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Research Article

Hydrogen Production Technologies Evaluation Based on Interval-Valued Intuitionistic Fuzzy Multiattribute Decision Making Method

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We establish a decision making model for evaluating hydrogen production technologies in China, based on interval-valued intuitionistic fuzzy set theory. First of all, we propose a series of interaction interval-valued intuitionistic fuzzy aggregation operators comparing them with some widely used and cited aggregation operators. In particular, we focus on the key issue of the relationships between the proposed operators and existing operators for clear understanding of the motivation for proposing these interaction operators. This research then studies a group decision making method for determining the best hydrogen production technologies using interval-valued intuitionistic fuzzy approach. The research results of this paper are more scientific for two reasons. First, the interval-valued intuitionistic fuzzy approach applied in this paper is more suitable than other approaches regarding the expression of the decision maker's preference information. Second, the results are obtained by the interaction between the membership degree interval and the nonmembership degree interval. Additionally, we apply this approach to evaluate the hydrogen production technologies in China and compare it with other methods.

1. Introduction

One of the important parts of multicriteria decision making is intuitionistic fuzzy multiattribute decision making, and it is an important branch of operations research and management sciences. Intuitionistic fuzzy set (IFS) is a useful technique to describe the fuzziness of the world and it was characterized by membership degree and nonmembership degree [1]. Three years later, Atanassov and Gargov [2] extended the IFS to a more generalized form and introduced the interval-valued intuitionistic fuzzy set (IIFS). IIFS is characterized by the membership degree range and nonmembership degree range. Therefore, IIFS is more powerful to depict the fuzziness of the world and has been utilized in many fields, especially in decision making [3–8].

During the interval-valued intuitionistic fuzzy multicriteria decision making process, the experts often provide their evaluation information which should be aggregated by using the proper aggregation methods. Interval-valued intuitionistic fuzzy aggregation operators play an important role in multicriteria decision making. Up to now, there are many aggregation operators for IIFNs; the most basic interval-valued intuitionistic fuzzy aggregation operators are interval-valued intuitionistic fuzzy weighted average (IIFWA) operator and interval-valued intuitionistic fuzzy weighted geometric (IIFWG) operator proposed by Xu [9], based on which, a lot of extended operators are proposed by researchers, such as generalized interval-valued intuitionistic fuzzy geometric operator [10], interval-valued intuitionistic fuzzy Einstein ordered weighted geometric (I-IVIFEOWG) operator proposed by Yang and Yuan [11], induced interval-valued intuitionistic fuzzy Hamacher ordered weighted geometric (I-IVIFHOWG) operator [12], the interval-valued intuitionistic fuzzy Einstein weighted geometric operator, interval-valued intuitionistic fuzzy Einstein ordered weighted geometric operator and interval-valued intuitionistic fuzzy Einstein hybrid weighted geometric operator [13], and induced generalized interval-valued intuitionistic fuzzy hybrid Shapley averaging (IG-IVIFHSA) operator [14].

However, the basic aggregation operators IIFWA and IIFWG for aggregating IIFNs are not perfect since they cannot deal with some special cases. For example, suppose $\tilde{\alpha}_i = ([a_i, b_i], [c_i, d_i])$ $(i = 1, 2, \ldots, n)$ are a group of IIFNs, when one of the IIFNs' nonmembership degree ranges reduce to $[0, 0]$, then the nonmembership degree of the aggregated IIFN (IIFWA($\tilde{\alpha}_i$)) must be $[0, 0]$ without the consideration of other $n - 1$ nonmembership degree ranges which is unreasonable. Inspired by the idea of He et al. [15, 16], we propose some interactive interval-valued intuitionistic fuzzy aggregation operators for aggregating IIFNs which are good complement of the existing interval-valued intuitionistic fuzzy aggregation operators.

Hydrogen technologies evaluations using multicriteria decision making method is an important research area in energy management and has attracted much attention from researchers [17, 18]. Afgan et al. [19] used the multicriteria assessment technology to select the hydrogen energy systems from the performance, environment, and market criteria. McDowall and Eames [20] introduced a new methodology to assess the alternative future hydrogen energy systems for the UK. Ren et al. [21] developed a novel fuzzy multiactor decision making approach to assess the hydrogen technologies; the feature of the proposed method is that it can deal with the uncertainties and imprecision. Lee et al. [22] combined the AHP and DEA approaches and proposed a two-stage multicriteria decision making method for efficiently allocating energy R&D resources. However, IIFS is more powerful to express the uncertainties and imprecision in evaluating the hydrogen technologies which are the focus of this paper.

The remainder of this paper is organized as follows. Section 2 reviews the basic concept of interval-valued intuitionistic fuzzy set and the operations for IIFNs. Section 3 presents some new interval-valued intuitionistic fuzzy aggregation operators and numeric examples are presented. Comparative studies of these operators with the interval-valued intuitionistic fuzzy aggregation operators proposed by Xu [9] are illustrated. In Section 4, we develop a decision making method for dealing with interval-valued intuitionistic fuzzy information and we apply this approach to evaluate the hydrogen production technologies in China and compare it with other methods. Conclusion and the future research directions are discussed in Section 5.

2. Some Basic Concepts

Intuitionistic fuzzy set (IFS) proposed by Atanassov [1] is characterized by the ability of defining the membership degree $\mu_A(x)$ and nonmembership degree $\nu_A(x)$ of an element to a set simultaneously, and the $\mu_A(x)$ and $\nu_A(x)$ are the real numbers belonging to a set $[0, 1]$. Interval-valued intuitionistic fuzzy set (IIFS), proposed by Atanassov and Gargov [2], can express the experts' preference information more effectively since it uses the interval number instead of real number to express the membership degree and nonmembership degree. The definition of the IIFS is shown as follows.

