

Research Article

Research on Third-Party Collecting Game Model with Competition in Closed-Loop Supply Chain Based on Complex Systems Theory

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This paper studied system dynamics characteristics of closed-loop supply chain using repeated game theory and complex system theory. It established decentralized decision-making game model and centralized decision-making game model and then established and analyzed the corresponding continuity system. Drew the region local stability of Nash equilibrium and Stackelberg equilibrium, and a series of chaotic system characteristics, have an detail analysis of the Lyapunov index which is under the condition of different parameter combination. According to the limited rational expectations theory, it established repeated game model based on collection price and marginal profits. Further, this paper analyzed the influence of the parameters by numerical simulations and concluded three conclusions. First, when the collection price is to a critical value, the system will be into chaos state. Second, when the sale price of remanufacturing products is more than a critical value, the system will be in chaos state. Last, the initial value of the collection price is sensitive, small changes may cause fluctuations of market price. These conclusions guide enterprises in making the best decisions in each phase to achieve maximize profits.

1. Introduction

The product life cycle becomes shorter with the rapid development of market economy. A lot of products have been washed out before life end. Such condition not only creates a tremendous waste of resources but also brings great harm to people. Therefore, the “resources-products-waste products-remanufacturing product” closed-loop type of economic growth mode appears. It realizes the economic development and utilization of resources and environmental protection in coordination strategy of sustainable development goals. Enterprise began to take a positive attitude to collect products from customers. The research on collecting control problem becomes inevitable.

This paper draws on and contributes to several streams of literature, each of whom we review below. A growing body of literature in operations management addresses reverse logistics management issues for remanufacturable products. References [1, 2] defined as the supply chain to reverse supply

chain which formed a complete closed-loop system (closed-loop supply chain, hereinafter referred to as CLSC.) Reference [3] studied the pricing methods under three collecting channels which manufacturer, retailer, and third-party collector collected products, respectively. Reference [4] studied manufacturers should be responsible for the management of that collecting based on Xerox corporation and Kodak corporation. Reference [5] studied three kinds of collecting channels of manufacturers collecting, retailer collecting, and the third-party collecting in closed-loop supply chain. They found that the collecting distance was closer from consumers and the collecting efforts were more effective. References [6–8] described reverse logistics management mode of the third party. In this collecting mode, manufacturers entrust a third party to perform extended producer responsibility and manage used products.

References [9, 10] studied pricing analysis of reverse supply chain using the game theory method under retailers collecting mode and concluded the optimal strategy of single

stage. Reference [11] established the demand model of new products and remanufacturing products from social environmental protection consciousness and consumers' utility function to new products and remanufacturing products and drew the conclusions. The first is the optimal pricing strategy in different collecting channels in decentralized decision-making. The second is the influence of social environmental protection consciousness. Reference [12] research shows that the channel of manufacturers collecting directly is more favorable to consumers and social environment. In the autoindustry, independent third parties are handling used-product collection activities for the original equipment manufacturers (OEMs). The "big three" automanufacturers in the United States started to invest in joint research and remanufacturing partnerships with dismantling centers to benefit from their scale economies and experience. Third parties such as GENCO Distribution System are also preferred by some consumer goods manufacturers for their experience in used-product collection. In references [13–15] Analyzed the complexity of model for a class of delay complex dynamics, and some valuable conclusions are obtained.

The existing literatures studied major in a single phase of the mathematical model using game theory under the assumption that the closed-loop supply chain contains a manufacturer and a retailer and concludes equilibrium in perfectly rational state. However, market uncertainty caused enterprise not to be able to make decisions in perfectly rational. At present, many manufacturers devote themselves to the core competitiveness; they contract the collection of used products to a third party. So this paper sets up multistage game model of closed-loop supply chain which consists of a manufacturer and two competition collectors business based on limited rationality with China's remanufacturing development and studies its complex dynamic characteristics.

2. Model Assumptions and Notation

2.1. Assumption. First, this paper considers only remanufacturing products market. That means new products and remanufacturing product do not form a competitive market. It is more in line with the actual conditions of China. The manufacturer manufactures new products and remanufacturing products.

Second, the closed-loop supply chain includes a manufacturer and two competitive collecting corporations, the collecting corporations collect waste products from consumers and return the manufacturer. The manufacturer transfers payments to the collectors, and she sales directly to consumers. The manufacturers and two collectors are independent decision makers, and their strategic space is to choose the best collecting price. Their goal is to maximize returns in discrete time period as $t = 0, 1, 2, \dots$

Third, the number of the collection is increasing function of collecting price. The collecting capability and manufacturing capability are unlimited. In order to simplify the problem and emphasize main parameters influence of the system, all the collecting products can be manufactured, as shown in the MCTM collecting mode (Figure 1).

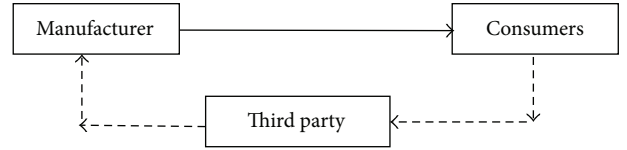


FIGURE 1: MCTM collecting mode.

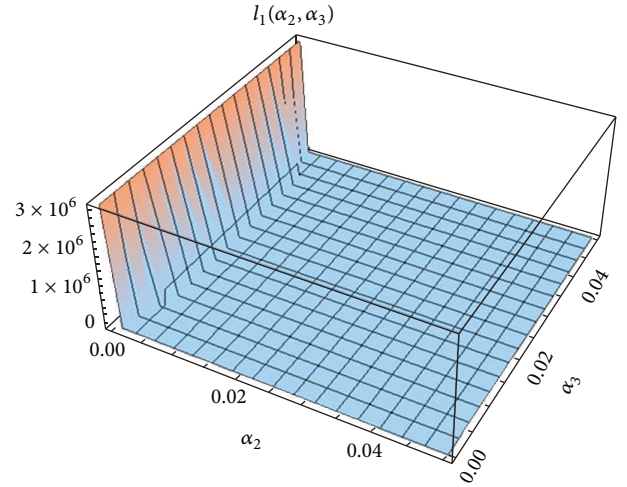


FIGURE 2: The change of $l_1(\alpha_1, \alpha_2, \alpha_3)$ with α_1, α_2 , and α_3 .

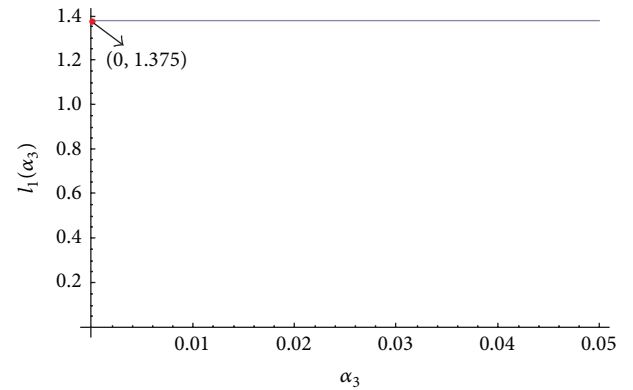


FIGURE 3: The change of $l_1(\alpha_3)$ with α_3 when $\alpha_1 = 0.02, \alpha_2 = 0.048$.

2.2. Notation. p_0 denotes the retail price of the product, a constant, and c_r the unit cost of remanufacturing a returned product into a new one, a constant. $p_m(t)$ is the collection price of the manufacturer transfer payment to collectors in the t period, a decision variable of the manufacturer. $p_{ri}(t)$ is the collection price of collectors in the t period. $r_i(t)$ is a decision variable of the collectors; that is, $p_{ri} = p_m(1 - r_i)$, $i = 1, 2$. A_i is the collecting cost of collectors including logistics cost and so forth, a constant. G_i is the collecting numbers for the waste product in the market as a function of collecting price, $G_i = k_i + \beta_i p_{ri}(t) - \delta_i p_{rj}(t)$, $i = 1, 2, j = 2, 1$, and subscript i, j will take values two collectors, where k is the waste products number of return voluntarily when the collectors' collecting price is zero; it denotes consumers

environmental awareness; β denotes consumers sensitive degree to collecting price; δ is competition coefficient in the two collectors, which satisfies $\beta \geq \delta$. The collecting function of the two collectors is, respectively, as follows:

$$\begin{aligned} G_1 &= k_1 + \beta_1 p_{r1}(t) - \delta_1 p_{r2}(t), \\ G_2 &= k_2 + \beta_2 p_{r2}(t) - \delta_2 p_{r1}(t). \end{aligned} \quad (1)$$

Π_m is the profits function of the manufacturer; its marginal profits are concluded by the first-order conditions, which are $\partial \Pi_m / \partial p_m(t)$. Π_m can be stated as

$$\begin{aligned} \prod_m &= (p_0 - p_m - c_r) \\ &\times (k_1 + \beta_1 p_{r1}(t) - \delta_1 p_{r2}(t)) \\ &\times (k_2 + \beta_2 p_{r2}(t) - \delta_2 p_{r1}(t)). \end{aligned} \quad (2)$$

Π_{ri} is the profits function of the two collectors; marginal profits are concluded by the first-order conditions, which are $\partial \Pi_{ri} / \partial r_i(t)$. Π_{ri} can be stated as

$$\begin{aligned} \prod_{r1} &= (p_m - p_{r1}(t) - A_1)(k_1 + \beta_1 p_{r1}(t) - \delta_1 p_{r2}(t)), \\ \prod_{r2} &= (p_m - p_{r2}(t) - A_2)(k_2 + \beta_2 p_{r2}(t) - \delta_2 p_{r1}(t)). \end{aligned} \quad (3)$$

Π is the profits function of the closed-loop supply chain, which can be stated as

$$\prod = \prod_m + \prod_{r1} + \prod_{r2}. \quad (4)$$

(p_m, r_1, r_2) is a pricing strategy, which can be obtained by (2) and (3). Furthermore they are satisfied with $\partial^2 \Pi_m / \partial p_m^2 = -2(1 - r_2)(\beta_2 - \delta_1) < 0$, $\partial^2 \Pi_{r1} / \partial r_1^2 = -2\beta_1 p_m^2(t) < 0$, and $\partial^2 \Pi_{r2} / \partial r_2^2 = -2\beta_2 p_m^2(t) < 0$. It can be proved that Π_i is concave function for p_i .

3. Decentralized Control Decision-Making Model

3.1. Nash Equilibrium

3.1.1. Model and Analysis

(1) *Model.* The manufacturer and collectors are equal in closed-loop supply chain; that is, the two sides make decision at the same time: manufacturer's decision is to choose the collection price p_i to maximize returns, and collectors' decision is to choose p_{ri} to maximize their returns.

Nash equilibrium can be concluded by the first-order conditions of three reaction functions, which meet $\partial \Pi_m / \partial p_m = 0$, $\partial \Pi_{ri} / \partial r_i = 0$ simultaneously. The Nash equilibrium is the optimal decision of a game in all kinds of may guess, thus make it to gain maximum benefit. The Nash equilibrium is

$$(p_m^*, r_1^*, r_2^*) = \left(\frac{-B + \sqrt{B^2 - 4AC}}{2A}, t_1 + \frac{t_2}{p_m^*}, t_3 + \frac{t_4}{p_m^*} \right), \quad (5)$$

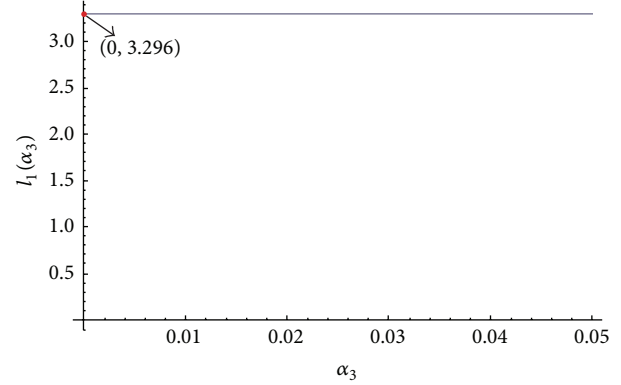


FIGURE 4: The change of $l_1(\alpha_3)$ with α_3 when $\alpha_1 = 0.02$, $\alpha_2 = 0.02$.

