Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2014, Article ID 760957, 9 pages
http://dx.doi.org/10.1155/2014/760957

Research Article

Variational Iteration Transform Method for Fractional Differential Equations with Local Fractional Derivative

Yong-Ju Yang and Liu-Qing Hua

School of Mathematics and Statistics, Nanyang Normal University, Nanyang 473061, China

Correspondence should be addressed to Yong-Ju Yang; tomjohn1007@126.com

Received 25 March 2014; Revised 15 May 2014; Accepted 16 May 2014; Published 18 June 2014

Academic Editor: Ali H. Bhrawy

We propose the variational iteration transform method in the sense of local fractional derivative, which is derived from the coupling method of local fractional variational iteration method and differential transform method. The method reduces the integral calculation of the usual variational iteration computations to more easily handled differential operation. And the technique is more orderly and easier to analyze computing result as compared with the local fractional variational iteration method. Some examples are illustrated to show the feature of the presented technique.

1. Introduction

Fractional differential equation has been considered with great importance due to its demonstrated applications in various areas such as electrical networks, fluid flow, biology, and dynamical systems [1–8]. And many classical differential equations possess a fractional analogy. As a result, a substantial number of methods for solving the fractional differential equations are developed to solve fractional differential equations [9–25].

Recently, local fractional derivative and calculus theory has been introduced by the researcher in [18, 19], which is set up on fractal geometry and which is the best method for describing the nondifferential function defined on Cantor sets. The physical explanation of the local fractional derivative can be seen in [26, 27]. A great deal of research work has been directed for the nondifferentiable phenomena in fractal domain concerning local fractional derivative, for example, [11, 12, 15, 18–25].

Motivated by the ongoing research method of local fractional differential equation, we present variational iteration transform method, which is the coupling method of local fractional variational iteration method and differential transform method. It is worth mentioning that since the method makes the complex iteration calculation more orderly, it may be considered as an efficient modification of variational iteration method. The rest of the paper is organized as follows. In Section 2, the basic mathematical fundamentals are presented briefly. In Section 3, the local fractional function differential transform method for solving the differential equations with local fractional derivative is investigated. In Section 4, several examples are illustrated. Finally, in Section 5, the conclusion is given.

2. Mathematical Fundamentals

In this section, we introduce the basic notions of local fractional derivative, local fractional integral, and local fractional differential transform method.

2.1. Local Fractional Derivatives and Integrals. The local fractional derivative of $u(x)$ of order α $(0 < \alpha \leq 1)$ at x_0 is defined by [18, 19]

$$\begin{aligned} u^{(\alpha)}(x_0) &= \lim_{x \to x_0} \frac{\Delta^{\alpha}(u(x) - u(x_0))}{(x - x_0)^{\alpha}} \\ &= D_x^{(\alpha)} u(x_0) = \left. \frac{d^{\alpha} u(x)}{dx^{\alpha}} \right|_{x = x_0}, \end{aligned} \tag{1}$$

where $\Delta^{\alpha}(u(x) - u(x_0)) \cong \Gamma(1 + \alpha)(u(x) - u(x_0))$.

In the interval $[a, b]$, local fractional integral of $u(x)$ of order α is given by [18, 19]

$$\begin{aligned} {}_aI_b^{(\alpha)} u(x) &= \frac{1}{\Gamma(1+\alpha)} \lim_{\Delta t \to 0} \sum_{j=0}^{j=N-1} u(t_j)(\Delta t_j)^\alpha \\ &= \frac{1}{\Gamma(1+\alpha)} \int_a^b u(t)(dt)^\alpha, \end{aligned} \tag{2}$$

where $\Delta t_j = t_{j+1} - t_j$, $\Delta t = \max\{\Delta t_1, \Delta t_2, \Delta t_j, \ldots\}$ and $[t_j, t_{j+1}]$, $j = 0, \ldots, N-1$, and $t_0 = a$, $t_N = b$ is a partition of the interval $[a, b]$.

2.2. Local Fractional Differential Transform Method. Similarly in [28–34], the local fractional different transform $U(x,t)$ (or $u(x)$, resp.) of the function $u(x,t)$ (or $u(x)$, resp.) is defined by the following formula:

$$\begin{aligned} &U(x,k) = \frac{1}{\Gamma(1+k\alpha)} \left[\frac{\partial^{k\alpha}}{\partial t^{k\alpha}} u(x,t) \right]_{t=0} \\ &\left(\text{or } U(k) = \frac{1}{\Gamma(1+k\alpha)} \left[\frac{d^{k\alpha} u(t)}{dt^{k\alpha}} \right]_{t=0}, \text{ resp.} \right). \end{aligned} \tag{3}$$

Obviously

$$\begin{aligned} &u(x,t) = \sum_{k=0}^{\infty} \left\{ U(x,k) t^{k\alpha} \right\} \\ &\left(\text{or } u(t) = \sum_{k=0}^{\infty} \left\{ U(k) t^{k\alpha} \right\}, \text{ resp.} \right). \end{aligned} \tag{4}$$

If we let

$$\begin{aligned} &U(x,k) = DT_k \{u(x,t)\} \\ &(\text{or } U(k) = DT_k \{u(t)\}, \text{ resp.}) \end{aligned} \tag{5}$$

and let

$$\begin{aligned} &DT_k^{-1} \{U(x,k)\} = \frac{u^{(k\alpha)}(x,0)}{\Gamma(1+k\alpha)} \frac{t^{k\alpha}}{\Gamma(1+k\alpha)} \\ &\left(\text{or } DT_k^{-1} \{U(k)\} = \frac{u^{(k\alpha)}(0)}{\Gamma(1+k\alpha)} \frac{t^{k\alpha}}{\Gamma(1+k\alpha)}, \text{ resp.} \right) \end{aligned} \tag{6}$$

then we have

$$\begin{aligned} &u(x,t) = \sum_{k=0}^{\infty} DT_k^{-1} \{U(x,k)\} \\ &\left(\text{or } u(t) = \sum_{k=0}^{\infty} DT_k^{-1} \{U(k)\}, \text{ resp.} \right). \end{aligned} \tag{7}$$

In Table 1, we list a few operations concerning local fractional differential transform, where m is a nonnegative integer.