Definition 1 (Atanassov and Gargov [2]). Let a set X be fixed; the concept of interval-valued intuitionistic fuzzy set (IIFS) $\widetilde{A}$ on X is defined as follows:

$$\widetilde{A} = \{\langle x, \tilde{\mu}_A(x), \tilde{\nu}_A(x)\rangle \mid x \in X\}, \quad (1)$$

where $\tilde{\mu}_A(x)$ and $\tilde{\nu}_A(x)$ are the degree ranges of membership and nonmembership and satisfy the following condition:

$$\tilde{\mu}_A(x) \subset [0, 1], \qquad \tilde{\nu}_A(x) \subset [0, 1]. \quad (2)$$

For convenience, an IIFN $\tilde{\alpha}$ can be denoted by $([a, b], [c, d])$, where

$$[a, b] \subset [0, 1], \qquad [c, d] \subset [0, 1], \quad b + d \le 1. \quad (3)$$

Definition 2 (Xu [9]). Let $\tilde{\alpha}_1 = ([a_1, b_1], [c_1, d_1])$ and $\tilde{\alpha}_2 = ([a_2, b_2], [c_2, d_2])$ be any two IIFNs; then some operations of $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ can be defined as

(1) $\tilde{\alpha}_1 \oplus \tilde{\alpha}_2 = ([a_1 + a_2 - a_1 a_2, b_1 + b_2 - b_1 b_2], [c_1 c_2, d_1 d_2])$;

(2) $\tilde{\alpha}_1 \otimes \tilde{\alpha}_2 = ([a_1 a_2, b_1 b_2], [c_1 + c_2 - c_1 c_2, d_1 + d_2 - d_1 d_2])$;

(3) $\lambda\tilde{\alpha}_1 = ([1 - (1 - a_1)^\lambda, 1 - (1 - b_1)^\lambda], [c_1^\lambda, d_1^\lambda]), \lambda > 0$;

(4) $\tilde{\alpha}_1^\lambda = ([a_1^\lambda, b_1^\lambda], [1 - (1 - c_1)^\lambda, 1 - (1 - d_1)^\lambda]), \lambda > 0$.

Xu [9] introduced the score function $s(\tilde{\alpha}) = (1/2)(a - c + b - d)$ to get the score of $\tilde{\alpha}$ and defined an accuracy function $h(\tilde{\alpha}) = (1/2)(a + b + c + d)$ to evaluate the accuracy degree of $\tilde{\alpha}$. Xu [9] gave an order relation between two IVIFNs, $\tilde{\alpha}$ and $\tilde{\beta}$.

If $s(\tilde{\alpha}) < s(\tilde{\beta})$, then $\tilde{\alpha} < \tilde{\beta}$;

If $s(\tilde{\alpha}) = s(\tilde{\beta})$, then

(i) If $h(\tilde{\alpha}) = h(\tilde{\beta})$, then $\tilde{\alpha} = \tilde{\beta}$;

(ii) If $h(\tilde{\alpha}) < h(\tilde{\beta})$, then $\tilde{\alpha} < \tilde{\beta}$.

It should be noted that Definition 2 and the comparing laws for any IIFNs proposed by Xu [9] have been used and cited widely [3, 23–28]. In the other words, they had produced main effect to the development of IIFS theory.

3. Interval-Valued Intuitionistic Fuzzy Interactive Aggregation Operators

3.1. The New Operations for IIFNs. Though the operations defined by Xu [9] have been used and cited widely, they still have some shortcomings. The following examples illustrated this phenomenon.

Example 3. Suppose $\tilde{\alpha}_1 = \langle[0.2, 0.4], [0.0, 0.0]\rangle$, $\tilde{\alpha}_2 = \langle[0.1, 0.2], [0.2, 0.4]\rangle$, $\tilde{\alpha}_3 = \langle[0.0, 0.1], [0.5, 0.8]\rangle$, and $\tilde{\alpha}_4 = \langle[0.1, 0.3], [0.4, 0.6]\rangle$ are four IIFNs; then, using operation (1) defined in Definition 2, we can get

$$\begin{aligned}\tilde{\alpha}_1 \oplus \tilde{\alpha}_2 &= \langle[0.28, 0.52], [0.0, 0.0]\rangle, \\ \tilde{\alpha}_1 \oplus \tilde{\alpha}_3 &= \langle[0.20, 0.46], [0.0, 0.0]\rangle, \\ \tilde{\alpha}_1 \oplus \tilde{\alpha}_4 &= \langle[0.28, 0.58], [0.0, 0.0]\rangle.\end{aligned} \quad (4)$$

Example 3 shows that the nonmembership degrees range of the sum of the two IIFNs is totally decided by the nonmembership degree range of $\tilde{\alpha}_1$ without any consideration of other IIFNs, which is not reasonable in reality.

Example 4. Suppose $\tilde{\alpha}_1 = \langle[0.0, 0.0], [0.3, 0.5]\rangle$, $\tilde{\alpha}_2 = \langle[0.3, 0.5], [0.4, 0.5]\rangle$, $\tilde{\alpha}_3 = \langle[0.2, 0.7], [0.1, 0.2]\rangle$, and $\tilde{\alpha}_4 = \langle[0.5, 0.9], [0.0, 0.1]\rangle$ are four IIFNs; then, using operation (2) defined in Definition 2, we can get

$$\begin{aligned} \tilde{\alpha}_1 \otimes \tilde{\alpha}_2 &= \langle[0.0, 0.0], [0.58, 0.75]\rangle, \\ \tilde{\alpha}_1 \otimes \tilde{\alpha}_3 &= \langle[0.0, 0.0], [0.37, 0.60]\rangle, \\ \tilde{\alpha}_1 \otimes \tilde{\alpha}_4 &= \langle[0.0, 0.0], [0.30, 0.55]\rangle. \end{aligned} \tag{5}$$

Example 4 shows that the membership degrees range of the product of the two IIFNs is totally decided by the membership degree range of $\tilde{\alpha}_1$ without any consideration of other IIFNs, which is not workable.

The above analysis indicates that the definition of IIFNs introduced by Xu [9] could be improved to some extent, and we defined some new operations for IIFNs motivated by the idea of He et al. [15, 16].