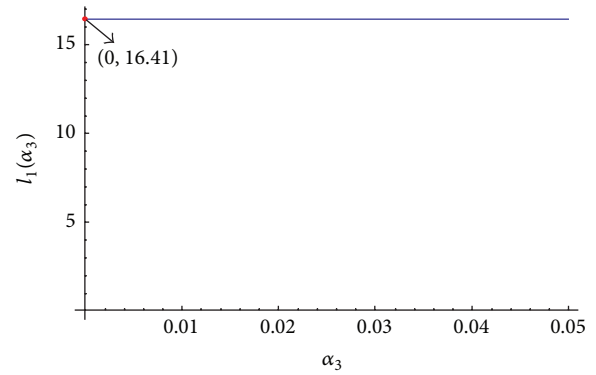


FIGURE 5: The change of $l_1(\alpha_3)$ with α_3 when $\alpha_1 = 0.02$, $\alpha_2 = 0.004$.

where A , B , and C are

$$\begin{aligned} A &= 2(\beta_1 + \beta_2 - \delta_1 - \delta_2)(p_0 - c_r) \\ &\quad - 2t_1(\beta_1 - \delta_2) - 2t_3(\beta_2 - \delta_1), \\ B &= k_1 + k_2 - (\beta_1 + \beta_2 - \delta_1 - \delta_2)(p_0 - c_r) \\ &\quad + (p_0 - c_r)(t_1(\beta_1 - \delta_2) + t_3(\beta_2 - \delta_1)) \\ &\quad - 2t_2(\beta_1 - \delta_2) - 2t_4(\beta_2 - \delta_1), \\ C &= (p_0 - c_r)t_2(\beta_1 - \delta_2) + t_4(\beta_2 - \delta_1). \end{aligned} \quad (6)$$

In formula (6), t_1, t_2, t_3, t_4 can be expressed as

$$\begin{aligned} t_1 &= \frac{2\beta_1\beta_2 - 2\beta_2\delta_1 + \delta_1\beta_2 - \delta_1\delta_2}{4\beta_1\beta_2 - \delta_1\delta_2}, \\ t_2 &= \frac{2\beta_2k_1 + 2\beta_2\beta_1A_1 + \delta_1k_2 - \delta_1\beta_2A_2}{4\beta_1\beta_2 - \delta_1\delta_2}, \\ t_3 &= \frac{2\beta_1\beta_2 - 2\beta_1\delta_2 + \delta_2\beta_1 - \delta_1\delta_2}{4\beta_1\beta_2 - \delta_1\delta_2}, \\ t_4 &= \frac{2\beta_1k_2 + 2\beta_2\beta_1A_2 + \delta_2k_1 - \delta_2\beta_1A_1}{4\beta_1\beta_2 - \delta_1\delta_2}. \end{aligned} \quad (7)$$

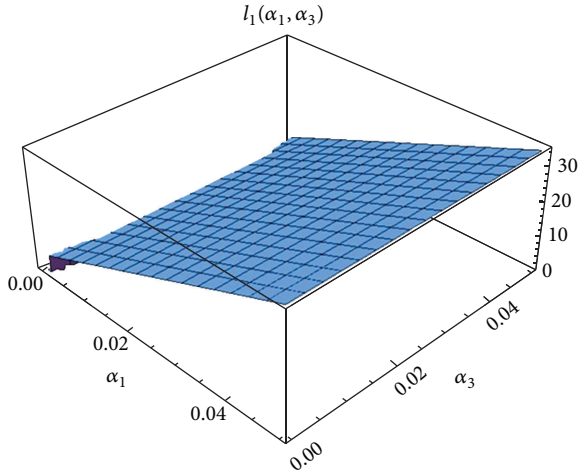


FIGURE 6: The change of $l_1(\alpha_1, \alpha_2, \alpha_3)$ with α_2 , α_1 , and α_3 .

In reality, the game between node enterprises in closed-loop supply chain is continuous, enterprises' decision-making is a long-term repeated process, and its action has long-term memory. And each node enterprise does not completely control the market information and also cannot fully expect future market changes, so based on limited rational expectations decision we adjust process with marginal gains. They can make the next-period price decision on the basis of the local estimate to his marginal profits in current period. Their price adjustment processes are

$$\begin{aligned}
 p_m(t+1) &= p_m(t) + \alpha_1 p_m(t) \\
 &\quad \times (((1 - r_1(t))(\beta_1 - \delta_2) \\
 &\quad + (1 - r_2(t))(\beta_2 - \delta_1)) \\
 &\quad \times (p_0 - 2p_m(t) - c_r) - k_1 - k_2), \\
 r_1(t+1) &= r_1(t) + \alpha_2 r_1(t) \\
 &\quad \times (-2\beta_1 r_1(t) p_m^2(t) + p_m(t) \\
 &\quad \times (k_1 + (\beta_1 - \delta_1 + \delta_1 r_2(t)) \\
 &\quad \times p_m(t) + \beta_1 A_1)), \\
 r_2(t+1) &= r_2(t) + \alpha_3 r_2(t) \\
 &\quad \times (-2\beta_2 r_2(t) p_m^2(t) + p_m(t) \\
 &\quad \times (k_2 + (\beta_2 - \delta_2 + \delta_2 r_1(t)) p_m(t) + \beta_2 A_2)), \quad (8)
 \end{aligned}$$

where $p_1(t+1)$, $r_1(t+1)$, $r_2(t+1)$, and $p_1(t)$, $r_1(t)$, $r_2(t)$ are used to present the price in the $(t+1)$ th period and in the (t) th period. Where α_i are positive parameters, $i = 1, 2, 3$, which denotes adjustment speed, respectively, for the manufacturer and two collectors.

From the system (8) we can conclude that the optimal collecting price of the manufacturer is related to the collection

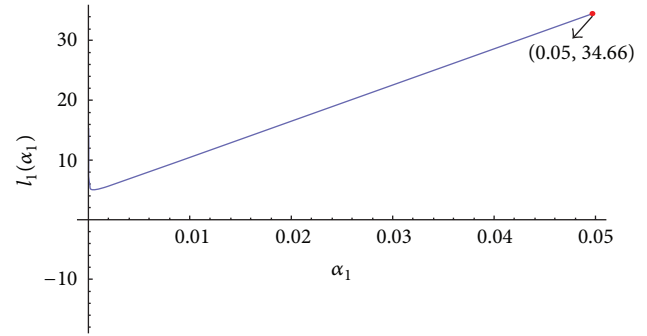


FIGURE 7: The change of $l_1(\alpha_1)$ with α_1 when $\alpha_2 = 0.004$, $\alpha_3 = 0.01$.

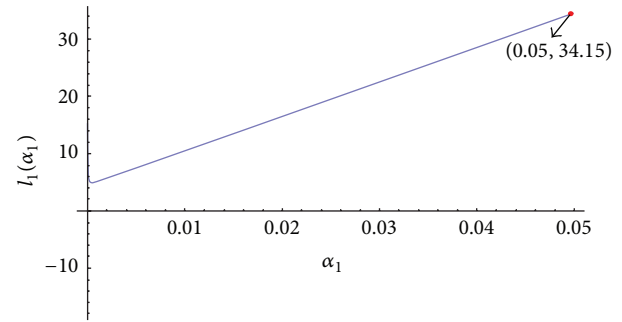


FIGURE 8: The change of $l_1(\alpha_1)$ with α_1 when $\alpha_2 = 0.004$, $\alpha_3 = 0.03$.

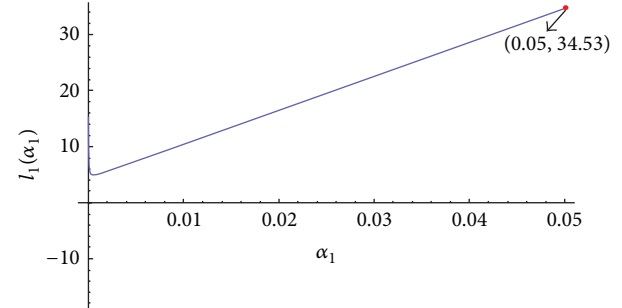


FIGURE 9: The change of $l_1(\alpha_1)$ with α_1 when $\alpha_2 = 0.004$, $\alpha_3 = 0.05$.

price adjustment coefficient, the two collectors' collecting price, sale price, and consumers' environmental protection awareness. Similarly, the optimal collecting price of collectors is related to collecting price adjustment parameters, collecting price of the manufacturer, collecting function, consumers's environmental protection awareness, and collecting costs.

(2) *Model Analysis.* In closed-loop supply chain, any enterprise decision-making is all according to the maximum profits, the equilibrium less than zero is not practical significance, such as manufacturers to make power mainly because of benefits, collecting business is also benefit with collecting. So only research system (8) is the equilibrium; system (8) of the eight fixed points, respectively, is $E_0(0, 0, 0)$, $E_1(p_m^1, 0, 0)$, $E_2(0, r_1^2, 0)$, $E_3(0, 0, r_2^3)$, $E_4(p_m^4, r_1^4, 0)$,

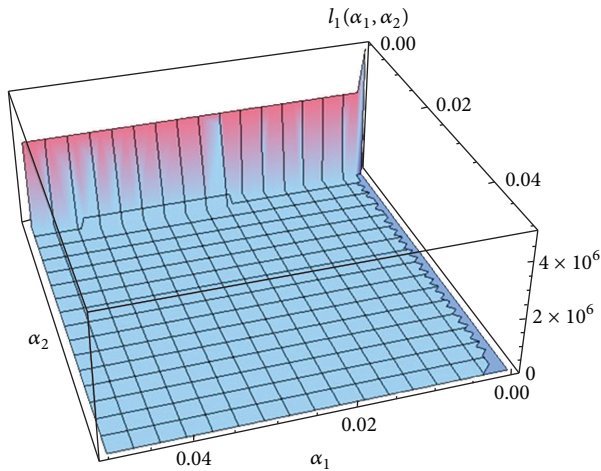


FIGURE 10: The change of $I_1(\alpha_1, \alpha_2, \alpha_3)$ with α_3 , α_1 , and α_2 .

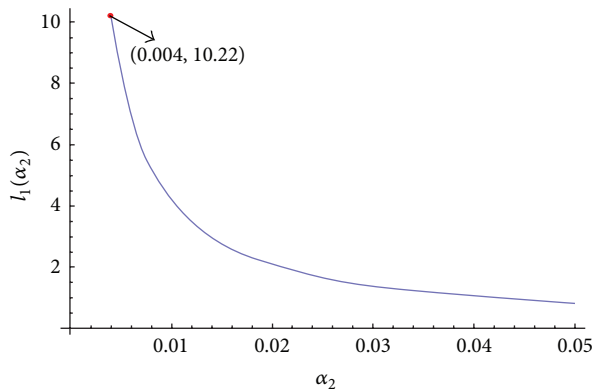


FIGURE 11: The change of $I_1(\alpha_2)$ with α_2 when $\alpha_1 = 0.01$, $\alpha_3 = 0.03$.