Table 1

Original function	Transformed function
$f(x) = x^{m\alpha}$	$F(k) = \begin{cases} 1, & k = m \\ 0, & k \neq m \end{cases}$
$u(x) v(x)$	$\sum_{r=0}^{k} \{U(r) V(k-r)\}$
$u^{(m\alpha)}(x)$	$\frac{\Gamma[1+(k+m)\alpha]}{\Gamma(1+k\alpha)} U(k+m); k \geq -m+1$
$f(x) = \underbrace{{}_0I_x^\alpha \cdots {}_0I_x^\alpha}_{k_0} u(\tau)$	$F(k) = \frac{\Gamma(1+(k-k_0)\alpha)}{\Gamma(1+k\alpha)} U(k-k_0)$

3. Local Fractional Variational Iteration Transform Method

To introduce the basic ideas of the variational iteration transform method, we consider the following nonlinear fractional differential equation in the sense of local fractional derivative:

$$\frac{\partial^{k_o\alpha}}{\partial t^{k_0}} T(x,t) + L_\alpha T(x,t) + R_\alpha T(x,t) = g(x,t), \tag{8}$$

where L_α is the linear local fractional operator of order less than k_0 $(k_0 \geq 2)$, R_α is the nonlinear local fractional operator of order less than k_0, and $g(x,t)$ is a source term of the nondifferential function.

According to the local fractional variational iteration method [20, 35], we can construct local fractional variational iteration algorithm:

$$\begin{aligned} T_{n+1}(x,t) = T_n(x,t) &+ {}_0I_t^{((k_o-1)\alpha)} \\ &\times \left\{ \frac{\lambda^\alpha(\tau)}{\Gamma(1+\alpha)} \left(\frac{\partial^{k_o\alpha}}{\partial t^{k_0}} T_n(x,\tau) + L_\alpha T_n(x,\tau) \right.\right. \\ &\left.\left. + R_\alpha \widetilde{T}_n(x,\tau) - g(x,\tau) \right) \right\}, \end{aligned} \tag{9}$$

where $\lambda^\alpha(\tau)/\Gamma(1+\alpha)$ is a fractal Lagrange multiplier.

Suppose $\delta^\alpha \widetilde{T}_n(x,\tau)$ is a restricted local fractional variation; that is, $\delta^\alpha \widetilde{T}_n(x,\tau) = 0$ [20, 35]; we get [21]

$$\frac{\lambda^\alpha(\tau)}{\Gamma(1+\alpha)} = \frac{(\tau - t)^{(k_0-1)\alpha}}{\Gamma[1+(k_0-1)\alpha]}. \tag{10}$$

Substituting (10) into (9), a fractional iteration procedure is constructed as follows:

$$\begin{aligned} T_{n+1}(x,t) = T_n(x,t) &+ {}_0I_t^{((k_o-1)\alpha)} \\ &\times \left\{ \frac{(x-t)^{(k_0-1)\alpha}}{\Gamma[1+(k_0-1)\alpha]} \right. \end{aligned}$$

$$\times\left\{\frac{\partial^{k_o\alpha}}{\partial t^{k_0}}T_n(x,\tau)+L_\alpha T_n(x,\tau)+R_\alpha T_n(x,\tau)-g(x,\tau)\right\}\Bigg\}. \tag{11}$$

Applying local fractional differential transform with respect to t on both sides of (11) and using local fractional derivative algorithm [18], we obtain

$$DT_k(T_{n+1}(x,t))=DT_k(T_0(x,t)),\quad (0\le k\le k_0-1) \tag{12}$$

and also

$$DT_k\{T_{n+1}(x,t)\}=-DT_k\left\{{}_0I_t^{(k\alpha)}\left(L_\alpha T_n(x,\tau)+R_\alpha T_n(x,\tau)-g(x,\tau)\right)\right\},\quad (k\ge k_0), \tag{13}$$

where $T_0(x,t)$ is an initial value.

By virtue of (5), (12), (13), and the operation displayed in Table 1, we can yield

$$\begin{aligned}T_{n+1}(x,t)&=-\sum_{k=k_0}^{\infty}DT_k^{-1}\left[\frac{\Gamma(1+(k-k_0)\alpha)}{\Gamma(1+k\alpha)}W_n(x,k-k_0)\right]+\sum_{k=0}^{k_0-1}DT_k^{-1}\{DT_k(T_0(x,k))\}\\&=-\sum_{k=k_0}^{\infty}DT_k^{-1}\left[\frac{\Gamma(1+(k-k_0)\alpha)}{\Gamma(1+k\alpha)}W_n(x,k-k_0)\right]+\sum_{k=0}^{k_0-1}\frac{T_0^{(k\alpha)}(x,0)}{\Gamma(1+k\alpha)}\frac{t^{k\alpha}}{\Gamma(1+k\alpha)},\end{aligned} \tag{14}$$

where

$$W_n(x,k)=DT_k^{-1}(w_n(x,t)) \tag{15}$$

and where

$$w_n(x,t)=L_\alpha T_n(x,t)+R_\alpha T_n(x,t)-g(x,t). \tag{16}$$

So, we have the exact solution of (8) in the form:

$$\begin{aligned}T(x,t)&=\lim_{n\to\infty}T_{n+1}(x,t)\\&=\lim_{n\to\infty}\left\{-\sum_{k=k_0}^{\infty}DT_k^{-1}\times\left[\frac{\Gamma(1+(k-k_0)\alpha)}{\Gamma(1+k\alpha)}W_n(x,k-k_0)\right]+\sum_{k=0}^{k_0-1}\frac{T_0^{(k\alpha)}(x,0)}{\Gamma(1+k\alpha)}\frac{t^{k\alpha}}{\Gamma(1+k\alpha)}\right\}.\end{aligned} \tag{17}$$

Now we discuss the solution of the following quadratic linear fractional differential equations which is the simple example of (8):

$$\frac{\partial^{2\alpha}T(x,t)}{\partial t^{2\alpha}}+p_1\frac{\partial^{2\alpha}T(x,t)}{\partial x^{2\alpha}}+p_2\frac{\partial^{\alpha}T(x,t)}{\partial t^{\alpha}}+p_3\frac{\partial^{\alpha}T(x,t)}{\partial x^{\alpha}}+p_4T(x,t)=0, \tag{18}$$

where p_1, p_2, p_3, and p_4 are all constant numbers.