Definition 5. Suppose $\tilde{\alpha} = ([a, b], [c, d])$, $\tilde{\alpha}_1 = ([a_1, b_1], [c_1, d_1])$, and $\tilde{\alpha}_2 = ([a_2, b_2], [c_2, d_2])$ are three IIFNs; some new operations were defined as follows:

(1) $\tilde{\alpha}_1 \oplus \tilde{\alpha}_2 = \langle[1-(1-a_1)(1-a_2), 1-(1-b_1)(1-b_2)], [(1-a_1)(1-a_2) - (1-(a_1+c_1))(1-(a_2+c_2)), (1-b_1)(1-b_2) - (1-(b_1+d_1))(1-(b_2+d_2))]\rangle$;

(2) $\tilde{\alpha}_1 \otimes \tilde{\alpha}_2 = \langle[(1-c_1)(1-c_2) - (1-(a_1+c_1))(1-(a_2+c_2)), (1-d_1)(1-d_2) - (1-(b_1+d_1))(1-(b_2+d_2))], [1-(1-c_1)(1-c_2), 1-(1-d_1)(1-d_2)]\rangle$;

(3) $\lambda\tilde{\alpha} = \langle[1-(1-a)^\lambda, 1-(1-b)^\lambda], [(1-a)^\lambda - (1-(a+c))^\lambda, (1-b)^\lambda - (1-(b+d))^\lambda]\rangle$;

(4) $(\tilde{\alpha})^\lambda = \langle[(1-c)^\lambda - (1-(a+c))^\lambda, (1-d)^\lambda - (1-(b+d))^\lambda], [1-(1-c)^\lambda, 1-(1-d)^\lambda]\rangle$.

Example 6. Suppose $\tilde{\alpha}_1 = \langle[0.2, 0.4], [0.0, 0.0]\rangle$, $\tilde{\alpha}_2 = \langle[0.1, 0.2], [0.2, 0.4]\rangle$, $\tilde{\alpha}_3 = \langle[0.0, 0.1], [0.5, 0.8]\rangle$, and $\tilde{\alpha}_4 = \langle[0.1, 0.3], [0.4, 0.6]\rangle$ are four IIFNs; then, using operation (1) defined in Definition 5, we can get

$$\begin{aligned} \tilde{\alpha}_1 \oplus \tilde{\alpha}_2 &= \langle[0.28, 0.52], [0.16, 0.24]\rangle, \\ \tilde{\alpha}_1 \oplus \tilde{\alpha}_3 &= \langle[0.20, 0.46], [0.40, 0.48]\rangle, \\ \tilde{\alpha}_1 \oplus \tilde{\alpha}_4 &= \langle[0.28, 0.58], [0.32, 0.36]\rangle. \end{aligned} \tag{6}$$

Example 6 shows that the drawbacks described in Example 3 disappeared.

Example 7. Suppose $\tilde{\alpha}_1 = \langle[0.0, 0.0], [0.3, 0.5]\rangle$, $\tilde{\alpha}_2 = \langle[0.3, 0.5], [0.4, 0.5]\rangle$, $\tilde{\alpha}_3 = \langle[0.2, 0.7], [0.1, 0.2]\rangle$, and $\tilde{\alpha}_4 = \langle[0.5, 0.9], [0.0, 0.1]\rangle$ are four IIFNs; then, using the operation (2) defined in Definition 5, we can get

$$\begin{aligned} \tilde{\alpha}_1 \otimes \tilde{\alpha}_2 &= \langle[0.21, 0.25], [0.58, 0.75]\rangle, \\ \tilde{\alpha}_1 \otimes \tilde{\alpha}_3 &= \langle[0.14, 0.35], [0.37, 0.60]\rangle, \\ \tilde{\alpha}_1 \otimes \tilde{\alpha}_4 &= \langle[0.35, 0.45], [0.30, 0.55]\rangle. \end{aligned} \tag{7}$$

Example 7 shows that the drawbacks described in Example 4 disappeared.

The new operational laws for IIFNs defined in Definition 5 satisfy Theorem 8.

Theorem 8. *Suppose $\tilde{\alpha} = ([a, b], [c, d])$, $\tilde{\alpha}_1 = ([a_1, b_1], [c_1, d_1])$, and $\tilde{\alpha}_2 = ([a_2, b_2], [c_2, d_2])$ are three IIFNs; the following equations are valid:*

(1) $\tilde{\alpha}_1 \oplus \tilde{\alpha}_2 = \tilde{\alpha}_2 \oplus \tilde{\alpha}_1$;

(2) $\tilde{\alpha}_1 \otimes \tilde{\alpha}_2 = \tilde{\alpha}_2 \otimes \tilde{\alpha}_1$;

(3) $\lambda(\tilde{\alpha}_1 \oplus \tilde{\alpha}_2) = \lambda\tilde{\alpha}_1 \oplus \lambda\tilde{\alpha}_2$;

(4) $(\tilde{\alpha}_1 \otimes \tilde{\alpha}_2)^\lambda = \tilde{\alpha}_1^\lambda \otimes \tilde{\alpha}_2^\lambda$;

(5) $\lambda_1\tilde{\alpha} \oplus \lambda_2\tilde{\alpha} = (\lambda_1 + \lambda_2)\tilde{\alpha}$;

(6) $\tilde{\alpha}^{\lambda_1} \otimes \tilde{\alpha}^{\lambda_2} = \tilde{\alpha}^{(\lambda_1+\lambda_2)}$.

Proof. The proof of Theorem 8 is very simple, omitted here. □

3.2. Interval-Valued Intuitionistic Fuzzy Interactive Aggregation Operators. In Section 3.1, we have introduced the new operations for IIFNs based on the analysis of the imperfections of the existing operations. The main advantage of the new operations is that it can handle the extreme cases better such as the nonmembership degree range or the membership degree range reduced to the [0, 0]. Furthermore, the new aggregation operators for IIFNs also need to be addressed. Therefore, we proposed a series of interaction interval-valued intuitionistic fuzzy aggregation operators for aggregating the IIFNs. The comparisons with the existing operators are also presented.

Definition 9. Suppose $(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n)$ is a group of IIFNs and $w = (w_1, w_2, \ldots, w_n)$ is the weight vector of them, such that $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$. Then,

$$\begin{aligned} &\mathrm{IIFIWA}(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n) \\ &\quad = \bigoplus_{j=1}^n w_j\tilde{\alpha}_j = w_1\tilde{\alpha}_1 \oplus w_2\tilde{\alpha}_2 \oplus \cdots \oplus w_n\tilde{\alpha}_n \end{aligned} \tag{8}$$

is named an interval-valued intuitionistic fuzzy interactive weighted average (IIFIWA) operator.