$E_5(p_m^5, 0, r_2^5)$, $E_6(0, r_1^6, r_2^6)$, $E_7(p_m^*, r_1^*, r_2^*)$. The analysis shows that by E_0 , E_1 , E_2 , E_3 , E_4 , E_5 and E_6 , is no real significance. Therefore, we discuss Nash equilibrium which is $E_7(p_m^*, r_1^*, r_2^*)$; E_7 is the reaction function intersection of the manufacturer and collectors, which means marginal profits of both sides are zero, but this does not mean that the result of the game will tend to equilibrium. Instead, a party rational behavior change may cause game process to occur very complex phenomenon. E_7 has local stability, this stability region by α_1 , α_2 , α_3 decided. The stability of E_7 will be studied. The first we calculate Jacobi matrix of system (8), means

$$J = \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix}. \quad (9)$$

We put $E_7(p_m^*, r_1^*, r_2^*)$ into formula (9), according to the Jury stability criterion, the sufficient and necessary conditions of E_7 asymptotic stability are that the zeros of characteristic polynomial $F(\lambda) = \lambda^3 + A\lambda^2 + B\lambda + C$ are in a circular

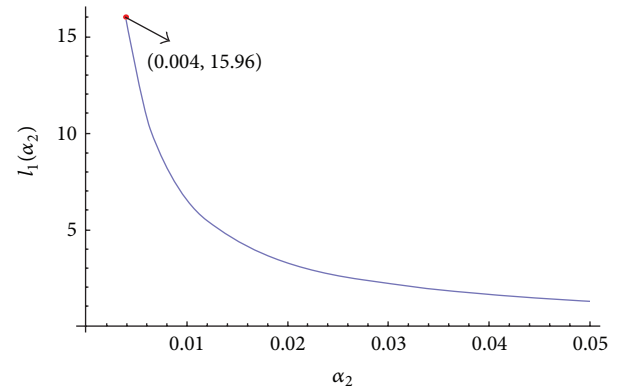


FIGURE 12: The change of $I_1(\alpha_2)$ with α_2 when $\alpha_1 = 0.02$, $\alpha_3 = 0.03$.

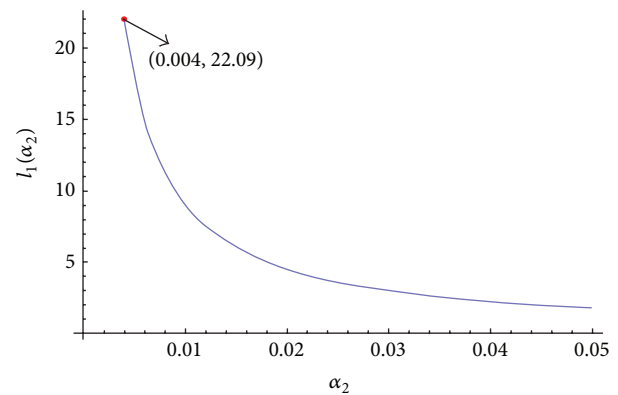
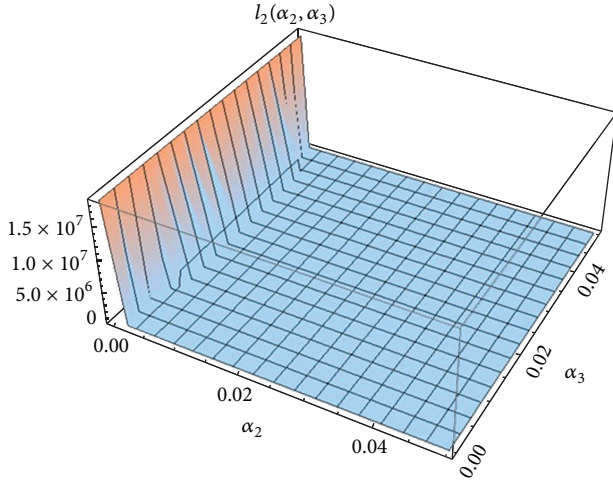
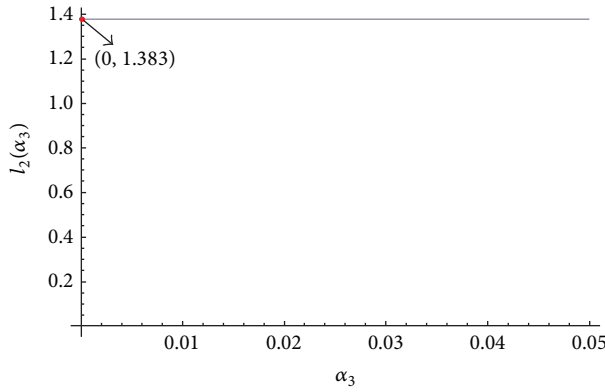


FIGURE 13: The change of $I_1(\alpha_2)$ with α_2 when $\alpha_1 = 0.03$, $\alpha_3 = 0.03$.

area. Moreover, the characteristic polynomial will satisfy the conditions as follows:

$$\begin{aligned} F(1) &= 1 + A + B + C > 0, \\ (-1)^3 F(-1) &= 1 - A + B - C > 0, \\ 1 &> |C|, \\ 1 - C^2 &> |B - AC|. \end{aligned} \quad (10)$$

At present, only a few simple dynamic systems are analyzed with analytical method, but complex dynamic system mainly uses the numerical analysis method. This paper processes numerical simulation on system (8) by Matlab to express the dynamic characteristics. Because remanufacturing is still initial stage in china, consumers environmental protection awareness is lower, and manufacturing cost (including the fixed and variable cost) is higher; for autparts collecting, the parameters of system (8) can be defined as $p_0 = 70$, $c_r = 30$, $A_1 = 1$, $A_2 = 0.8$, $k_1 = 1$, $k_2 = 1$, $\beta_1 = 0.6$, $\beta_2 = 0.55$, $\delta_1 = 0.2$, and $\delta_2 = 0.2$ to study the local stability of equilibrium point. Get the parameter value into $E_7(p_m^*, r_1^*, r_2^*)$ and formula (10) to conclude the value for (17.385, 0.491, 0.489).

FIGURE 14: The change of $l_2(\alpha_1, \alpha_2, \alpha_3)$ with α_3 , α_1 , and α_2 .FIGURE 15: The change of $l_2(\alpha_3)$ with α_3 when $\alpha_1 = 0.02$, $\alpha_2 = 0.048$.

Change the discrete system (8) into a continuous system and the points of Jacobi matrix for

$$J = \begin{bmatrix} -13.317\alpha_1 & -17.755\alpha_1 & -15.562\alpha_1 \\ -0.775\alpha_2 & -177.888\alpha_2 & 29.68\alpha_2 \\ -0.821\alpha_3 & 29.599\alpha_3 & -164.65\alpha_3 \end{bmatrix}. \quad (11)$$

Consider the continuous-time nonlinear dynamical system

$$\dot{x} = Ax + N(x), \quad A = (a_{ij})_{n \times n}, \quad x \in \mathbb{R}^n. \quad (12)$$

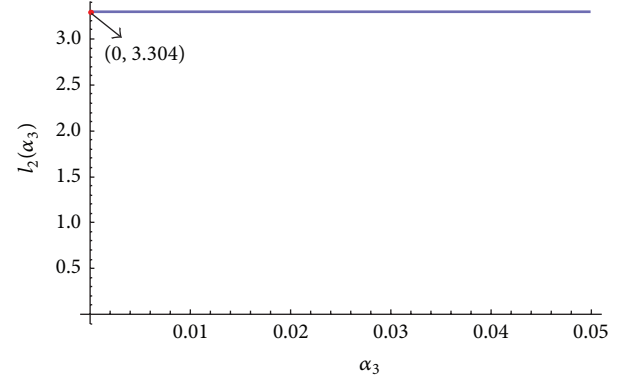
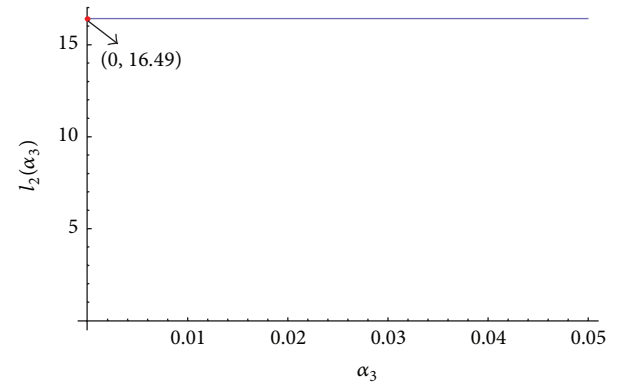
Let the function $N(x)$ be written as

$$N(x) = \frac{1}{2}B(x, x) + \frac{1}{6}C(x, x, x) + O(\|x\|^4), \quad (13)$$

where $B(x, y)$ and $C(x, y, z)$ are bilinear and trilinear functions, respectively. In coordinates, we have

$$B_i(x, y) = \sum_{j,k=1}^n \frac{\partial^2 N_i(\xi)}{\partial \xi_j \partial \xi_k} \Big|_{\xi=0} x_j y_k, \quad (14)$$

$$C_i(x, y, z) = \sum_{j,k,l=1}^n \frac{\partial^3 N_i(\xi)}{\partial \xi_j \partial \xi_k \partial \xi_l} \Big|_{\xi=0} x_j y_k z_l.$$

FIGURE 16: The change of $l_2(\alpha_3)$ with α_3 when $\alpha_1 = 0.02$, $\alpha_2 = 0.02$.FIGURE 17: The change of $l_2(\alpha_3)$ with α_3 when $\alpha_1 = 0.02$, $\alpha_2 = 0.004$.

Supposing that A has a pair of complex eigenvalues on the imaginary axis, they are $\lambda_{1,2} = \pm i\omega$ ($\omega > 0$), and these eigenvalues are the only eigenvalues with $\text{Re } \lambda = 0$. Let $q \in \mathbb{C}^n$ be a complex eigenvector to $\lambda_1 = i\omega$:

$$Aq = i\omega q, \quad A\bar{q} = -i\omega\bar{q}. \quad (15)$$

And the adjoint eigen vector $p \in \mathbb{C}^n$ admits the properties

$$A^T p = -i\omega p, \quad A^T \bar{p} = i\omega \bar{p} \quad (16)$$

and satisfies the normalization $\langle q, p \rangle = 1$.

The first Lyapunov coefficient at the origin is defined by

$$l_1 = \frac{1}{2\omega} \text{Re} \left[\langle p, C(q, q, \bar{q}) \rangle - 2 \langle p, B(q, A^{-1}B(q, \bar{q})) \rangle \right. \\ \left. + \langle p, B(\bar{q}, (2i\omega E - A)^{-1}B(q, q)) \rangle \right]. \quad (17)$$

Next, we calculate $i\omega q$, $J\bar{q} = -i\omega\bar{q}$, $J^T p = -i\omega p$, and $J^T \bar{p} = i\omega \bar{p}$; If $q = (q_1, q_2, q_3)^T \neq 0$, calculate q, \bar{q}, p, \bar{p} :

$$\begin{bmatrix} -13.317\alpha_1 - i\omega & -17.755\alpha_1 & -15.562\alpha_1 \\ -0.775\alpha_2 & -177.888\alpha_2 - i\omega & 29.68\alpha_2 \\ -0.821\alpha_3 & 29.599\alpha_3 & -164.65\alpha_3 - i\omega \end{bmatrix} \\ \times \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (18)$$

having nonzero solutions.

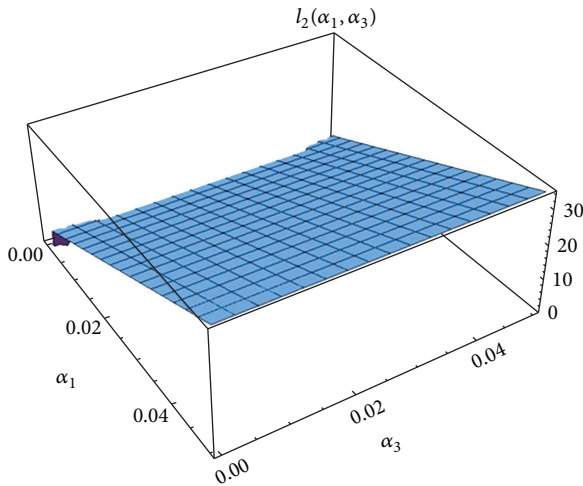


FIGURE 18: The change of $l_2(\alpha_1, \alpha_2, \alpha_3)$ with α_2 , α_1 , and α_3 .