According to (9), the correction iteration algorithm is given as follows:

$$T_{n+1}(x,t)=T_n(x,t)+{}_0I_t^{\alpha}\times\left\{\frac{(\tau-t)^{(\alpha)}}{\Gamma(1+\alpha)}\left\{L_{tt}^{(2\alpha)}T_n+p_1L_{xx}^{(2\alpha)}T_n+p_2L_t^{(\alpha)}T_n+p_3L_x^{(\alpha)}T_n+p_4T_n\right\}\right\}. \tag{19}$$

Taking local fractional differential transform with respect to t on (19), we deduce

$$\begin{aligned}&DT_k(T_{n+1}(x))=-DT_{k-2}\left(p_1L_{xx}^{(2\alpha)}T_n+p_2L_t^{(\alpha)}T_n+p_3L_x^{(\alpha)}T_n+p_4T_n\right),\quad(k\ge 2)\\&DT_k(T_{n+1}(x))=DT_k(T_0(x)),\quad(k=0,1),\end{aligned} \tag{20}$$

where

$$T_n(x,t)=u_n(t)f_n(x) \tag{21}$$

and where

$$U_n(k)=DT_k(u_n(t)). \tag{22}$$

By using (20), (21), (22), and the results included in Table 1, we obtain

$$\begin{aligned}U_{n+1}(k)f_{n+1}(x)&=-\left(p_1f_n^{(2\alpha)}(x)+p_3f_n^{(\alpha)}(x)+p_4f_n(x)\right)\times DT_k\left\{{}_0I_t^{(2\alpha)}(u_n)\right\}-p_2f_n(x)DT_k\left\{{}_0I_t^{(\alpha)}(u_n)\right\}\\&=-\left(p_1f_n^{(2\alpha)}(x)+p_3f_n^{(\alpha)}(x)+p_4f_n(x)\right)\frac{U(k-2)}{k\alpha(k-1)\alpha}-p_2f_n(x)\frac{U(k-1)}{k\alpha},\quad(k\ge 2)\end{aligned} \tag{23}$$

and also

$$U_{n+1}(k)f_{n+1}(x)=U_0(k)f_{n+1}(x),\quad(0\le k\le 1). \tag{24}$$

If we let

$$p_1 f_0^{(2\alpha)}(x) + p_3 f_0^{(\alpha)}(x) + p_4 f_0(x) = p_5 f_0(x), \quad (25)$$

then

$$f_n(x) = f_0(x). \quad (26)$$

By (23), (24), and (26), we have

$$U_{n+1}(k) = -p_5 \frac{U_n(k-2)}{k\alpha(k-1)\alpha} - p_2 \frac{U_n(k-1)}{k\alpha}, \quad (k \geq 2)$$
$$U_{n+1}(k) = U_0(k), \quad (k = 0, 1). \quad (27)$$

According to (14) and (27), we get

$$u_{n+1}(t) = \sum_{k=2}^{\infty} DT_k^{-1}\left[-p_5 \frac{U_n(k-2)}{k\alpha(k-1)\alpha} - p_2 \frac{U_n(k-1)}{k\alpha}\right] + \sum_{k=0}^{1} \frac{u_0^{(k\alpha)}(0)}{\Gamma(1+k\alpha)} \frac{t^{k\alpha}}{\Gamma(1+k\alpha)}. \quad (28)$$

Taking the limit, we arrive at

$$u(t) = \lim_{n\to\infty} u_{n+1}(t) = \lim_{n\to\infty}\left\{\sum_{k=2}^{\infty} DT_k^{-1}\left[-p_5 \frac{U_n(k-2)}{k\alpha(k-1)\alpha} - p_2 \frac{U_n(k-1)}{k\alpha}\right] + \sum_{k=0}^{1} \frac{u_0^{(k\alpha)}(0)}{\Gamma(1+k\alpha)} \frac{t^{k\alpha}}{\Gamma(1+k\alpha)}\right\}. \quad (29)$$

Hence, under the condition equation (25), the exact solution of (18) reads as follows:

$$T(x,t) = u(t) f_0(x). \quad (30)$$

4. Illustrative Examples

In porous media, there are fractals [35]. In fractal media, there are lots of mathematical physical models by the local differential fractional equation, for example, [36, 37]. In this section, we will present the solution of several equations or system of equations to assess the validity of variational iteration transform method.

Case 1. Consider the following nonlinear local fractional wave-like differential equation with variable coefficients:

$$\frac{\partial^{2\alpha} T}{\partial t^{2\alpha}} - \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{\partial^{\alpha}\left(D_x^{(\alpha)}(T) D_x^{(2\alpha)}(T)\right)}{\partial x^{\alpha}} + \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\left(\frac{\partial^{2\alpha} T}{\partial x^{2\alpha}}\right)^2 + T = 0, \quad (t > 0,\ 0 < x < 1), \quad (31)$$

where

$$T = T(x,t), \quad (32)$$

subject to the initial conditions

$$T(x,0) = 0, \qquad \frac{\partial^{\alpha}}{\partial t^{\alpha}} T(x,0) = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}. \quad (33)$$

From (33) we take the initial value, which reads

$$T_0(x,t) = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{t^{\alpha}}{\Gamma(1+\alpha)}. \quad (34)$$

By using (31), we structure a local fractional iteration procedure as follows:

$$T_{n+1} = T_n + {}_0I_t^{(\alpha)} \times \left\{\frac{(\tau - t)^{\alpha}}{\Gamma(1+\alpha)}\left(\frac{\partial^{2\alpha} T_n}{\partial t^{2\alpha}} - \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \times \frac{\partial^{\alpha}\left(D_x^{(\alpha)}(T_n) D_x^{(2\alpha)}(T_n)\right)}{\partial x^{\alpha}} + \frac{x^{\alpha}}{\Gamma(1+\alpha)}\left(\frac{\partial^{2\alpha} T_n}{\partial x^{2\alpha}}\right)^2 + T_n\right)\right\}, \quad (35)$$

where

$$T_n = T_n(x,t). \quad (36)$$

Let

$$w_n(x,\tau) = -\frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{\partial^{\alpha}\left(D_x^{(\alpha)}(T_n) D_x^{(2\alpha)}(T_n)\right)}{\partial x^{\alpha}} + \frac{x^{\alpha}}{\Gamma(1+\alpha)}\left(\frac{\partial^{2\alpha} T_n}{\partial x^{2\alpha}}\right)^2 + T_n. \quad (37)$$