Theorem 10. *Suppose $(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n)$ is a group of IIFNs; then their aggregated value by using IIFIWA operator is*

$$\begin{aligned} &\mathit{IIFIWA}(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n) \\ &\quad = \bigoplus_{j=1}^n w_j\tilde{\alpha}_j \\ &\quad = \left(\left[1 - \prod_{j=1}^n (1-a_j)^{w_j}, 1 - \prod_{j=1}^n (1-b_j)^{w_j}\right],\right. \end{aligned}$$

$$\left[\prod_{j=1}^{n}\left(1-a_j\right)^{w_j}-\prod_{j=1}^{n}\left(1-\left(a_j+c_j\right)\right)^{w_j},\right.$$
$$\left.\left.\prod_{j=1}^{n}\left(1-b_j\right)^{w_j}-\prod_{j=1}^{n}\left(1-\left(b_j+d_j\right)\right)^{w_j}\right]\right), \tag{9}$$

where $w = (w_1, w_2, \ldots, w_n)^T$ *is the weight vector of* $(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n)$ *with* $w_j \in [0, 1]$ *and* $\sum_{j=1}^{n} w_j = 1$.

Proof. The mathematical induction method is applied to prove (9).

(1) When $n = 2$, according to the Definition 5, we have

$$\begin{aligned}
w_1\tilde{\alpha}_1 = {} & \left(\left[1-(1-a_1)^{w_1}, 1-(1-b_1)^{w_1}\right],\right.\\
& \left[(1-a_1)^{w_1}-(1-(a_1+c_1))^{w_1},\right.\\
& \left.\left.(1-b_1)^{w_1}-(1-(b_1+d_1))^{w_1}\right]\right),\\
w_2\tilde{\alpha}_2 = {} & \left(\left[1-(1-a_2)^{w_2}, 1-(1-b_2)^{w_2}\right],\right.\\
& \left[(1-a_2)^{w_2}-(1-(a_2+c_2))^{w_2},\right.\\
& \left.\left.(1-b_2)^{w_2}-(1-(b_2+d_2))^{w_2}\right]\right),
\end{aligned}$$

$$\begin{aligned}
& \text{IIFIWA}\left(\tilde{\alpha}_1, \tilde{\alpha}_2\right)\\
& = \bigoplus_{j=1}^{2} w_j\tilde{\alpha}_j = w_1\tilde{\alpha}_1 \oplus w_2\tilde{\alpha}_2\\
& = \left(\left[1-(1-a_1)^{w_1}, 1-(1-b_1)^{w_1}\right],\right.\\
& \quad \left[(1-a_1)^{w_1}-(1-(a_1+c_1))^{w_1},\right.\\
& \quad \left.\left.(1-b_1)^{w_1}-(1-(b_1+d_1))^{w_1}\right]\right)\\
& \quad \oplus\left(\left[1-(1-a_2)^{w_2}, 1-(1-b_2)^{w_2}\right],\right.\\
& \quad \left[(1-a_2)^{w_2}-(1-(a_2+c_2))^{w_2},\right.\\
& \quad \left.\left.(1-b_2)^{w_2}-(1-(b_2+d_2))^{w_2}\right]\right)\\
& = \left(\left[1-\prod_{j=1}^{2}\left(1-a_j\right)^{w_j}, 1-\prod_{j=1}^{2}\left(1-b_j\right)^{w_j}\right],\right.\\
& \quad \left[\prod_{j=1}^{2}\left(1-a_j\right)^{w_j}-\prod_{j=1}^{2}\left(1-\left(a_j+c_j\right)\right)^{w_j},\right.\\
& \quad \left.\left.\prod_{j=1}^{2}\left(1-b_j\right)^{w_j}-\prod_{j=1}^{2}\left(1-\left(b_j+d_j\right)\right)^{w_j}\right]\right).
\end{aligned} \tag{10}$$

This portrays (9) is valid when $n = 2$.

(2) If (8) holds for $n = k$, that is,

$$\begin{aligned}
& \text{IIFIWA}\left(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_k\right)\\
& = \bigoplus_{j=1}^{k} w_j\tilde{\alpha}_j = \left(\left[1-\prod_{j=1}^{k}\left(1-a_j\right)^{w_j}, 1-\prod_{j=1}^{k}\left(1-b_j\right)^{w_j}\right],\right.\\
& \quad \left[\prod_{j=1}^{k}\left(1-a_j\right)^{w_j}-\prod_{j=1}^{k}\left(1-\left(a_j+c_j\right)\right)^{w_j},\right.\\
& \quad \left.\left.\prod_{j=1}^{k}\left(1-b_j\right)^{w_j}-\prod_{j=1}^{k}\left(1-\left(b_j+d_j\right)\right)^{w_j}\right]\right),
\end{aligned} \tag{11}$$

then, when $n = k + 1$, we have

$$\begin{aligned}
& \text{IIFIWA}\left(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_k, \tilde{\alpha}_{k+1}\right)\\
& = \bigoplus_{j=1}^{k+1} w_j\tilde{\alpha}_j = \bigoplus_{j=1}^{k} w_j\tilde{\alpha}_j \oplus w_{k+1}\tilde{\alpha}_{k+1}\\
& = \left(\left[1-\prod_{j=1}^{k}\left(1-a_j\right)^{w_j}, 1-\prod_{j=1}^{k}\left(1-b_j\right)^{w_j}\right],\right.\\
& \quad \left[\prod_{j=1}^{k}\left(1-a_j\right)^{w_j}-\prod_{j=1}^{k}\left(1-\left(a_j+c_j\right)\right)^{w_j},\right.\\
& \quad \left.\left.\prod_{j=1}^{k}\left(1-b_j\right)^{w_j}-\prod_{j=1}^{k}\left(1-\left(b_j+d_j\right)\right)^{w_j}\right]\right)\\
& \quad \oplus\left(\left[1-(1-a_{k+1})^{w_{k+1}}, 1-(1-b_{k+1})^{w_{k+1}}\right],\right.\\
& \quad \left[(1-a_{k+1})^{w_{k+1}}-(1-(a_{k+1}+c_{k+1}))^{w_{k+1}},\right.\\
& \quad \left.\left.(1-b_{k+1})^{w_{k+1}}-(1-(b_{k+1}+d_{k+1}))^{w_{k+1}}\right]\right)\\
& = \left(\left[1-\prod_{j=1}^{k+1}\left(1-a_j\right)^{w_j}, 1-\prod_{j=1}^{k+1}\left(1-b_j\right)^{w_j}\right],\right.\\
& \quad \left[\prod_{j=1}^{k+1}\left(1-a_j\right)^{w_j}-\prod_{j=1}^{k+1}\left(1-\left(a_j+c_j\right)\right)^{w_j},\right.\\
& \quad \left.\left.\prod_{j=1}^{k+1}\left(1-b_j\right)^{w_j}-\prod_{j=1}^{k+1}\left(1-\left(b_j+d_j\right)\right)^{w_j}\right]\right).
\end{aligned} \tag{12}$$

