


FIGURE 5: State responses for Case B under (7).

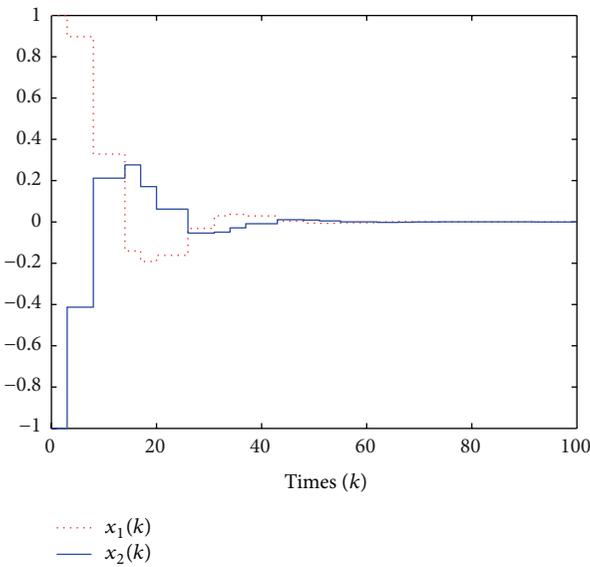


FIGURE 4: State responses for Case B under (2).

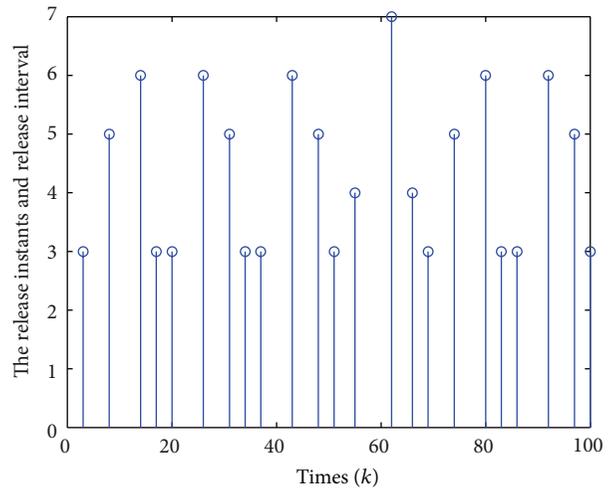


FIGURE 6: Release instants and release interval for Case B under (2).

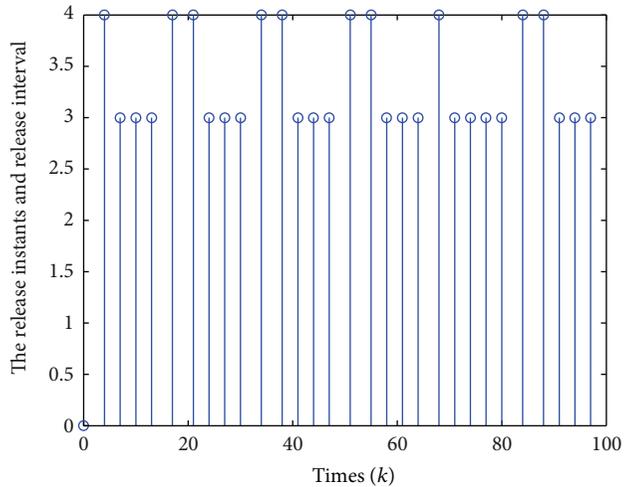


FIGURE 7: Release instants and release interval for Case B under (7).

(2). Figure 7 shows the release instants and release interval due to the event-triggering condition (7). It can be seen from Figure 6 that the number of sensor data transmissions caused by the event-triggering condition (2) is much smaller than the total number of sensor data transmissions with event-triggering condition (7) and periodic release scheme, which implies the network bandwidth is saved for other traffic. Moreover, in order to further clearly demonstrate the relationship between the parameter σ and the number of sensor data transmissions N and the average transmission rate Δ for given $d_1 = 1$, $d_3 = 5$, and $\lambda = 40$, some computation results are listed in Table 2 below (the other parameter matrices are omitted for simplicity).

TABLE 2: The number of sensor data transmissions N due to condition (7).

σ	0	0.1	0.2	0.3	0.4	0.5
N	101	51	45	37	34	30
Δ	1	1.9	2.2	2.7	2.9	3.2

Furthermore, from Table 2, by simple calculation, it is found that the sensor with event-triggering scheme ($\sigma = 0.1, 0.2, 0.3, 0.4, 0.5$) transmits only 50%, 45%, 37%, 34%, and 30% of samples produced by periodic release scheme ($\sigma = 0$), respectively. In other words, the resource utilization by the event-triggering scheme can be obtained with 50%, 55%, 63%, 66%, and 70% improvements, respectively.

Based on above analysis, it is concluded that the introduction of the event-triggering scheme in NCSs will reduce the network load while the system performance is not degraded.

5. Conclusion

This paper has presented an event-triggering scheme and a fuzzy controller codesign method for the stabilization of nonlinear NCSs and the improvement in resource utilization. By the proposed event-triggering scheme, the current sensor data is transmitted only when the current sensor value and the previously transmitted one satisfy a certain quantitative relation. Considering the effect of signal transmission delay and using the well-known Takagi-Sugeno (T-S) fuzzy model, a T-S fuzzy delay feedback control system is proposed to describe the nonlinear NCSs, and the codesign fuzzy state-feedback controller and event-triggering condition guarantee that NCSs are asymptotically stable. In contrast to the time-triggered scheme (or periodic release scheme), the proposed event-triggering scheme improves the resource utilization with acceptable performance degradation. Also, it is worth mentioning that, in this paper, data packet dropout is not considered. Simultaneous consideration of transmission delay and data packet dropout may be more meaningful since they are the main two network-induced characteristics. However, it may add much more difficulties to mathematic modeling and analysis of the NCSs, and this remains to be a future research topic. Moreover, in practice, the current state is not usually available in its entirety, and the problem of the output feedback may be tackled by using the idea proposed in [45].

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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