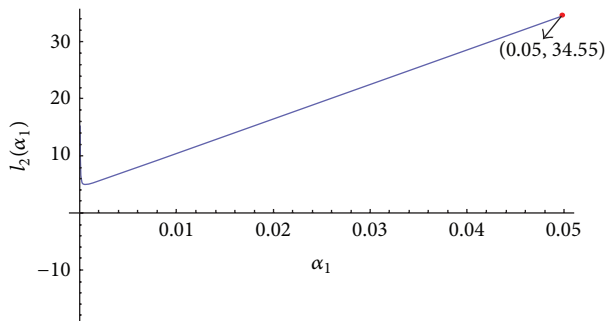


FIGURE 19: The change of $l_2(\alpha_1)$ with α_1 when $\alpha_2 = 0.004$, $\alpha_2 = 0.01$.

It means

$$\begin{vmatrix} -13.317\alpha_1 - iw & -17.755\alpha_1 & -15.562\alpha_1 \\ -0.775\alpha_2 & -177.888\alpha_2 - iw & 29.68\alpha_2 \\ -0.821\alpha_3 & 29.599\alpha_3 & -164.65\alpha_3 - iw \end{vmatrix} = 0. \quad (19)$$

It can be got from calculating

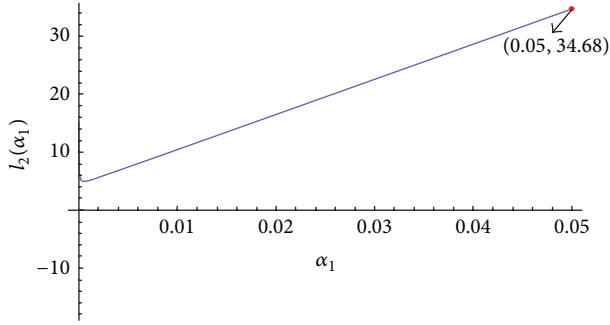
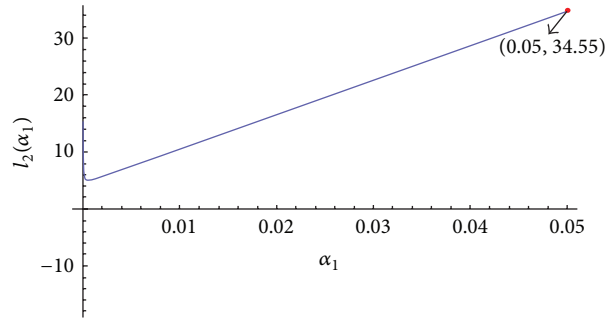
$$\begin{aligned} f_1(w) &= -375716.354\alpha_1\alpha_2\alpha_3 + 117.77755\alpha_2^2\alpha_3 \\ &\quad + 13.317\alpha_1w^2 + 177.888\alpha_2w^2 + 164.65\alpha_3w^2 = 0, \\ f_2(w) &= -2368.9345\alpha_1\alpha_2w + 0.6\alpha_2^2w \\ &\quad - 28410.761\alpha_2\alpha_3w - 2179.87\alpha_1\alpha_3w + w^3 = 0. \end{aligned} \quad (20)$$

(1) When $\begin{cases} w = \alpha_1 \\ f_1(w) = 0 \\ f_2(w) = 0 \end{cases}$, matrix J has pure imaginary eigenvalues $\pm i\alpha_1$. Calculate q, \bar{q}, p, \bar{p} .

The result is as follows.

$$\begin{aligned} L_2(\alpha_1, \alpha_2, \alpha_3) &= \frac{1}{2\omega} [\langle p, C(q, q, \bar{q}) \rangle - 2 \langle p, B(q, A^{-1}B(q, \bar{q})) \rangle \\ &\quad + \langle p, B(\bar{q}, (2i\omega E - A)^{-1}B(q, q)) \rangle] \\ &= \frac{(408.75 - 29.6i)\alpha_1\alpha_2}{(0.82i\alpha_1 - 168.9853\alpha_2)\alpha_2} \\ &\quad - 2 \left[\frac{(408.75 - 29.6i)\alpha_1\alpha_2}{(0.82i\alpha_1 - 168.9853\alpha_2)\alpha_2} \right. \\ &\quad \left. - \frac{2179.87\alpha_1}{375716.354\alpha_1\alpha_2 - 117.7775\alpha_2^2} \right. \\ &\quad \left. + \left((61899.185 - 4510.5i)\alpha_2 \right. \right. \\ &\quad \left. \left. \times (5846897.898i\alpha_1^2 \right. \right. \\ &\quad \left. \left. - (1.24 \times 10^9 + 1832.854i)\alpha_1\alpha_2 \right. \right. \\ &\quad \left. \left. + 388107.8\alpha_2^2 \right)^{-1} \right] \\ &\quad + \left(\{\alpha_1 [(-4 + 26.634i)\alpha_1 \right. \right. \\ &\quad \left. \left. + (2179.87 + 329.3i)\alpha_3] \right\} \right. \\ &\quad \left. \times \left(\{(-53 - 8i)\alpha_1^3 \right. \right. \\ &\quad \left. \left. + \alpha_1^2 [(-711.55 + 4737.8i)\alpha_2 \right. \right. \\ &\quad \left. \left. - (658.6 - 4359.7i)\alpha_3] - 117.8\alpha_2^2\alpha_3 \right. \right. \\ &\quad \left. \left. + \alpha_1\alpha_2(1.2i\alpha_2 + (375716.354 + 56821.5i)\alpha_3) \right\} \right)^{-1} \\ &\quad + \{(-407.31 + 29.7i)\alpha_2\} \\ &\quad \times (-1.55i\alpha_1\alpha_2 - 151.97\alpha_2\alpha_3) \} \\ &\quad \times \left(\{ (15.6i\alpha_1 + 3295.3\alpha_2) \right. \right. \\ &\quad \left. \left. \times \{(-53.3 - 8i)\alpha_1^3 \right. \right. \\ &\quad \left. \left. + \alpha_1^2 [(-711.552 + 4737.9i)\alpha_2 \right. \right. \\ &\quad \left. \left. - (658.6 - 4359.74i)\alpha_3] \right. \right. \\ &\quad \left. \left. - 117.77755\alpha_2^2\alpha_3 + \alpha_1\alpha_2 \right. \right. \\ &\quad \left. \left. \times [1.2i\alpha_2 + (375716.354 + 56821.52i)\alpha_3] \right\} \right)^{-1} \\ &= l_1(\alpha_1, \alpha_2, \alpha_3) + m_1(\alpha_1, \alpha_2, \alpha_3), \end{aligned} \quad (21)$$

where $L_1(\alpha_1, \alpha_2, \alpha_3)$ is the Lyapunov exponent function about α_1, α_2 , and α_3 and $l_1(\alpha_1, \alpha_2, \alpha_3)$ is the real part. The following is the figure of $l_1(\alpha_1, \alpha_2, \alpha_3)$.

FIGURE 20: The change of $l_2(\alpha_1)$ with α_1 when $\alpha_2 = 0.004$, $\alpha_3 = 0.03$.FIGURE 21: The change of $l_2(\alpha_1)$ with α_1 when $\alpha_2 = 0.004$, $\alpha_3 = 0.05$.

- (1) Fix the value of α_1 and $\alpha_2 \in (0, 0.05)$, $\alpha_3 \in (0, 0.05)$.
 - (i) $\alpha_1 = 0.02$, $\alpha_2 = 0.048$, and $\alpha_3 \in (0, 0.05)$ (see Figure 3).
 - (ii) $\alpha_1 = 0.02$, $\alpha_2 \in (0, 0.05)$, and $\alpha_3 \in (0, 0.05)$ (see Figure 2).

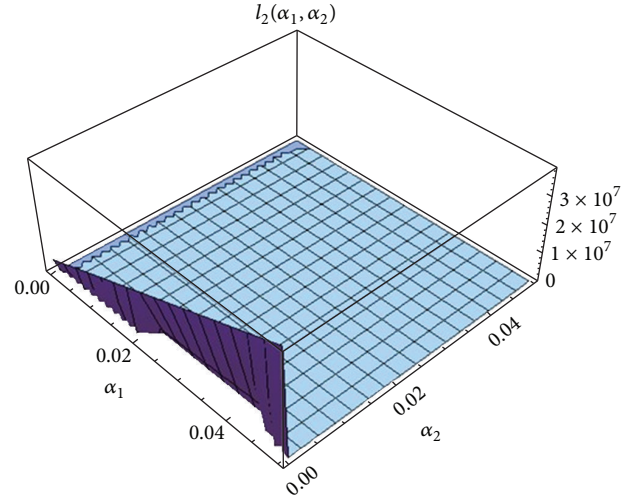
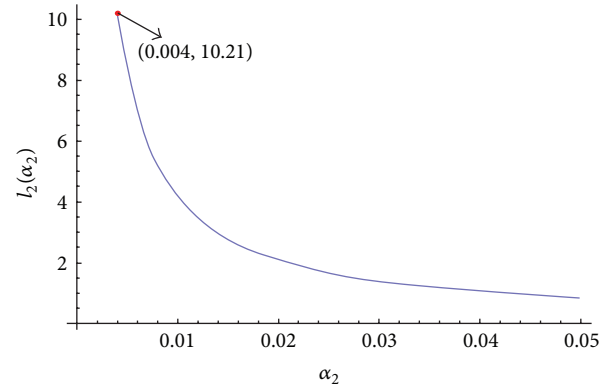
We can know from Figure 3 that when $\alpha_1 = 0.02$, $\alpha_2 = 0.048$, and α_3 is between zero and 0.05, $l_1(\alpha_3)$ is always equal to 1.375.

- (iii) $\alpha_1 = 0.02$, $\alpha_2 = 0.02$, and $\alpha_3 \in (0, 0.05)$ (see Figure 4).
- (iv) $\alpha_1 = 0.02$, $\alpha_2 = 0.004$, and $\alpha_3 \in (0, 0.05)$ (see Figure 5).

From Figures 4 and 5 we can observe the same change of $l_1(\alpha_3)$ as in Figures 2 and 3.

- (2) Fix the value of α_2 and $\alpha_1 \in (0, 0.05)$, $\alpha_3 \in (0, 0.05)$.
 - $\alpha_2 = 0.004$, $\alpha_1 \in (0, 0.05)$, and $\alpha_3 \in (0, 0.05)$ (see Figure 6).
 - $\alpha_2 = 0.004$, $\alpha_3 = 0.01$, and $\alpha_1 \in (0, 0.05)$ (see Figure 7).
 - $\alpha_2 = 0.004$, $\alpha_3 = 0.03$, and $\alpha_1 \in (0, 0.05)$ (see Figure 8).
 - $\alpha_2 = 0.004$, $\alpha_3 = 0.05$, and $\alpha_1 \in (0, 0.05)$ (see Figure 9).

- (3) Fix the value of α_3 and $\alpha_1 \in (0, 0.05)$, $\alpha_2 \in (0, 0.05)$.
 - (i) $\alpha_3 = 0.03$, $\alpha_1 \in (0, 0.05)$, and $\alpha_2 \in (0, 0.05)$ (see Figure 10).
 - (ii) $\alpha_3 = 0.03$, $\alpha_1 = 0.01$, and $\alpha_2 \in (0, 0.05)$ (see Figure 11).
 - (iii) $\alpha_3 = 0.03$, $\alpha_1 = 0.02$, and $\alpha_2 \in (0, 0.05)$ (see Figure 12).
 - (iv) $\alpha_3 = 0.03$, $\alpha_1 = 0.03$, and $\alpha_2 \in (0, 0.05)$ (see Figure 13).

FIGURE 22: The change of $l_2(\alpha_1, \alpha_2, \alpha_3)$ with α_2 , α_1 , and α_3 .FIGURE 23: The change of $l_2(\alpha_2)$ with α_2 when $\alpha_1 = 0.01$, $\alpha_3 = 0.03$.