According to (14), we can derive

$$\begin{aligned} T_1(x,t) &= \sum_{k=0}^{1} \frac{T_0^{(k\alpha)}(x,0)}{\Gamma(1+k\alpha)} \frac{t^{k\alpha}}{\Gamma(1+k\alpha)} - \sum_{k=2}^{\infty} DT_k^{-1}\left[\frac{1}{k\alpha\cdot(k-1)\alpha} W_0(x,k-2)\right] \\ &= T_0(x,t) - \sum_{k=2}^{\infty} DT_k^{-1}\left\{\frac{1}{k\alpha\cdot(k-1)\alpha} W_0(x,k-2)\right\} \\ &= T_0(x,t) - DT_2^{-1}\left(\frac{W_0(x,1)}{\Gamma(1+3\alpha)}\right) \\ &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\left(\frac{t^{\alpha}}{\Gamma(1+\alpha)} - \frac{t^{3\alpha}}{\Gamma(1+3\alpha)}\right); \end{aligned}$$

$$T_2(x,t) = \sum_{k=0}^{1} \frac{T_0^{(k\alpha)}(x,0)}{\Gamma(1+k\alpha)} \frac{t^{k\alpha}}{\Gamma(1+k\alpha)} - \sum_{k=2}^{\infty} DT_k^{-1}\left[\frac{1}{k\alpha\cdot(k-1)\alpha} W_1(x,k-2)\right]$$
$$= T_1(x,t) - DT_4^{-1}\left\{\frac{1}{5\alpha\cdot 4\alpha} W_1(x,3)\right\}$$
$$= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\left(\frac{t^{\alpha}}{\Gamma(1+\alpha)} - \frac{t^{3\alpha}}{\Gamma(1+3\alpha)} + \frac{t^{5\alpha}}{\Gamma(1+5\alpha)}\right);$$
$$\vdots \quad (38)$$

Proceeding in this manner, we get

$$T_n(x,t) = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\left(\frac{t^{\alpha}}{\Gamma(1+\alpha)} - \frac{t^{3\alpha}}{\Gamma(1+3\alpha)} + \frac{t^{5\alpha}}{\Gamma(1+5\alpha)} - \cdots + (-1)^{n+1}\frac{t^{(2n-1)\alpha}}{\Gamma(1+(2n-1)\alpha)}\right). \quad (39)$$

Thus, the exact solution of (31) is given in the form:

$$T(x,t) = \lim_{n\to\infty} T_n(x,t) = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \times \left(\frac{t^{\alpha}}{\Gamma(1+\alpha)} - \frac{t^{3\alpha}}{\Gamma(1+3\alpha)} + \frac{t^{5\alpha}}{\Gamma(1+5\alpha)} - \cdots + (-1)^{n+1}\frac{t^{(2n-1)\alpha}}{\Gamma(1+(2n-1)\alpha)} + \cdots\right)$$
$$= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \sin_\alpha(t^\alpha). \quad (40)$$

Case 2. Consider the following local fractional equation:

$$\frac{\partial^{2\alpha} T(x,t)}{\partial t^{2\alpha}} + 2^{\alpha}\frac{\partial^{2\alpha} T(x,t)}{\partial x^{2\alpha}} + 9^{\alpha} T(x,t) = 0 \quad (41)$$

subject to the initial conditions

$$T(x,0) = \sin_\alpha(2x^\alpha), \quad \frac{\partial^\alpha}{\partial t^\alpha} T(x,0) = 0. \quad (42)$$

According to (41), we structure the following iteration formula:

$$T_{n+1}(x,t) = T_n(x,t) + {}_0I_t^{(\alpha)} \times \left\{\frac{(\tau-t)^\alpha}{\Gamma(1+\alpha)}\left(\frac{\partial^{2\alpha} T(x,\tau)}{\partial t^{2\alpha}} + 2^{\alpha}\frac{\partial^{2\alpha} T(x,\tau)}{\partial x^{2\alpha}} + 9^{\alpha} T(x,\tau)\right)\right\}. \quad (43)$$

By using (25) and (42), we take an initial value as

$$T_0(x,t) = \sin_\alpha(2x^\alpha). \quad (44)$$

According to (23) and (43), we can get

$$U_{n+1}(k) = -\frac{U_n(k-2)}{k\alpha\cdot(k-1)\alpha}, \quad (k \geq 2) \quad (45)$$

and moreover

$$T_{n+1}(x,t) = \sin_\alpha(2x^\alpha)\sum_{k=0}^{1} \frac{u_0^{(k\alpha)}(x,0)}{\Gamma(1+k\alpha)} \frac{t^{k\alpha}}{\Gamma(1+k\alpha)} + \sin_\alpha(2x^\alpha)\sum_{k=2}^{\infty} DT_k^{-1}\left(\frac{-U_n(k-2)}{(k-1)\alpha\cdot k\alpha}\right)$$
$$= T_0(x,t) + \sin_\alpha(2x^\alpha)\sum_{k=2}^{\infty} DT_k^{-1}\left(\frac{-U_n(k-2)}{(k-1)\alpha\cdot k\alpha}\right). \quad (46)$$

Thus we have

$$T_1(x,t) = T_0(x,t) + \sin_\alpha(2x^\alpha)\sum_{k=2}^{\infty} DT_k^{-1}\left(\frac{-U_0(k-2)}{(k-1)\alpha\cdot k\alpha}\right)$$
$$= \sin_\alpha(2x^\alpha) - \sin_\alpha(2x^\alpha) DT_2^{-1}\left(\frac{U_0(0)}{\alpha\cdot 2\alpha}\right)$$
$$= \sin_\alpha(2x^\alpha)\left(1^\alpha - \frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right);$$
$$T_2(x,t) = T_0(x,t) + \sin_\alpha(2x^\alpha)\sum_{k=2}^{\infty} DT_k^{-1}\left(\frac{-U_1(k-2)}{(k-1)\alpha\cdot k\alpha}\right)$$
$$= \sin_\alpha(2x^\alpha)\left(1^\alpha - \frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right) - DT_4^{-1}\left(E_\alpha(x^\alpha)\frac{U_1(2)}{4\alpha\cdot 3\alpha}\right)$$
$$= \sin_\alpha(2x^\alpha)\left(1^\alpha - \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{t^{4\alpha}}{\Gamma(1+4\alpha)}\right);$$
$$T_3(x,t) = \sin_\alpha(2x^\alpha) \times \left(1^\alpha - \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{t^{4\alpha}}{\Gamma(1+4\alpha)} - \frac{t^{6\alpha}}{\Gamma(1+6\alpha)}\right);$$
$$\vdots \quad (47)$$

Continuing in this manner, we obtain

$$T_n(x,t) = \sin_\alpha(2x^\alpha)\left(\sum_{k=0}^{\infty} \frac{(-1)^k t^{2k\alpha}}{\Gamma(1+2k\alpha)}\right). \quad (48)$$

Thus, the expression of the exact solution of (41) is given by

$$\begin{aligned} T(x,t) &= \lim_{n\to\infty} T_n(x,t) \\ &= \sin_\alpha(2x^\alpha)\left(\sum_{k=0}^{\infty}\frac{(-1)^k t^{2k\alpha}}{\Gamma(1+2k\alpha)}\right) \\ &= \sin_\alpha(2x^\alpha)\cos_\alpha(-t^\alpha). \end{aligned} \tag{49}$$

According to the ideas of local fractional variational iteration transform method, we can also deal with fractional differential equation system with a similar method.