In other words, (9) is valid when $n = k + 1$. Therefore, (9) is valid for all n. Then

$$\begin{aligned}
& \text{IIFIWA}\left(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n\right)\\
& = \bigoplus_{j=1}^{n} w_j\tilde{\alpha}_j = \left(\left[1-\prod_{j=1}^{n}\left(1-a_j\right)^{w_j}, 1-\prod_{j=1}^{n}\left(1-b_j\right)^{w_j}\right],\right.
\end{aligned}$$

$$\left[\prod_{j=1}^{n}\left(1-a_j\right)^{w_j}-\prod_{j=1}^{n}\left(1-\left(a_j+c_j\right)\right)^{w_j},\right.$$

$$\left.\left.\prod_{j=1}^{n}\left(1-b_j\right)^{w_j}-\prod_{j=1}^{n}\left(1-\left(b_j+d_j\right)\right)^{w_j}\right]\right). \tag{13}$$

It should be noted that the above proof is largely inspired by the idea of Zhao et al. [29] and He et al. [15, 16]. □

Example 11 shows the application of Theorem 10 in IIFNs aggregation problem.

Example 11. Let $\tilde{\alpha}_1 = \langle[0.1, 0.2], [0.0, 0.0]\rangle$, $\tilde{\alpha}_2 = \langle[0.2, 0.4], [0.2, 0.4]\rangle$, $\tilde{\alpha}_3 = \langle[0.2, 0.3], [0.4, 0.7]\rangle$, and $\tilde{\alpha}_4 = \langle[0.1, 0.4], [0.4, 0.6]\rangle$ be four IIFNs and $w = (0.14, 0.36, 0.32, 0.18)$ their weight. Use the IIFWA operator [9] to aggregate the four IIFNs; the result can be obtained as follows:

$$\begin{aligned}\tilde{\alpha}^* &= \text{IIFWA}\left(\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3, \tilde{\alpha}_4\right)\\ &= \left(\left[1-\prod_{j=1}^{4}\left(1-a_j\right)^{w_j}, 1-\prod_{j=1}^{4}\left(1-b_j\right)^{w_j}\right],\right.\\ &\qquad \left.\left[\prod_{j=1}^{n}\left(c_j\right)^{w_j}, \prod_{j=1}^{n}\left(d_j\right)^{w_j}\right]\right)\\ &= \langle[0.1693, 0.3438], [0, 0]\rangle.\end{aligned} \tag{14}$$

The aggregated result based on the IIFWA operator is $\langle[0.1693, 0.3438], [0, 0]\rangle$, and the nonmembership degree range is $[0, 0]$ which is totally determined by the nonmembership degree of IIFN $\tilde{\alpha}_1$. Obviously, it is unreasonable.

Utilize the IIFIWA operator (Definition 9 and Theorem 10); the aggregated result is as follows:

$$\begin{aligned}\tilde{\alpha}^* &= \text{IIFIWA}\left(\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3, \tilde{\alpha}_4\right)\\ &= \left(\left[1-\prod_{j=1}^{4}\left(1-a_j\right)^{w_j}, 1-\prod_{j=1}^{4}\left(1-b_j\right)^{w_j}\right],\right.\\ &\qquad \left[\prod_{j=1}^{4}\left(1-a_j\right)^{w_j}-\prod_{j=1}^{4}\left(1-\left(a_j+c_j\right)\right)^{w_j},\right.\\ &\qquad \left.\left.\prod_{j=1}^{4}\left(1-b_j\right)^{w_j}-\prod_{j=1}^{4}\left(1-\left(b_j+d_j\right)\right)^{w_j}\right]\right)\\ &= \langle[0.1693, 0.3438], [0.1750, 0.4799]\rangle.\end{aligned} \tag{15}$$

The aggregated result based on the IIFIWA operator is $\langle[0.1693, 0.3438], [0.1750, 0.4799]\rangle$, and the nonmembership degree range is $[0.1750, 0.4799]$ which is not totally determined by the nonmembership degree of one of the single IIFNs. Obviously, the result is more reasonable than the result obtained by IIFWA operator.

Definition 12. Suppose $(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n)$ is a group of IIFNs and $w = (w_1, w_2, \dots, w_n)$ is the weight vector of them, such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Then,

$$\text{IIFIWG}\left(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n\right) = \bigotimes_{j=1}^{n} \tilde{\alpha}_j^{w_j} = \tilde{\alpha}_1^{w_1} \otimes \tilde{\alpha}_2^{w_2} \otimes \cdots \otimes \tilde{\alpha}_n^{w_n} \tag{16}$$

is named an interval-valued intuitionistic fuzzy interactive weighted geometric (IIFIWG) operator.

Theorem 13. *Suppose $(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n)$ is a group of IIFNs; then their aggregated value by using IIFIWG operator is*

$$\begin{aligned}&\mathit{IIFIWG}\left(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n\right)\\ &= \bigotimes_{j=1}^{n} \tilde{\alpha}_j^{w_j} = \left(\left[\prod_{j=1}^{n}\left(1-c_j\right)^{w_j}-\prod_{j=1}^{n}\left(1-\left(a_j+c_j\right)\right)^{w_j},\right.\right.\\ &\qquad \left.\prod_{j=1}^{n}\left(1-d_j\right)^{w_j}-\prod_{j=1}^{n}\left(1-\left(b_j+d_j\right)\right)^{w_j}\right],\\ &\qquad \left.\left[1-\prod_{j=1}^{n}\left(1-c_j\right)^{w_j}, 1-\prod_{j=1}^{n}\left(1-d_j\right)^{w_j}\right]\right),\end{aligned} \tag{17}$$

where $w = (w_1, w_2, \dots, w_n)^T$ is the weight vector of $(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n)$ with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

Like Example 11, here we illustrate Example 14 to show the application of IIFIWG operator in aggregating the IIFNs.