- (2) When $\begin{cases} w = \alpha_2 \\ f_1(\alpha_2) = 0 \\ f_2(\alpha_2) = 0 \end{cases}$, matrix J has pure imaginary eigenvalues $\pm i\alpha_2$. Calculate q, \bar{q}, p, \bar{p} .
The result is as follows:

$$\begin{aligned}
 L_2(\alpha_1, \alpha_2, \alpha_3) &= \frac{1}{2\omega} \left[\langle p, C(q, q, \bar{q}) \rangle - 2 \langle p, B(q, A^{-1}B(q, \bar{q})) \rangle \right. \\
 &\quad \left. + \langle p, B(\bar{q}, (2i\omega E - A)^{-1}B(q, q)) \rangle \right] \\
 &= (0.000851 - 0.175i) \\
 &\quad + \frac{37.57 + 0.18i}{375716.354\alpha_1 - 117.8\alpha_2} + \frac{(2.42 + 0.012i)\alpha_1}{\alpha_2} \\
 &\quad + \frac{(0.01293 - 2.74i)\alpha_2}{375716.354\alpha_1^2 - 117.78\alpha_1\alpha_2}
 \end{aligned}$$

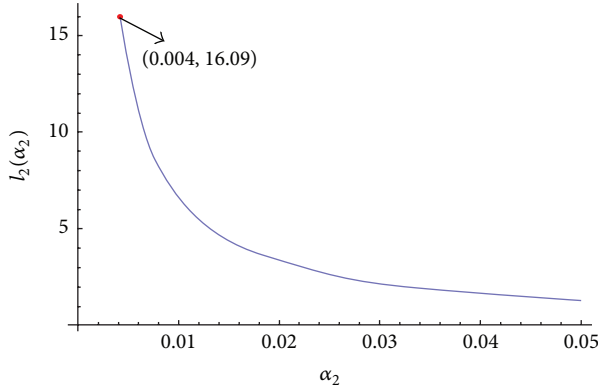


FIGURE 24: The change of $l_2(\alpha_2)$ with α_2 when $\alpha_1 = 0.02, \alpha_3 = 0.03$.

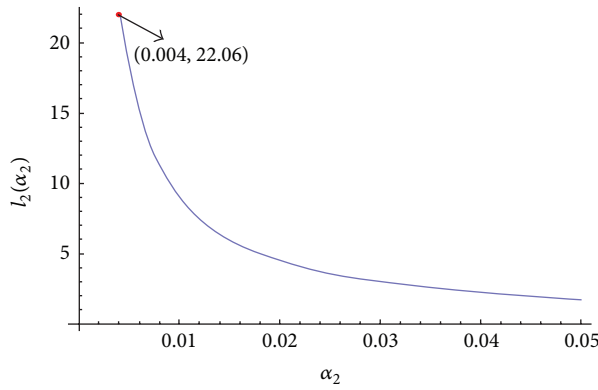


FIGURE 25: The change of $l_2(\alpha_2)$ with α_2 when $\alpha_1 = 0.03, \alpha_3 = 0.03$.

$$\begin{aligned}
 & + \frac{4359.74\alpha_1}{375716.354\alpha_1\alpha_2 - 117.77755\alpha_2^2} \\
 & + \left\{ \alpha_2^2 [(-0.014 + 0.000066i)\alpha_2 + (0.0065 + 1.37i)\alpha_3] \right. \\
 & \quad + \alpha_1\alpha_2 [(-4 + 0.19i)\alpha_2 + (18.784 + 329.2i)\alpha_3] \\
 & \quad \left. + \alpha_1^2 (26.6i\alpha_2 + 2179.87\alpha_3) \right\} \\
 & \times \{ \alpha_1\alpha_2 [(-53.268 + 4737.87i)\alpha_1\alpha_2 \\
 & \quad - (711.552 + 9.2i)\alpha_2^2 \\
 & \quad + (375716.354 + 4359.74i)\alpha_1\alpha_3 \\
 & \quad - (776.38 - 56821.52i)\alpha_2\alpha_3] \}^{-1} \\
 & = l_2(\alpha_1, \alpha_2, \alpha_3) + m_2(\alpha_1, \alpha_2, \alpha_3), \tag{22}
 \end{aligned}$$

where $L_2(\alpha_1, \alpha_2, \alpha_3)$ is the Lyapunov exponent function about α_1, α_2 , and α_3 and $l_2(\alpha_1, \alpha_2, \alpha_3)$ is the real part. The following is the figure of $l_2(\alpha_1, \alpha_2, \alpha_3)$.

(1) Fix the value of α_1 and $\alpha_2 \in (0, 0.05), \alpha_3 \in (0, 0.05)$.

(i) $\alpha_1 = 0.02, \alpha_2 \in (0, 0.05)$, and $\alpha_3 \in (0, 0.05)$ (see Figure 14).

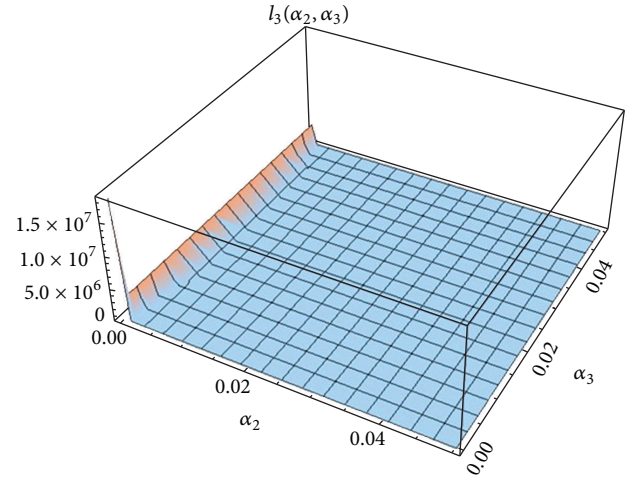


FIGURE 26: The change of $l_3(\alpha_1, \alpha_2, \alpha_3)$ with α_1, α_2 , and α_3 .

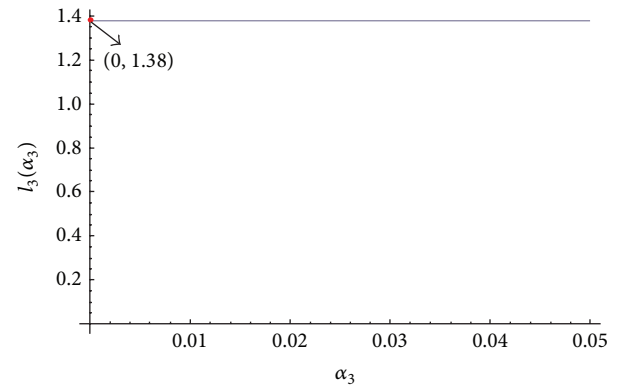


FIGURE 27: The change of $l_2(\alpha_3)$ with α_3 when $\alpha_1 = 0.02, \alpha_2 = 0.048$.

(ii) $\alpha_1 = 0.02, \alpha_2 = 0.048$, and $\alpha_3 \in (0, 0.05)$ (see Figure 15).

(iii) $\alpha_1 = 0.02, \alpha_2 = 0.02$, and $\alpha_3 \in (0, 0.05)$ (see Figure 16).

(iv) $\alpha_1 = 0.02, \alpha_2 = 0.004$, and $\alpha_3 \in (0, 0.05)$ (see Figure 17).

(2) Fix the value of α_2 and $\alpha_1 \in (0, 0.05), \alpha_3 \in (0, 0.05)$.

(i) $\alpha_2 = 0.004, \alpha_1 \in (0, 0.05)$, and $\alpha_3 \in (0, 0.05)$ (see Figure 18).

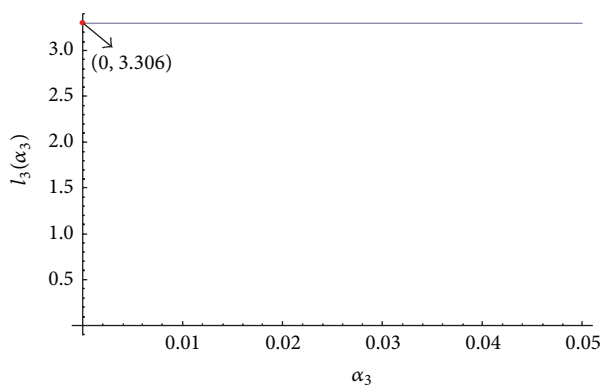
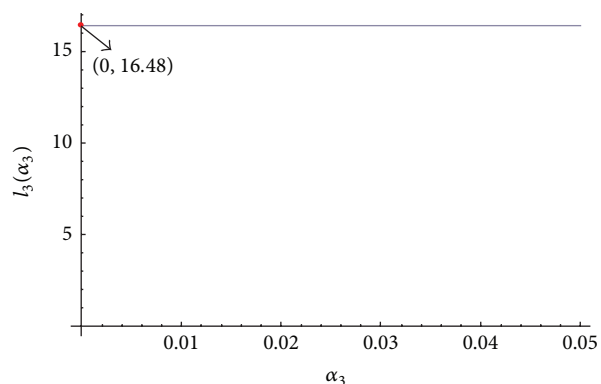
(ii) $\alpha_2 = 0.004, \alpha_3 = 0.01$, and $\alpha_1 \in (0, 0.05)$ (see Figure 19).

(iii) $\alpha_2 = 0.004, \alpha_1 \in (0, 0.05)$, and $\alpha_3 = 0.03$ (see Figure 20).

(iv) $\alpha_2 = 0.004, \alpha_3 = 0.05$, and $\alpha_1 \in (0, 0.05)$ (see Figure 21).

(3) Fix the value of α_3 and $\alpha_1 \in (0, 0.05), \alpha_2 \in (0, 0.05)$.

(i) $\alpha_3 = 0.03, \alpha_1 \in (0, 0.05)$, and $\alpha_2 \in (0, 0.05)$ (see Figure 22).

FIGURE 28: The change of $l_2(\alpha_3)$ with α_3 when $\alpha_1 = 0.02$, $\alpha_2 = 0.02$.FIGURE 29: The change of $l_2(\alpha_3)$ with α_3 when $\alpha_1 = 0.02$, $\alpha_2 = 0.004$.

(ii) $\alpha_3 = 0.03$, $\alpha_1 = 0.01$, and $\alpha_2 \in (0, 0.05)$ (see Figure 23).

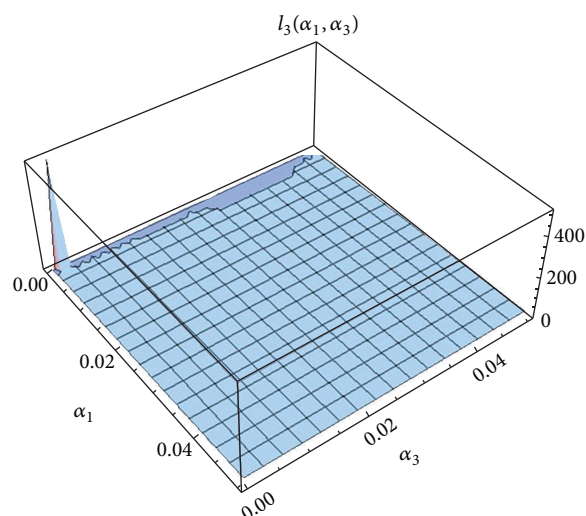
(iii) $\alpha_3 = 0.03$, $\alpha_1 = 0.02$, and $\alpha_2 \in (0, 0.05)$ (see Figure 24).

(iv) $\alpha_3 = 0.03$, $\alpha_1 = 0.03$, and $\alpha_2 \in (0, 0.05)$ (see Figure 25).

(3) When $\begin{cases} w = \alpha_3 \\ f_1(\alpha_3) = 0 \\ f_2(\alpha_3) = 0 \end{cases}$, matrix J has pure imaginary eigenvalues $\pm i\alpha_3$. Calculate q, \bar{q}, p, \bar{p} .