Case 3. Consider the system of local fractional coupled Helmholtz equations

$$\begin{aligned} &\frac{\partial^{2\alpha}u_1(x,t)}{\partial t^{2\alpha}} + \frac{\partial^{2\alpha}u_2(x,t)}{\partial x^{2\alpha}} - u_1(x,t) = 0, \\ &\frac{\partial^{2\alpha}u_2(x,t)}{\partial t^{2\alpha}} + \frac{\partial^{2\alpha}u_1(x,t)}{\partial x^{2\alpha}} - u_2(x,t) = 0 \end{aligned} \tag{50}$$

with the initial conditions

$$\frac{\partial^\alpha}{\partial t^\alpha}u_1(x,0) = E_\alpha(x^\alpha), \qquad u_1(x,0) = 0. \tag{51}$$

Now we can structure the similar local fractional iteration procedure.

Using (51), we take an initial value as

$$u_{1,0}(x,t) = \frac{t^\alpha E_\alpha(x^\alpha)}{\Gamma(1+\alpha)}. \tag{52}$$

According to variational iteration method [38], we can similarly construct local fractional variational iteration formula

$$\begin{aligned} u_{1,n+1}(x,t) &= u_{1,n}(x,t) \\ &+ {}_0I_t^{(\alpha)}\left\{\frac{(\tau-t)^\alpha}{\Gamma(1+\alpha)}\left(\frac{\partial^{2\alpha}u_{1,n}(x,\tau)}{\partial\tau^{2\alpha}} + \frac{\partial^{2\alpha}u_{2,n}(x,\tau)}{\partial x^{2\alpha}} - u_{1,n}\right)\right\}, \\ u_{2,n+1}(x,t) &= u_{2,n}(x,t) \\ &+ {}_0I_t^{(\alpha)}\left\{\frac{(\tau-t)^\alpha}{\Gamma(1+\alpha)}\left(\frac{\partial^{2\alpha}u_{2,n}(x,\tau)}{\partial\tau^{2\alpha}} + \frac{\partial^{2\alpha}u_{1,n}(x,\tau)}{\partial x^{2\alpha}} - u_{2,n}\right)\right\}. \end{aligned} \tag{53}$$

Applying local fractional differential transform and integral transformation on (53), we can easily get

$$\begin{aligned} &DT_k\{u_{1,n+1}(x,t)\} \\ &\quad = -DT_k\left\{{}_0I_t^{(\alpha)}\ {}_0I_t^{(\alpha)}\left\{\frac{\partial^{2\alpha}u_{2,n}(x,\tau)}{\partial x^{2\alpha}} - u_{1,n}(x,\tau)\right\}\right\}, \\ &DT_k\{u_{2,n+1}(x,t)\} \\ &\quad = -DT_k\left\{{}_0I_t^{(\alpha)}\ {}_0I_t^{(\alpha)}\left\{\frac{\partial^{2\alpha}u_{1,n}(x,\tau)}{\partial x^{2\alpha}} - u_{2,n}(x,\tau)\right\}\right\}, \\ &\qquad (k \ge 2). \end{aligned} \tag{54}$$

Analyzing (52) and (54), we can deduce

$$\begin{aligned} &DT_k\{u_{1,n+1}(x,t)\} = -DT_k\{u_{2,n+1}(x,t)\}, \\ &DT_k\{u_{2,n+1}(x,t)\} = 2DT_k\left\{{}_0I_t^{(\alpha)}\ {}_0I_t^{(\alpha)}(u_{2,n}(x,t))\right\}. \end{aligned} \tag{55}$$

According to (54) and (55), we have

$$U_{n+1}(x,k) = 2\frac{U_n(x,k-2)}{k\alpha\cdot(k-1)\alpha}, \quad (k \ge 2). \tag{56}$$

From (52), we can deduce

$$\begin{aligned} &U_{n+1}(x,k) = 2\frac{U_n(x,k-2)}{k\alpha\cdot(k-1)\alpha} \quad \left(k \ge 2,\ k = 2k'-1,\ k' \in N\right), \\ &U_{n+1}(x,k) = 0, \quad \left(k \ge 2,\ k = 2k',\ k' \in N\right). \end{aligned} \tag{57}$$

Then we get

$$\begin{aligned} u_{1,n+1}(x,t) &= \sum_{k=2}^{\infty} DT_k^{-1}\left(\frac{2U_{1,n}(x,k-2)}{k\alpha(k-1)\alpha}\right) \\ &+ \sum_{k=0}^{1}\frac{u_{1,0}^{(k\alpha)}(x,k)}{\Gamma(1+k\alpha)}\frac{t^{k\alpha}}{\Gamma(1+k\alpha)}. \end{aligned} \tag{58}$$

By virtue of (58), we have

$$\begin{aligned} &u_{1,1}(x,t) \\ &\quad = \sum_{k=0}^{1}\frac{u_{1,0}^{(k\alpha)}(x,k)}{\Gamma(1+k\alpha)}\frac{t^{k\alpha}}{\Gamma(1+k\alpha)} + \sum_{k=2}^{\infty}DT_k^{-1}\left(\frac{2U_{1,0}(x,k-2)}{k\alpha\cdot(k-1)\alpha}\right) \\ &\quad = \frac{t^\alpha E_\alpha(x^\alpha)}{\Gamma(1+\alpha)} + DT_3^{-1}\left(\frac{2U_{1,0}(x,1)}{3\alpha\cdot 2\alpha}\right) \\ &\quad = E_\alpha(x^\alpha)\left(\frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{2t^{3\alpha}}{\Gamma(1+3\alpha)}\right); \end{aligned}$$

$$\begin{aligned} &u_{1,2}(x,t) \\ &\quad = \sum_{k=0}^{1}\frac{u_{1,0}^{(k\alpha)}(x,k)}{\Gamma(1+k\alpha)}\frac{t^{k\alpha}}{\Gamma(1+k\alpha)} + \sum_{k=2}^{\infty}DT_k^{-1}\left(\frac{2U_{1,1}(x,k-2)}{k\alpha(k-1)\alpha}\right) \end{aligned}$$