Example 14. Let $\tilde{\alpha}_1 = \langle[0.0, 0.0], [0.2, 0.3]\rangle$, $\tilde{\alpha}_2 = \langle[0.1, 0.3], [0.5, 0.6]\rangle$, $\tilde{\alpha}_3 = \langle[0.2, 0.5], [0.1, 0.4]\rangle$, and $\tilde{\alpha}_4 = \langle[0.2, 0.3], [0.2, 0.6]\rangle$ be four IIFNs and $w = (0.19, 0.23, 0.34, 0.24)$ their weight. Use the IIFWG operator [9] to aggregate the four IIFNs; the result can be obtained as follows:

$$\begin{aligned}\tilde{\alpha}^* &= \text{IIFWG}\left(\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3, \tilde{\alpha}_4\right)\\ &= \left(\left[\prod_{j=1}^{n}\left(a_j\right)^{w_j}, \prod_{j=1}^{n}\left(b_j\right)^{w_j}\right],\right.\\ &\qquad \left.\left[1-\prod_{j=1}^{n}\left(1-c_j\right)^{w_j}, 1-\prod_{j=1}^{n}\left(1-d_j\right)^{w_j}\right]\right)\\ &= \langle[0, 0], [0.2526, 0.4894]\rangle.\end{aligned} \tag{18}$$

From Example 14, we can find out that the aggregated result based on the IIFWG operator is $\langle[0, 0], [0.2526, 0.4894]\rangle$ and the membership degree range is $[0, 0]$ which is totally determined by the membership degree of IIFN $\tilde{\alpha}_1$. This was obviously an unreasonable calculated result.

Based on the IIFIWG operator (Definition 12 and Theorem 13), the aggregated result is as follows:

$$\tilde{\alpha}^* = \text{IIFIWG}(\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3, \tilde{\alpha}_4)$$
$$= \left(\left[\prod_{j=1}^{4}(1-c_j)^{w_j} - \prod_{j=1}^{4}(1-(a_j+c_j))^{w_j}, \prod_{j=1}^{4}(1-d_j)^{w_j} - \prod_{j=1}^{4}(1-(b_j+d_j))^{w_j}\right], \left[1-\prod_{j=1}^{4}(1-c_j)^{w_j}, 1-\prod_{j=1}^{4}(1-d_j)^{w_j}\right]\right)$$
$$= \langle [0.1390, 0.3659], [0.2526, 0.4894]\rangle. \quad (19)$$

Obviously, the membership degree range is [0.1390, 0.3659] rather than [0, 0] and was more reasonable than the result obtained by IIFWG operator.

3.3. Interval-Valued Intuitionistic Fuzzy Interactive Ordered Weighted Operator. In many real situations, the data should be ordered before application. In many sports events, such as gymnastics and diving, the biggest and the smallest evaluation results given by the experts should be deleted and the other evaluation results will be aggregated. In these situations, the evaluation results should be ordered. OWA operator, proposed by Yager [30], is a very useful aggregation technique to deal with this situation. The OWA operator has attracted the interest of many researchers [31–44]. In the following, based on the idea of OWA operator, we extended the IIFIWA and IIFIWG operators and proposed the IIFIOWA and IIFIOWG operators.

Definition 15. Suppose is a group of IIFNs expressed as $(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n)$; the interval-valued intuitionistic fuzzy interactive ordered weighted average (IIFIOWA) operator and interval-valued intuitionistic fuzzy interactive ordered weighted geometric (IIFIOWG) operator are defined as follows:

$$\text{IIFIOWA}(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n) = \bigoplus_{j=1}^{n} \omega_j \tilde{\alpha}_{\delta(j)} = \omega_1 \tilde{\alpha}_{\delta(1)} \oplus \omega_2 \tilde{\alpha}_{\delta(2)} \oplus \cdots \oplus \omega_n \tilde{\alpha}_{\delta(n)},$$
$$\text{IIFIOWG}(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n) = \bigotimes_{j=1}^{n} \tilde{\alpha}_{\delta(j)}^{\omega_j} = \tilde{\alpha}_{\delta(1)}^{\omega_1} \otimes \tilde{\alpha}_{\delta(2)}^{\omega_2} \otimes \cdots \otimes \tilde{\alpha}_{\delta(n)}^{\omega_n}, \quad (20)$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the associated weight vector such that $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$. $\delta : (1, 2, \ldots, n) \to (1, 2, \ldots, n)$, $\tilde{\alpha}_{\delta(j)}$ is the jth largest of $\tilde{\alpha}_j$.

Theorem 16. *Suppose $(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n)$ is a group of IIFNs; then their aggregated value by using IIFIOWA operator or IIFIOWG operator is*

$$\text{IIFIOWA}(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n) = \bigoplus_{j=1}^{n} \omega_j \tilde{\alpha}_{\delta(j)}$$
$$= \left(\left[1-\prod_{j=1}^{n}(1-a_{\delta(j)})^{\omega_j}, 1-\prod_{j=1}^{n}(1-b_{\delta(j)})^{\omega_j}\right], \left[\prod_{j=1}^{n}(1-a_{\delta(j)})^{\omega_j} - \prod_{j=1}^{n}(1-(a_{\delta(j)}+c_{\delta(j)}))^{\omega_j}, \prod_{j=1}^{n}(1-b_{\delta(j)})^{\omega_j} - \prod_{j=1}^{n}(1-(b_{\delta(j)}+d_{\delta(j)}))^{\omega_j}\right]\right),$$
$$\text{IIFIOWG}(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n) = \bigotimes_{j=1}^{n} \tilde{\alpha}_j^{\omega_j}$$
$$= \left(\left[\prod_{j=1}^{n}(1-c_{\delta(j)})^{\omega_j} - \prod_{j=1}^{n}(1-(a_{\delta(j)}+c_{\delta(j)}))^{\omega_j}, \prod_{j=1}^{n}(1-d_{\delta(j)})^{\omega_j} - \prod_{j=1}^{n}(1-(b_{\delta(j)}+d_{\delta(j)}))^{\omega_j}\right], \left[1-\prod_{j=1}^{n}(1-c_{\delta(j)})^{\omega_j}, 1-\prod_{j=1}^{n}(1-d_{\delta(j)})^{\omega_j}\right]\right), \quad (21)$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector of $(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n)$ with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