The result is as follows:

$$\begin{aligned} L_3(\alpha_1, \alpha_2, \alpha_3) &= \frac{1}{2\omega} \left[\langle p, C(q, q, \bar{q}) \rangle - 2 \langle p, B(q, A^{-1}B(q, \bar{q})) \rangle \right. \\ &\quad \left. + \langle p, B(\bar{q}, (2i\omega E - A)^{-1}B(q, q)) \rangle \right] \\ &= \frac{408.75\alpha_1\alpha_2 - 29.6i\alpha_2\alpha_3}{\alpha_2(-168.9853\alpha_2 + 0.821i\alpha_3)} \\ &\quad + \{(7183233.84 + 87766i)\alpha_1^2\alpha_2 \\ &\quad + (61899.185 + 631.33i)\alpha_1\alpha_2^2 \end{aligned}$$

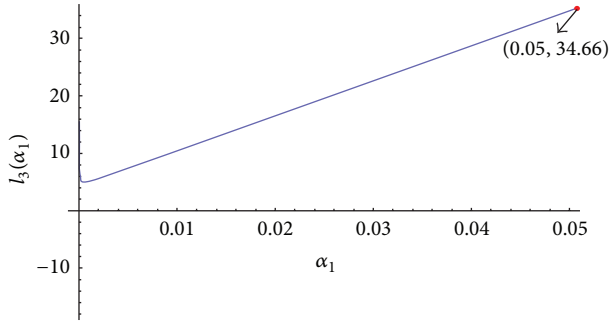
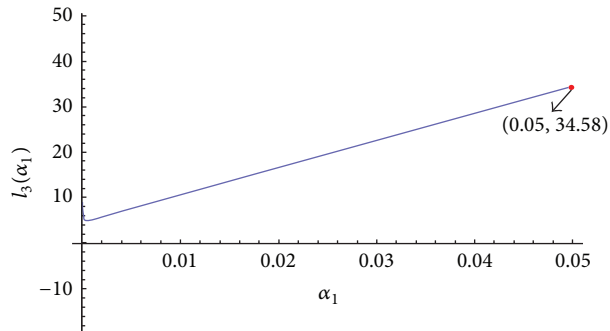
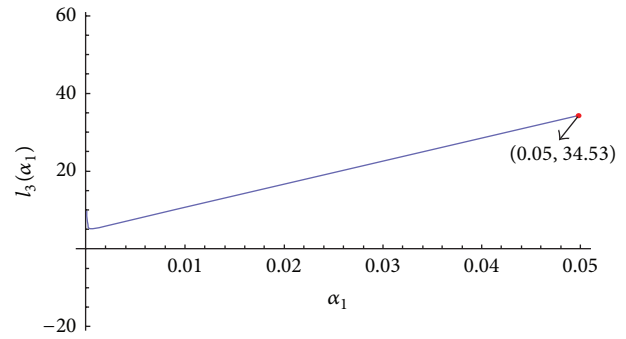
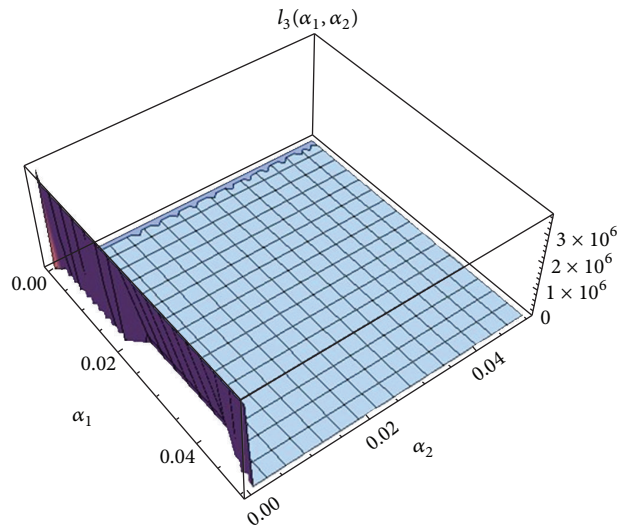
FIGURE 30: The change of $l_3(\alpha_1, \alpha_2, \alpha_3)$ with α_1, α_2 , and α_3 .

$$\begin{aligned} &- (414.48 - 33923.1i)\alpha_1^2\alpha_3 \\ &- (13181 - 1085129.6i)\alpha_1\alpha_2\alpha_3 \\ &- (46 - 4510.5i)\alpha_2^2\alpha_3 - (5124.6 + 62.25i)\alpha_1\alpha_3^2 \} \\ &\times \left\{ (3295.26\alpha_1\alpha_2 + 15.562i\alpha_1\alpha_3) \right. \\ &\times \left[(375716.354 + 4737.87i)\alpha_1\alpha_2 \right. \\ &\quad - (117.78 + 1.2i)\alpha_2^2 - (53.3 - 4359.74i)\alpha_1\alpha_3 \\ &\quad - (711.552 - 56821.5i)\alpha_2\alpha_3 \\ &\quad \left. \left. - (658.6 + 8i)\alpha_3^2 \right] \right\}^{-1} \\ &- 2 \left[-\frac{2179.867\alpha_1}{375716.354\alpha_1\alpha_2 - 117.78\alpha_2^2} \right. \\ &\quad + \frac{408.75\alpha_1\alpha_2 - 29.6i\alpha_2\alpha_3}{\alpha_2(-168.9853\alpha_2 + 0.821i\alpha_3)} \\ &\quad + (61899.185\alpha_1\alpha_2 - 4510.5i\alpha_2\alpha_3) \\ &\quad \times ((-375716.354\alpha_1 + 117.78\alpha_2) \\ &\quad \left. \times (3295.26\alpha_1\alpha_2 - 15.562i\alpha_1\alpha_3))^{-1} \right] \\ &= l_3(\alpha_1, \alpha_2, \alpha_3) + m_3(\alpha_1, \alpha_2, \alpha_3), \end{aligned} \quad (23)$$

where $L_3(\alpha_1, \alpha_2, \alpha_3)$ is the Lyapunov exponent function about α_1, α_2 , and α_3 and $l_3(\alpha_1, \alpha_2, \alpha_3)$ is the real part. The following is the figure of $l_3(\alpha_1, \alpha_2, \alpha_3)$.

(1) Fix the value of α_1 and $\alpha_2 \in (0, 0.05)$, $\alpha_3 \in (0, 0.05)$.

(i) $\alpha_1 = 0.02$, $\alpha_2 \in (0, 0.05)$, and $\alpha_3 \in (0, 0.05)$ (see Figure 26).

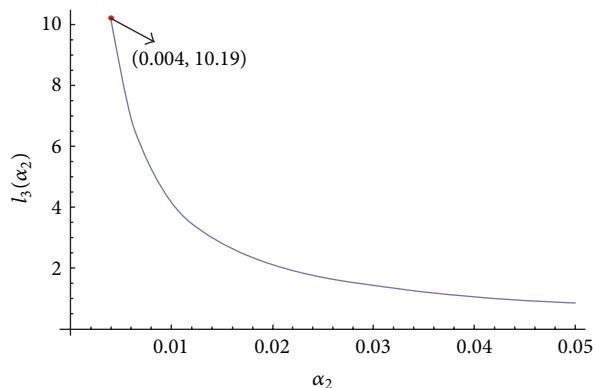
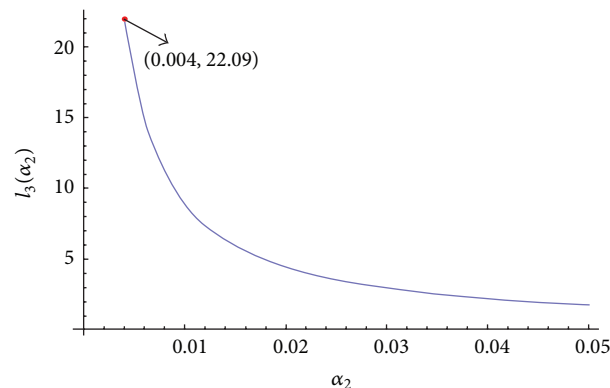
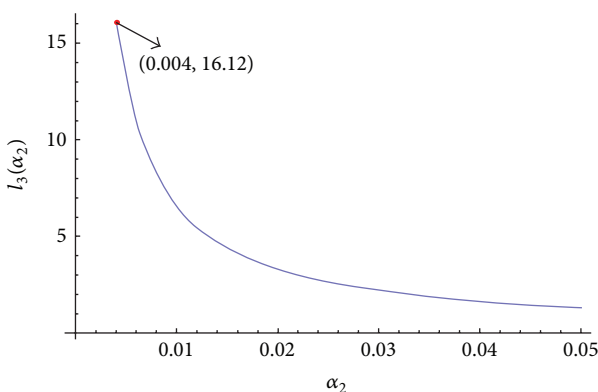
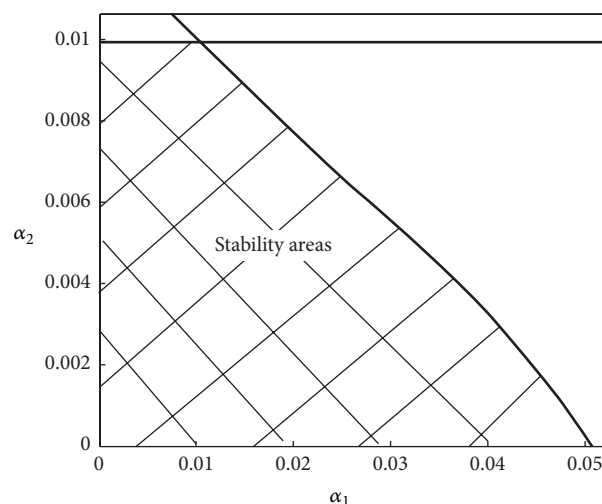
FIGURE 31: The change of $l_3(\alpha_1)$ with α_1 when $\alpha_2 = 0.004$, $\alpha_3 = 0.01$.FIGURE 32: The change of $l_3(\alpha_1)$ with α_1 when $\alpha_2 = 0.004$, $\alpha_3 = 0.03$.FIGURE 33: The change of $l_3(\alpha_1)$ with α_1 when $\alpha_2 = 0.004$, $\alpha_3 = 0.05$.FIGURE 34: The change of $l_3(\alpha_1, \alpha_2, \alpha_3)$ with α_1 , α_2 , and α_3 .

- (ii) $\alpha_1 = 0.02$, $\alpha_2 = 0.048$, and $\alpha_3 \in (0, 0.05)$ (see Figure 27).
- (iii) $\alpha_1 = 0.02$, $\alpha_2 = 0.02$, and $\alpha_3 \in (0, 0.05)$ (see Figure 28).
- (iv) $\alpha_1 = 0.02$, $\alpha_2 = 0.004$, and $\alpha_3 \in (0, 0.05)$ (see Figure 29).
- (2) Fix the value of α_2 and $\alpha_1 \in (0, 0.05)$, $\alpha_3 \in (0, 0.05)$.
 - (i) $\alpha_2 = 0.004$, $\alpha_1 \in (0, 0.05)$, and $\alpha_3 \in (0, 0.05)$ (see Figure 30).
 - (ii) $\alpha_2 = 0.004$, $\alpha_3 = 0.01$, and $\alpha_1 \in (0, 0.05)$ (see Figure 31).
 - (iii) $\alpha_2 = 0.004$, $\alpha_3 = 0.03$, $\alpha_1 \in (0, 0.05)$ (see Figure 32).
 - (iv) $\alpha_2 = 0.004$, $\alpha_3 = 0.05$, and $\alpha_1 \in (0, 0.05)$ (see Figure 33).
- (3) Fix the value of α_3 and $\alpha_1 \in (0, 0.05)$, $\alpha_2 \in (0, 0.05)$.
 - (i) $\alpha_3 = 0.03$, $\alpha_1 \in (0, 0.05)$, and $\alpha_2 \in (0, 0.05)$ (see Figure 34).
 - (ii) $\alpha_3 = 0.03$, $\alpha_1 = 0.01$, and $\alpha_2 \in (0, 0.05)$ (see Figure 35).
 - (iii) $\alpha_3 = 0.03$, $\alpha_1 = 0.02$, and $\alpha_2 \in (0, 0.05)$ (see Figure 36).
 - (iv) $\alpha_3 = 0.03$, $\alpha_1 = 0.03$, and $\alpha_2 \in (0, 0.05)$ (see Figure 37).