$$
\begin{aligned}
&= E_\alpha(x^\alpha)\left(\frac{t^\alpha}{\Gamma(1+\alpha)}+\frac{2t^{3\alpha}}{\Gamma(1+3\alpha)}\right)\\
&\quad + DT_5^{-1}\left(\frac{2U_{1,1}(x,3)}{4\alpha\cdot 5\alpha}\right)\\
&= E_\alpha(x^\alpha)\left(\frac{t^\alpha}{\Gamma(1+\alpha)}+\frac{2t^{3\alpha}}{\Gamma(1+3\alpha)}+\frac{4t^{5\alpha}}{\Gamma(1+5\alpha)}\right);\\
&u_{1,3}(x,t)\\
&= E_\alpha(x^\alpha)\left(\frac{t^\alpha}{\Gamma(1+\alpha)}+\frac{2t^{3\alpha}}{\Gamma(1+3\alpha)}\right.\\
&\qquad\left.+\frac{4t^{5\alpha}}{\Gamma(1+5\alpha)}+\frac{8t^{7\alpha}}{\Gamma(1+7\alpha)}\right);\\
&\vdots
\end{aligned}
\tag{59}
$$

Then we derive the following result:

$$u_{1,n}(x,t) = E_\alpha(x^\alpha)\left(\sum_{k=0}^{n}\frac{2^k t^{(2k+1)\alpha}}{\Gamma(1+(2k+1)\alpha)}\right). \tag{60}$$

Obviously

$$u_{2,n}(x,t) = -u_{1,n}(x,t) = -E_\alpha(x^\alpha)\left(\sum_{k=0}^{n}\frac{2^k t^{(2k+1)\alpha}}{\Gamma(1+(2k+1)\alpha)}\right). \tag{61}$$

Thus, the expression of the final solution of (50) reads as follows:

$$
\begin{aligned}
u_1(x,t) &= \lim_{n\to\infty} u_{1,n}(x,t) = E_\alpha(x^\alpha)\frac{\sinh_\alpha\left(\sqrt{2}t\right)^\alpha}{\sqrt{2}},\\
u_2(x,t) &= \lim_{n\to\infty} u_{2,n}(x,t) = -E_\alpha(x^\alpha)\frac{\sinh_\alpha\left(\sqrt{2}t\right)^\alpha}{\sqrt{2}}.
\end{aligned}
\tag{62}
$$

Case 4. Consider the system of local fractional coupled Burger's equations

$$
\begin{aligned}
&\frac{\partial^\alpha u_1(x,t)}{\partial t^\alpha}+\frac{\partial^{2\alpha}u_1(x,t)}{\partial x^{2\alpha}}-2\frac{\partial^\alpha u_1(x,t)}{\partial x^\alpha}u_1(x,t)\\
&\quad+\frac{\partial^\alpha\left[u_1(x,t)u_2(x,t)\right]}{\partial x^\alpha}=0,\\
&\frac{\partial^\alpha u_2(x,t)}{\partial t^\alpha}+\frac{\partial^{2\alpha}u_2(x,t)}{\partial x^{2\alpha}}-2\frac{\partial^\alpha u_2(x,t)}{\partial x^\alpha}u_2(x,t)\\
&\quad+\frac{\partial^\alpha\left[u_1(x,t)u_2(x,t)\right]}{\partial x^\alpha}=0
\end{aligned}
\tag{63}
$$

with the initial conditions

$$
\begin{aligned}
u_1(x,0) &= u_2(x,0) = E_\alpha(x^\alpha),\\
u_1(x,0) &= u_1(x,0) = E_\alpha(x^\alpha).
\end{aligned}
\tag{64}
$$

Now we can structure the similar local fractional iteration procedure.

Using (64), we take an initial value as

$$u_{1,0}(x,t) = u_{2,0}(x,t) = E_\alpha(t^\alpha). \tag{65}$$

According to variational iteration method [38], we can similarly construct local fractional variational iteration formula

$$
\begin{aligned}
&u_{1,n+1}(x,t)\\
&= u_{1,n}(x,t)\\
&\quad+ {}_0I_x^{(\alpha)}\left\{\frac{(\tau-x)^\alpha}{\Gamma(1+\alpha)}\left(\frac{\partial^\alpha u_{1,n}}{\partial t^\alpha}+\frac{\partial^{2\alpha}u_{1,n}}{\partial x^{2\alpha}}\right.\right.\\
&\qquad\left.\left.-2\frac{\partial^\alpha u_{1,n}}{\partial x^\alpha}u_{1,n}+\frac{\partial^\alpha(u_{1,n}u_{2,n})}{\partial x^\alpha}\right)\right\},\\
&u_{2,n+1}(x,t)\\
&= u_{2,n}(x,t)\\
&\quad+ {}_0I_x^{(\alpha)}\left\{\frac{(\tau-x)^\alpha}{\Gamma(1+\alpha)}\left(\frac{\partial^\alpha u_{2,n}}{\partial t^\alpha}+\frac{\partial^{2\alpha}u_{2,n}}{\partial x^{2\alpha}}\right.\right.\\
&\qquad\left.\left.-2\frac{\partial^\alpha u_{2,n}}{\partial x^\alpha}u_{2,n}+\frac{\partial^\alpha(u_{1,n}u_{2,n})}{\partial x^\alpha}\right)\right\}.
\end{aligned}
\tag{66}
$$

Applying local fractional differential transform and integral transformation on (66), we can easily get

$$
\begin{aligned}
&DT_k\{u_{1,n+1}(x,t)\}\\
&= -DT_k\left\{{}_0I_x^{(\alpha)}\ {}_0I_x^{(\alpha)}\right.\\
&\qquad\left.\times\left\{\frac{\partial^\alpha u_{1,n}}{\partial t^\alpha}-2\frac{\partial^\alpha u_{1,n}}{\partial x^\alpha}u_{1,n}+\frac{\partial^\alpha(u_{1,n}u_{2,n})}{\partial x^\alpha}\right\}\right\},\\
&DT_k\{u_{2,n+1}(x,t)\}\\
&= -DT_k\left\{{}_0I_x^{(\alpha)}\ {}_0I_x^{(\alpha)}\right.\\
&\qquad\left.\times\left\{\frac{\partial^\alpha u_{2,n}}{\partial t^\alpha}-2\frac{\partial^\alpha u_{2,n}}{\partial x^\alpha}u_{2,n}+\frac{\partial^\alpha(u_{1,n}u_{2,n})}{\partial x^\alpha}\right\}\right\},\\
&\qquad (k\geq 2).
\end{aligned}
\tag{67}
$$