Example 17. Let $\tilde{\alpha}_1 = \langle[0.2, 0.3], [0.5, 0.6]\rangle$, $\tilde{\alpha}_2 = \langle[0.5, 0.6], [0.2, 0.4]\rangle$, $\tilde{\alpha}_3 = \langle[0.3, 0.4], [0.0, 0.0]\rangle$, and $\tilde{\alpha}_4 = \langle[0.1, 0.2], [0.3, 0.5]\rangle$ be four IIFNs and $\omega = (0.14, 0.36, 0.32, 0.18)$ be the associate weight of IIFIOWA and IIFIOWG operators.

Since

$$S(\tilde{\alpha}_1) = \frac{1}{2}(0.0 + 0.0 - 0.5 - 0.6) = -0.55,$$
$$S(\tilde{\alpha}_2) = \frac{1}{2}(0.5 + 0.6 - 0.2 - 0.4) = 0.25,$$
$$S(\tilde{\alpha}_3) = \frac{1}{2}(0.3 + 0.4 - 0.0 - 0.0) = 0.35, \quad (22)$$
$$S(\tilde{\alpha}_4) = \frac{1}{2}(0.1 + 0.2 - 0.3 - 0.5) = -0.25,$$

Table 1: The interval-valued intuitionistic fuzzy decision matrix $\widetilde{A}$.

	c_1	c_2	c_3	c_4
A_1	([0.1, 0.2], [0.0, 0.0])	([0.1, 0.3], [0.5, 0.7])	([0.1, 0.2], [0.6, 0.7])	([0.0, 0.0], [0.7, 0.8])
A_2	([0.2, 0.3], [0.1, 0.2])	([0.3, 0.4], [0.2, 0.3])	([0.2, 0.3], [0.3, 0.5])	([0.1, 0.2], [0.4, 0.5])
A_3	([0.3, 0.5], [0.2, 0.3])	([0.1, 0.3], [0.4, 0.6])	([0.3, 0.4], [0.4, 0.6])	([0.2, 0.4], [0.3, 0.6])

then,

$$S(\widetilde{\alpha}_3) > S(\widetilde{\alpha}_2) > S(\widetilde{\alpha}_4) > S(\widetilde{\alpha}_1), \quad \widetilde{\alpha}_{\sigma(1)} = \widetilde{\alpha}_3, \quad \widetilde{\alpha}_{\sigma(2)} = \widetilde{\alpha}_2, \quad \widetilde{\alpha}_{\sigma(3)} = \widetilde{\alpha}_4, \quad \widetilde{\alpha}_{\sigma(4)} = \widetilde{\alpha}_1. \tag{23}$$

Based on the IIFOWA operator proposed by Xu [9], we can get

$$\widetilde{\alpha}^* = \text{IIFOWA}(\widetilde{\alpha}_1, \widetilde{\alpha}_2, \widetilde{\alpha}_3, \widetilde{\alpha}_4) = \langle [0.2834, 0.3767], [0.0, 0.0] \rangle. \tag{24}$$

This aggregation result indicates that the nonmembership degree range of the $\widetilde{\alpha}^*$ is determined by the IIFN $\widetilde{\alpha}_3$.

Based on the IIFIOWA operator proposed in this paper, we can get

$$\widetilde{\alpha}^* = \text{IIFIOWA}(\widetilde{\alpha}_1, \widetilde{\alpha}_2, \widetilde{\alpha}_3, \widetilde{\alpha}_4) = \langle [0.2834, 0.3767], [0.2644, 0.5652] \rangle. \tag{25}$$

Obviously, this result seems more reasonable.

Based on the IIFOWG operator proposed by Xu [9], we can get

$$\widetilde{\alpha}^* = \text{IIFOWG}(\widetilde{\alpha}_1, \widetilde{\alpha}_2, \widetilde{\alpha}_3, \widetilde{\alpha}_4) = \langle [0.2644, 0.5652], [0.0, 0.0] \rangle. \tag{26}$$

This aggregation result indicates that the membership degree range of the $\widetilde{\alpha}^*$ is determined by the IIFN $\widetilde{\alpha}_1$.

Based on the IIFIOWG operator proposed in this paper, we can get

$$\widetilde{\alpha}^* = \text{IIFIOWA}(\widetilde{\alpha}_1, \widetilde{\alpha}_2, \widetilde{\alpha}_3, \widetilde{\alpha}_4) = \langle [0.2644, 0.5652], [0.2733, 0.4348] \rangle. \tag{27}$$

Obviously, this result seems more reasonable.

4. Application of the Proposed Operators to Evaluate the Hydrogen Production Technologies

With China's sustained and rapid economic and social development, energy resources, and increasing pressure on the environment, developing light pollution and renewable energy is of great significance to China's sustainable development. Hydrogen is recognized as clean energy, low carbon, and zero carbon energy source which has attracted wide attention in various countries [45–47]. Hydrogen technologies evaluation involves multiattribute decision making and many attribute should be evaluated, such as environment, economic, and social [48].

One high-tech development company in Zhejiang Province, China, intends to invest in the hydrogen energy production. Three kinds of hydrogen production technologies have been identified according to their own business situation and the famous energy expert's suggestions, such as nuclear based high temperature electrolysis technology (NHTET), electrolysis of water technology by hydropower, and coal gasification technology, expressed by A_1, A_2, and A_3. The company wants to find out the most suitable technique from the three alternatives mainly according to environment performance C_1, economic performance C_2, social performance C_3, and the support degree of government policies C_4. Meanwhile, the four attributes have different importance weight and could be determined by many effective methods, such as AHP. Here we suppose the weight of the four attributes is $(0.14, 0.36, 0.32, 0.18)^T$. The performance of the three alternatives on the four attributes is expressed by IIFNs and is shown in Table 1.