Premising $\alpha_3 = 0.01$, the stable area in the α_1, α_2 plane is determined by inequality group (10). Figure 38 shows the system fixed point in the area of the asymptotic stability.

3.1.2. Numerical Simulation. Using Matlab, parameters influence on the system (8) can be analyzed through numerical simulation.

(1) α_1 , α_2 , and α_3 Influence on the Collecting Market. The second collector on the assumption that the manufacturer and the first collector stem variables parameters is fixed with $\alpha_1 = 0.01$, $\alpha_3 = 0.01$, her system variables parameters α_2 is at $[0, 0.05]$. Manufacturers' initial collecting price p_m , collector 1 and 2 for manufacturers of the collecting price of depreciate rate for 20, 0.35, respectively, 0.3. And, as shown in Figure 39 changes Figures 39(a), 39(b), and 39(c) shows, the corresponding Lyapunov index as shown in Figure 39(d). Figure 39 showing, when $0 < \alpha_2 < 0.0097$, the system is in stable state. after multi game, p_m , r_1 and r_2 is stable at the point of (17.4, 0.4904, 0.4825). When $\alpha_1 = 0.01$, $\alpha_2 = 0.0097$, $\alpha_3 = 0.0097$, the system (8) occurs the first bifurcation, then after cycle 2, 4 cycle, and so forth, the system is gradually into the chaotic state. Figure 39(d) is Lyapunov index spectrum distribution which can also confirmed the phenomena. at $\alpha_2 = 0.0099$, the max Lyapunov index is zero, the system is in critical condition. At $\alpha_2 = 0.0133$, most of Lyapunov index greater than zero which explains system at chaos state (Figure 39 and Figure 44).

FIGURE 35: The change of $I_3(\alpha_2)$ with α_2 when $\alpha_1 = 0.01$, $\alpha_3 = 0.03$.FIGURE 37: The change of $I_3(\alpha_2)$ with α_2 when $\alpha_1 = 0.03$, $\alpha_3 = 0.03$.FIGURE 36: The change of $I_3(\alpha_2)$ with α_2 when $\alpha_1 = 0.02$, $\alpha_3 = 0.03$.FIGURE 38: $\alpha_3 = 0.01$, the stable area in α_1, α_2 plane.

When initial setup and other parameters are the same and α_1 , α_2 , and α_3 are adjusted in the interval $[0, 0.05]$ simultaneously, system variables of the manufacturers and two collectors change as shown in Figures 40(a), 40(b), and 40(c), and Lyapunov index is as in Figure 40(d). When $0 < \alpha_2 < 0.0098$, the system is in stable state at the point of $(17.4, 0.4904, 0.4825)$. When the value of α_1 , α_2 , and α_3 is 0.0098, the system (8) occurs the first bifurcation, then after cycle 2, 4 cycle, and so forth, the system is gradually into the chaotic state. Figure 40(d) is Lyapunov index spectrum distribution which can also confirm the phenomena. At $\alpha_2 = 0.0099$, the max Lyapunov index is zero, and the system is in critical condition. Comparing with Figure 39, system steady equilibrium does not change, keeps for $(17.4, 0.4904, 0.4825)$.

(2) *System Variables Power Spectrum.* According to the numerical simulation, not only system variables graph in the phase space along with time can be drawn, but also power spectrum graph of system variables can be estimated by period chart method. When $\alpha_1 = 0.01$, $\alpha_2 = 0.014$, and $\alpha_3 = 0.01$, power spectrum graph of the manufacturer and two collectors is shown as in Figures 41(a), 41(b), and 41(c).

According to the numerical results, no matter how large the collection price adjustment parameters are, the collection price traverses the whole value area over time, but p_m , r_1 , and r_2 in the system (10) always limit in a certain range, which also

verifies boundedness and ergodicity of chaos. Because $\alpha_2 > \alpha_3$ and r_1 amplitude is larger than r_2 , the adjustment of system variables must be limited in a certain range.

(3) *The Influence of Initial Setup.* The butterfly effect is an important symbol of chaotic motions, which is sensitive to initial state, and small changes of the initial conditions may cause the adjacent orbital evolution to index form separate after multiple episodes. For the above conditions, when $\alpha_1 = 0.01$, $\alpha_2 = 0.014$, and $\alpha_3 = 0.01$, sensitive dependence on initial value of p_m , r_1 , and r_2 is as shown in Figures 42(a), 42(b), and 42(c). p_m value is, respectively, taken 20 and 20.001, after 96 cycles iteration, p_m difference value is 0.1335 which is 133.5 times than the initial difference value. r_1 and r_2 value is, respectively, taken 0.35 and 0.351, 0.3 and 0.301, respectively, experience in 44 and 45 iteration, r_1 and r_2 difference values is respective 0.2826 and 0.1364 which are 282 times and 136 times than the initial difference values. Furthermore, from Figure 42 we can conclude that system variables values are approximate in the first period, with increase of iterations the difference increase obviously.

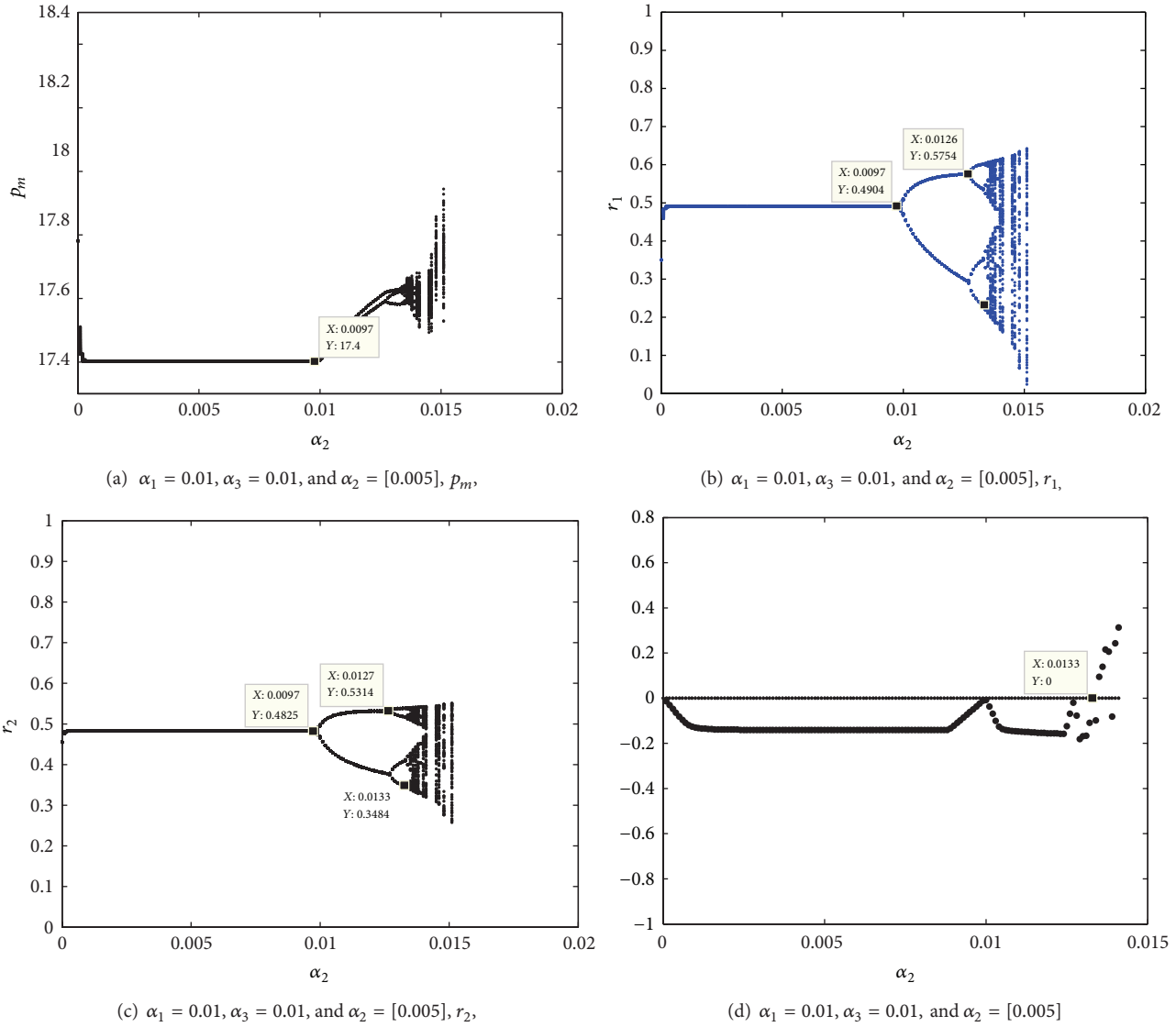


FIGURE 39: The max Lyapunov index.

3.2. Stackelberg Equilibrium

3.2.1. Model and the Analysis. Suppose that the manufacturer and collectors have principal and subordinate relationship, the manufacturer is Stackelberg leader, and collectors are followers; then they process sequential dynamic game; the game equilibrium is Stackelberg equilibrium. In this game, the manufacturer makes the decision of sales price and collection price according to the market informa Lyapunov exponent function abouttution; then two collectors make decision according to the decision-making of the manufacture:

$$\begin{aligned} \max \prod_{r1} &= (p_m - p_{r1}(t) - A_1)(k_1 + \beta_1 p_{r1}(t) - \delta_1 p_{r2}(t)) \\ \max \prod_{r2} &= (p_m - p_{r2}(t) - A_2)(k_2 + \beta_2 p_{r2}(t) - \delta_2 p_{r1}(t)) \end{aligned}$$

$$\begin{aligned} \text{s.t. } p_m &\in \arg \max \prod_{r1} \\ \text{s.t. } p_m &\in \arg \max \prod_{r2}. \end{aligned} \quad (24)$$

The result is

$$\begin{aligned} r_1^{**} &= (p_m (2\beta_1\beta_2 - 2\beta_2\delta_1 + \delta_1\beta_2 - \delta_1\delta_2) \\ &\quad + 2\beta_2k_1 + 2\beta_1\beta_2A_1 + k_2\delta_1 + \beta_2A_2\delta_1) \\ &\quad \times (p_m (4\beta_1\beta_2 - \delta_1\delta_2))^{-1}, \end{aligned}$$