Analyzing (64) and (67), we can deduce

$$
\begin{aligned}
DT_k\{u_{1,n+1}(x,t)\} &= DT_k\{u_{2,n+1}(x,t)\},\\
DT_k\{u_{2,n+1}(x,t)\} &= -DT_k\left\{{}_0I_x^{(\alpha)}\ {}_0I_x^{(\alpha)}(u_{2,n}(x,t))\right\}.
\end{aligned}
\tag{68}
$$

According to (67) and (68), we have

$$U_{1,n+1}(k,t) = -\frac{U_{1,n}(k-2,t)}{k\alpha\cdot(k-1)\alpha},\quad (k\geq 2). \tag{69}$$

From (64), we can deduce

$$U_{1,n+1}(k,t) = -\frac{U_{1,n}(k-2,t)}{k\alpha\cdot(k-1)\alpha} \quad \left(k\geq 2,\ k=2k',\ k'\in N\right),$$
$$U_{n+1}(k,t) = 0, \quad \left(k\geq 2,\ k=2k'-1,\ k'\in N\right). \tag{70}$$

Then we get

$$u_{1,n+1}(x,t) = \sum_{k=2}^{\infty} DT_k^{-1}\left(\frac{-U_{1,n}(k-2,t)}{k\alpha(k-1)\alpha}\right) + \sum_{k=0}^{1}\frac{u_{1,0}^{(k\alpha)}(k,t)}{\Gamma(1+k\alpha)}\frac{x^{k\alpha}}{\Gamma(1+k\alpha)}. \tag{71}$$

By virtue of (71), we have

$$\begin{aligned}
u_{1,1}(x,t) &= \sum_{k=0}^{1}\frac{u_{1,0}^{(k\alpha)}(k,t)}{\Gamma(1+k\alpha)}\frac{x^{k\alpha}}{\Gamma(1+k\alpha)} + \sum_{k=2}^{\infty}DT_k^{-1}\left(\frac{-U_{1,0}(k-2,t)}{k\alpha\cdot(k-1)\alpha}\right)\\
&= E_\alpha(t^\alpha) + DT_2^{-1}\left(\frac{-U_{1,0}(0,t)}{2\alpha\cdot\alpha}\right)\\
&= E_\alpha(t^\alpha)\left(1^\alpha - \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\right);\\
u_{1,2}(x,t) &= \sum_{k=0}^{1}\frac{u_{1,0}^{(k\alpha)}(k,t)}{\Gamma(1+k\alpha)}\frac{x^{k\alpha}}{\Gamma(1+k\alpha)} + \sum_{k=2}^{\infty}DT_k^{-1}\left(\frac{-U_{1,1}(k-2,t)}{k\alpha(k-1)\alpha}\right)\\
&= E_\alpha(t^\alpha)\left(1^\alpha - \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\right) + DT_4^{-1}\left(\frac{-U_{1,1}(2,t)}{4\alpha\cdot 3\alpha}\right)\\
&= E_\alpha(t^\alpha)\left(1^\alpha - \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{x^{4\alpha}}{\Gamma(1+4\alpha)}\right);\\
u_{1,3}(x,t) &= E_\alpha(t^\alpha)\left(1^\alpha - \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{x^{4\alpha}}{\Gamma(1+4\alpha)} - \frac{x^{6\alpha}}{\Gamma(1+6\alpha)}\right);\\
&\vdots
\end{aligned} \tag{72}$$

Then we derive the following result:

$$u_{1,n}(x,t) = E_\alpha(t^\alpha)\left(\sum_{k=0}^{n}(-1)^k\frac{x^{2k\alpha}}{\Gamma(1+2k\alpha)}\right). \tag{73}$$

Obviously

$$u_{2,n}(x,t) = u_{1,n}(x,t) = E_\alpha(t^\alpha)\left(\sum_{k=0}^{n}(-1)^k\frac{x^{2k\alpha}}{\Gamma(1+2k\alpha)}\right). \tag{74}$$

Thus, the expression of the final solution of (63) reads as follows:

$$u_1(x,t) = \lim_{n\to\infty}u_{1,n}(x,t) = E_\alpha(t^\alpha)\cos_\alpha(x^\alpha),$$
$$u_2(x,t) = \lim_{n\to\infty}u_{2,n}(x,t) = E_\alpha(t^\alpha)\cos_\alpha(x^\alpha). \tag{75}$$

5. Conclusions

In this work, we proposed the local fractional iteration transform method. The applications of the methods for solving fractional differential equations with local fractional derivative are discussed in several cases. The method provides the variational iteration formula in the form of polynomial or Maclaurin's series, which is more easily to deal with convergence and approximate problem. Of course, the new problems are beyond the scope of the present work.

Conflict of Interests

The authors declare that they have no conflict of interests regarding this paper.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (no. 609040410), the Foundation and Advanced Technology Research Program of Henan province (112300410300), and the Nanyang Normal University (NYNU2005k37). The authors would like to thank the editor and the reviewers for their constructive comments and suggestions to improve the quality of the paper.

References

[1] R. D. Driver, *Ordinary and Delay Differential Equations*, Springer, Berlin, Germany, 1977.

[2] I. Podlubny, *Fractional Differential Equations*, Academic Press, New York, NY, USA, 1999.

[3] S. G. Samko, A. A. Kilbas, and O. I. Marichev, *Fractional Integrals and Derivatives: Theory and Applications*, Gordon and Breach, Yverdon, Switzerland, 1993.

[4] A. M. Wazwaz, *Differential Equations: Methods and Applications*, Balkema, Rotterdam, The Netherlands, 2002.

[5] I. Podlubny, "Geometric and physical interpretation of fractional integration and fractional differentiation," *Fractional Calculus & Applied Analysis*, vol. 5, no. 4, pp. 367–386, 2002.

[6] N. Heymans and I. Podlubny, "Physical interpretation of initial conditions for fractional differential equations with Riemann-Liouville fractional derivatives," *Rheologica Acta*, vol. 45, no. 5, pp. 765–771, 2006.

[7] D. Baleanu, K. Diethelm, E. Scalas, and J. J. Trujillo, *Fractional Calculus: Models and Numerical Methods*, Series on Complexity,

Nonlinearity and Chaos, World Scientific Publishing, Boston, Mass, USA, 2012.

[8] H. Karami, A. Babakhani, and D. Baleanu, "Existence results for a class of fractional differential equations with periodic boundary value conditions and with delay," *Abstract and Applied Analysis*, vol. 2013, Article ID 176180, 8 pages, 2013.