First, we use the IIFIWA operator to aggregate the performance of the four attributes for three kinds of hydrogen production technologies, respectively,

$$\begin{aligned} \widetilde{\alpha}_1 &= \text{IIFIWA}(\widetilde{\alpha}_{11}, \widetilde{\alpha}_{12}, \ldots, \widetilde{\alpha}_{14}) = ([0.0828, 0.2063], [0.0799, 0.3301]), \\ \widetilde{\alpha}_2 &= \text{IIFIWA}(\widetilde{\alpha}_{21}, \widetilde{\alpha}_{22}, \ldots, \widetilde{\alpha}_{24}) = ([0.2212, 0.3217], [0.2158, 0.3197]), \\ \widetilde{\alpha}_3 &= \text{IIFIWA}(\widetilde{\alpha}_{31}, \widetilde{\alpha}_{32}, \ldots, \widetilde{\alpha}_{34}) = ([0.2151, 0.3817], [0.2176, 0.4326]). \end{aligned} \tag{28}$$

Next, according to the scores function of IIFNs given in Section 2, the scores $s(\widetilde{\alpha}_i)$ $(i = 1, 2, 3)$ can be calculated as follows:

$$s(\widetilde{\alpha}_1) = -0.0604, \quad s(\widetilde{\alpha}_2) = 0.0037, \quad s(\widetilde{\alpha}_3) = -0.0267. \tag{29}$$

Since

$$s(\widetilde{\alpha}_2) > s(\widetilde{\alpha}_3) > s(\widetilde{\alpha}_1), \tag{30}$$

then

$$A_2 \succ A_3 \succ A_1. \tag{31}$$

Therefore, the most suitable hydrogen production technology is A_2.

Table 2: The detailed comparison of A_1 and A_2.

	c_1	c_2	c_3	c_4
A_1	([0.1, 0.2], [0.0, 0.0])	([0.1, 0.3], [0.5, 0.7])	([0.1, 0.2], [0.6, 0.7])	([0.0, 0.0], [0.7, 0.8])
A_2	([0.2, 0.3], [0.1, 0.2])	([0.3, 0.4], [0.2, 0.3])	([0.2, 0.3], [0.3, 0.5])	([0.1, 0.2], [0.4, 0.5])
Membership degree range	$[a_{11}, b_{11}] \prec [a_{21}, b_{21}]$	$[a_{12}, b_{12}] \prec [a_{22}, b_{22}]$	$[a_{13}, b_{13}] \prec [a_{23}, b_{23}]$	$[a_{14}, b_{14}] \prec [a_{24}, b_{24}]$
Nonmembership degree range	$[c_{11}, d_{11}] \prec [c_{21}, d_{21}]$	$[c_{12}, d_{12}] \succ [c_{22}, d_{22}]$	$[c_{13}, d_{13}] \succ [c_{23}, d_{23}]$	$[c_{14}, d_{14}] \succ [c_{24}, d_{24}]$

4.1. Systematic Comparison with Other Research Results. Based on the IIFWA operator proposed by Xu [9], the result is inconsistent with the method in this paper

$$\begin{aligned} \tilde{\alpha}'_1 &= \text{IIFWA}\left(\tilde{\alpha}'_{11}, \tilde{\alpha}'_{12}, \dots, \tilde{\alpha}'_{14}\right) \\ &= ([0.0828, 0.2063], [0.0, 0.0]), \\ \tilde{\alpha}'_2 &= \text{IIFWA}\left(\tilde{\alpha}'_{21}, \tilde{\alpha}'_{22}, \dots, \tilde{\alpha}'_{24}\right) \\ &= ([0.2212, 0.3217], [0.4298, 0.5555]), \\ \tilde{\alpha}'_3 &= \text{IIFWA}\left(\tilde{\alpha}'_{31}, \tilde{\alpha}'_{33}, \dots, \tilde{\alpha}'_{34}\right) \\ &= ([0.2151, 0.3817], [0.5218, 0.6817]). \end{aligned} \quad (32)$$

Next, according to the scores function of IIFNs given in Section 2, the scores $s(\tilde{\alpha}'_i)$ $(i = 1, 2, 3)$ can be calculated as follows:

$$s\left(\tilde{\alpha}'_1\right) = 0.1445, \qquad s\left(\tilde{\alpha}'_2\right) = -0.2212, \qquad s\left(\tilde{\alpha}'_3\right) = -0.3034. \quad (33)$$

Since

$$s\left(\tilde{\alpha}'_1\right) > s\left(\tilde{\alpha}'_2\right) > s\left(\tilde{\alpha}'_3\right), \quad (34)$$

then

$$A_1 \succ A_2 \succ A_3. \quad (35)$$

Therefore, the most suitable alternative is A_1.

The optimal selection of two different methods is changed. From the above research results, we can find that the most suitable alternative is A_1 when the IIFWA operator is selected and the most suitable alternative is A_2 when the IIFIWA operator is involved.

Table 2 showed the detailed comparison of A_1 and A_2. From Table 2, we can easily find that the membership degree range of A_1 is worse than A_2 regarding the four attributes. Meanwhile, three of the four nonmembership degree ranges of A_1 are bigger than A_2 regarding the four attributes. Therefore, it is hard to accept the result that A_1 is better than A_2. The main reason for this result is that the nonmembership degree range of A_1 regarding criteria c_1 is $[0, 0]$. This indicates that the IIFWA operator proposed by Xu [9] is too sensitive to the situation where the nonmembership degree is reduced to $[0, 0]$.

5. Concluding Remarks and Future Works

In this paper, we have introduced some new aggregation operators for aggregating IIFNs, based on which a new MADM method has been proposed. Furthermore, we have used the MADM method to solve the problem of the evaluation of hydrogen production technologies. In order to find the effectiveness and superiority of the MADM method, we compared it with some existing methods. The MADM method proposed in this paper is meaningful because it can be used to solve some actual evaluation problems. However, just like all the existing MADM method, the MADM method proposed in this paper cannot be applied to deal with all decision making problems. Our method can be adapted from many aspects, such as considering the interconnection between the attributes. From the author's point of view, the future research should be the application of the MADM proposed in this paper with some necessary modifications, which is more suitable for concrete research problems.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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