[9] A. H. Bhrawy and A. S. Alofi, "The operational matrix of fractional integration for shifted Chebyshev polynomials," *Applied Mathematics Letters*, vol. 26, no. 1, pp. 25–31, 2013.

[10] D. Baleanu, A. H. Bhrawy, and T. M. Taha, "Two efficient generalized Laguerre spectral algorithms for fractional initial value problems," *Abstract and Applied Analysis*, vol. 2013, Article ID 546502, 10 pages, 2013.

[11] Y.-J. Yang, D. Baleanu, and X.-J. Yang, "A local fractional variational iteration method for laplace equation within local fractional operators," *Abstract and Applied Analysis*, vol. 2013, Article ID 202650, 6 pages, 2013.

[12] Y.-J. Yang, D. Baleanu, and X.-J. Yang, "Analysis of fractal wave equations by local fractional Fourier series method," *Advances in Mathematical Physics*, vol. 2013, Article ID 632309, 6 pages, 2013.

[13] A. H. Bhrawy and M. A. Alghamdi, "A shifted Jacobi-Gauss-Lobatto collocation method for solving nonlinear fractional Langevin equation involving two fractional orders in different intervals," *Boundary Value Problems*, vol. 2012, article 62, 2012.

[14] A. H. Bhrawy, M. M. Alghamdi, and T. M. Taha, "A new modified generalized Laguerre operational matrix of fractional integration for solving fractional differential equations on the half line," *Advances in Difference Equations*, vol. 2012, article 179, 2012.

[15] A.-M. Yang, X.-J. Yang, and Z.-B. Li, "Local fractional series expansion method for solving wave and diffusion equations on Cantor sets," *Abstract and Applied Analysis*, vol. 2013, Article ID 351057, 5 pages, 2013.

[16] A. H. Bhrawy and M. M. Al-Shomrani, "A shifted Legendre spectral method for fractional-order multi-point boundary value problems," *Advances in Difference Equations*, vol. 2012, article 8, 2012.

[17] M. Yaseen and M. Samraiz, "The modified new iterative method for solving linear and nonlinear Klein-Gordon equations," *Applied Mathematical Sciences*, vol. 6, no. 57–60, pp. 2979–2987, 2012.

[18] X. J. Yang, *Fractional Functional Analysis and Its Applications*, Asian Academic, Hong Kong, 2011.

[19] X. J. Yang, *Local Fractional Calculus and Its Applications*, World Science Publisher, New York, NY, USA, 2012.

[20] X.-J. Yang and D. Baleanu, "Fractal heat conduction problem solved by local fractional variation iteration method," *Thermal Science*, vol. 17, no. 2, pp. 625–628, 2013.

[21] C.-F. Liu, S.-S. Kong, and S.-J. Yuan, "Reconstructive schemes for variational iteration method within Yang-Laplace transform with application to fractal heat conduction problem," *Thermal Science*, vol. 17, no. 3, pp. 715–721, 2013.

[22] Y.-J. Hao, H. M. Srivastava, H. Jafari, and X.-J. Yang, "Helmholtz and diffusion equations associated with local fractional derivative operators involving the Cantorian and Cantor-type cylindrical coordinates," *Advances in Mathematical Physics*, vol. 2013, Article ID 754248, 5 pages, 2013.

[23] S.-Q. Wang, Y.-J. Yang, and H. K. Jassim, "Local fractional function decomposition method for solving inhomogeneous wave equations with local fractional derivative," *Abstract and Applied Analysis*, vol. 2014, Article ID 176395, 7 pages, 2014.

[24] M.-S. Hu, R. P. Agarwal, and X.-J. Yang, "Local fractional Fourier series with application to wave equation in fractal vibrating string," *Abstract and Applied Analysis*, vol. 2012, Article ID 567401, 15 pages, 2012.

[25] Y. Zhang, A. Yang, and X.-J. Yang, "1-D heat conduction in a fractal medium: a solution by the local fractional Fourier series method," *Thermal Science*, vol. 17, no. 3, pp. 953–956, 2013.

[26] J.-H. He, S. K. Elagan, and Z. B. Li, "Geometrical explanation of the fractional complex transform and derivative chain rule for fractional calculus," *Physics Letters A: General, Atomic and Solid State Physics*, vol. 376, no. 4, pp. 257–259, 2012.

[27] J.-H. He, "A new fractal derivation," *Thermal Science*, vol. 15, supplement 1, pp. S145–S147, 2011.

[28] A. S. V. R. Kanth and K. Aruna, "Solution of singular two-point boundary value problems using differential transformation method," *Physics Letters A: General, Atomic and Solid State Physics*, vol. 372, no. 26, pp. 4671–4673, 2008.

[29] F. Ayaz, "Solutions of the system of differential equations by differential transform method," *Applied Mathematics and Computation*, vol. 147, no. 2, pp. 547–567, 2004.

[30] S.-H. Chang and I.-L. Chang, "A new algorithm for calculating one-dimensional differential transform of nonlinear functions," *Applied Mathematics and Computation*, vol. 195, no. 2, pp. 799–808, 2008.

[31] M. El-Shahed, "Application of differential transform method to non-linear oscillatory systems," *Communications in Nonlinear Science and Numerical Simulation*, vol. 13, no. 8, pp. 1714–1720, 2008.

[32] S. Momani and V. S. Ertürk, "Solutions of non-linear oscillators by the modified differential transform method," *Computers & Mathematics with Applications*, vol. 55, no. 4, pp. 833–842, 2008.

[33] S. Momani, "Analytical approximate solutions of nonlinear oscillators by the modified decomposition method," *International Journal of Modern Physics C*, vol. 15, no. 7, pp. 967–979, 2004.

[34] Z. M. Odibat, "Differential transform method for solving Volterra integral equation with separable kernels," *Mathematical and Computer Modelling*, vol. 48, no. 7-8, pp. 1144–1149, 2008.

[35] B. Yu and J. Li, "Some fractal characters of porous media," *Fractals*, vol. 9, no. 3, pp. 365–372, 2001.

[36] V. Z. Povstenko, "Fractional heat conduction equation and associated thermal stress," *Journal of Thermal Stresses*, vol. 28, no. 1, pp. 83–102, 2005.

[37] J. Hristov, "Approximate solutions to fractional subdiffusion equations," *European Physical Journal: Special Topics*, vol. 193, no. 1, pp. 229–243, 2011.

[38] J.-H. He, "Variational iteration method for autonomous ordinary differential systems," *Applied Mathematics and Computation*, vol. 114, no. 2-3, pp. 115–123, 2000.