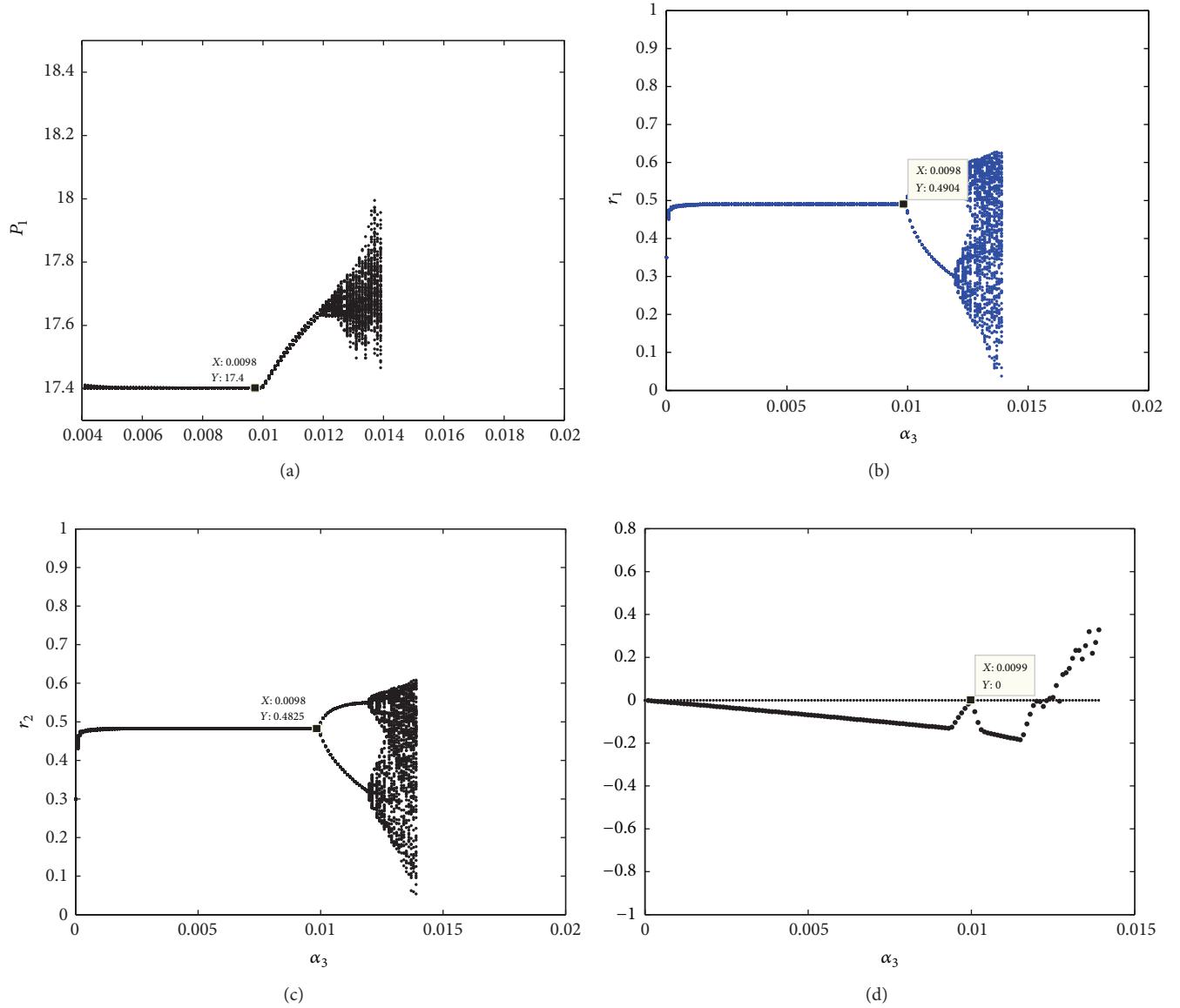


FIGURE 40: (a) $\alpha_1, \alpha_2, \alpha_3$ is $[0, 0.05]$, p_1 , (b) $\alpha_1, \alpha_2, \alpha_3$ is $[0, 0.05]$, r_1 , (c) $\alpha_1, \alpha_2, \alpha_3$ is $[0, 0.05]$, r_2 , and (d) $\alpha_1, \alpha_2, \alpha_3$ is $[0, 0.05]$, the max Lyapunov index.

$$\begin{aligned}
 r_2^{**} = & (p_m (2\beta_1\beta_2 - 2\beta_1\delta_2 + \delta_2\beta_1 - \delta_1\delta_2) \\
 & + 2\beta_1k_2 + 2\beta_1\beta_2A_2 + k_1\delta_2 + \beta_1A_2\delta_2) \\
 & \times (p_m (4\beta_1\beta_2 - \delta_1\delta_2))^{-1} \\
 & - (4\beta_1\beta_2 - \delta_1\delta_2) (k_1 + k_2)) \\
 & \times ((2\beta_1\beta_2 + 2\beta_2\delta_1 - \delta_1\beta_2) (\beta_2 - \delta_1) \\
 & + (2\beta_1\beta_2 + 2\beta_1\delta_2 - \delta_2\beta_1) (\beta_1 - \delta_2))^{-1}.
 \end{aligned} \tag{25}$$

Formula (25) is reaction function of the collectors. Put formula (25) into formula (2) and the following can be obtained:

$$\begin{aligned}
 p_m^{**} = & \frac{(p_0 - c_r)}{2} \\
 & - ((\beta_1 - \delta_2) (2\beta_2k_1 + 2\beta_2\beta_1A_1 + \delta_1k_2 + \delta_1\beta_2A_2) \\
 & + (\beta_2 - \delta_1) (2\beta_1k_2 + 2\beta_2\beta_1A_2 + \delta_2k_1 - \delta_2\beta_1A_1))
 \end{aligned}$$

With Nash equilibrium value the parameters values are $p_0 = 70, c_r = 30, A_1 = 1, A_2 = 0.8, k_1 = 1, k_2 = 1, \beta_1 = 0.6, \beta_2 = 0.55, \delta_1 = 0.2, \delta_2 = 0.2$. Then we put the values into Stackelberg equilibrium $(p_m^{**}, r_1^{**}, r_2^{**})$; the equilibrium value is $(19.17, 0.48, 0.48)$.

The manufacturer makes decision based on limited rational expectations. She adjusts the game process on the basis of marginal gains. If the marginal profits of t period are positive, in the $t + 1$ period this strategy action will be used. And if

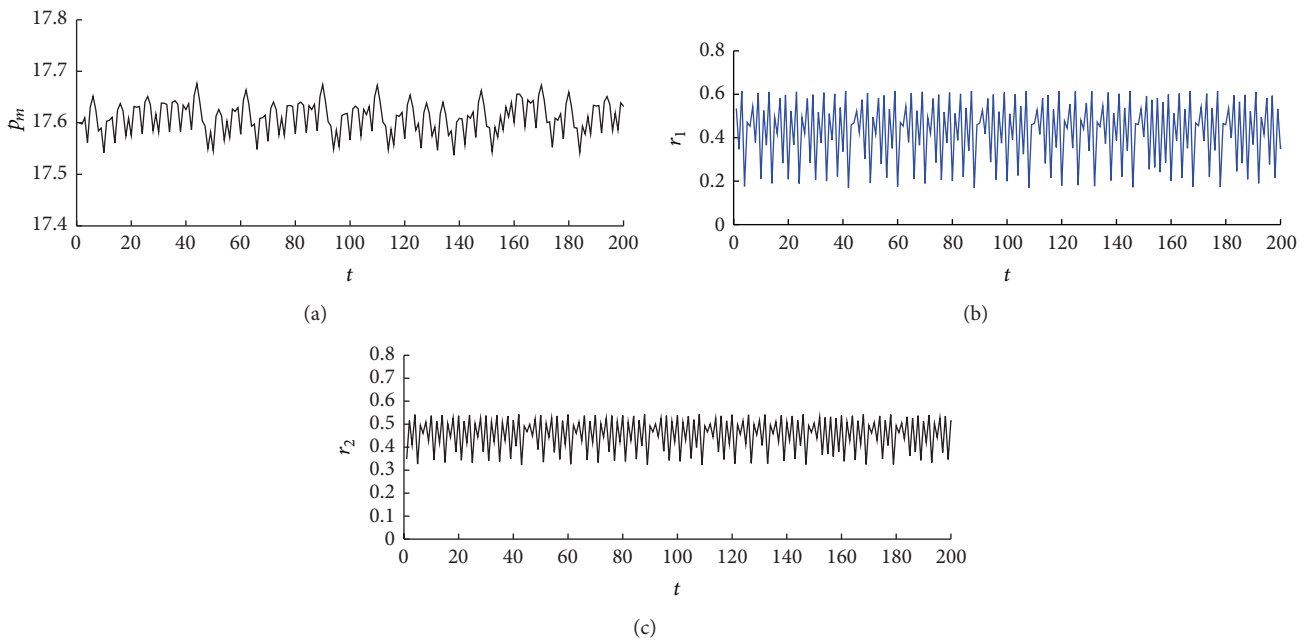


FIGURE 41: (a) $\alpha_1 = 0.01$, $\alpha_2 = 0.014$, and $\alpha_3 = 0.01$, power spectrum of p_m ; (b) $\alpha_1 = 0.01$, $\alpha_2 = 0.014$, and $\alpha_3 = 0.01$, power spectrum of r_1 ; and (c) $\alpha_1 = 0.01$, $\alpha_2 = 0.014$, and $\alpha_3 = 0.01$, power of spectrum r_2 .

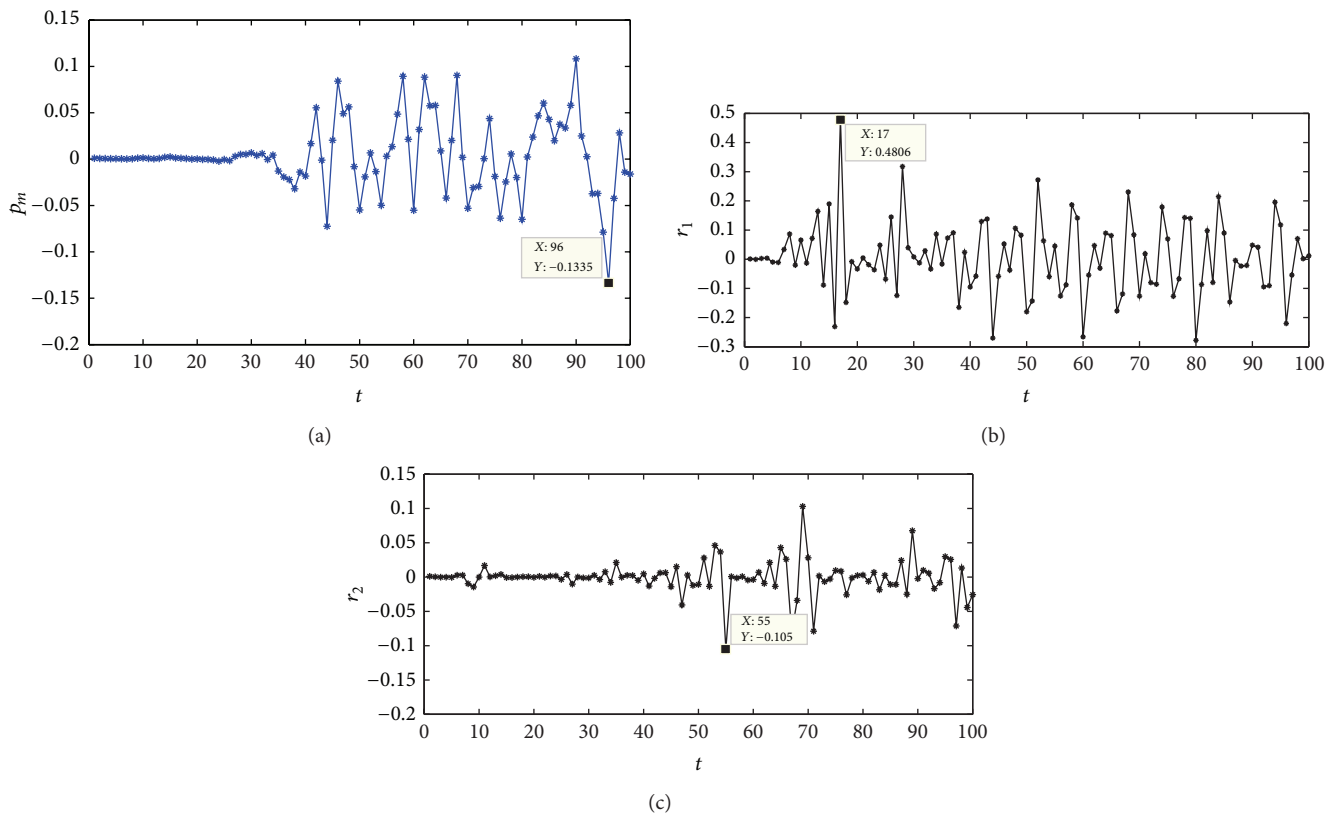


FIGURE 42: (a) $\alpha_1 = 0.01$, $\alpha_2 = 0.014$, and $\alpha_3 = 0.01$, the initial value sensitivity of p_m ; (b) $\alpha_1 = 0.01$, $\alpha_2 = 0.014$, and $\alpha_3 = 0.01$, the initial value sensitivity of r_1 ; and (c) $\alpha_1 = 0.01$, $\alpha_2 = 0.014$, and $\alpha_3 = 0.01$, the initial value sensitivity r_2 .

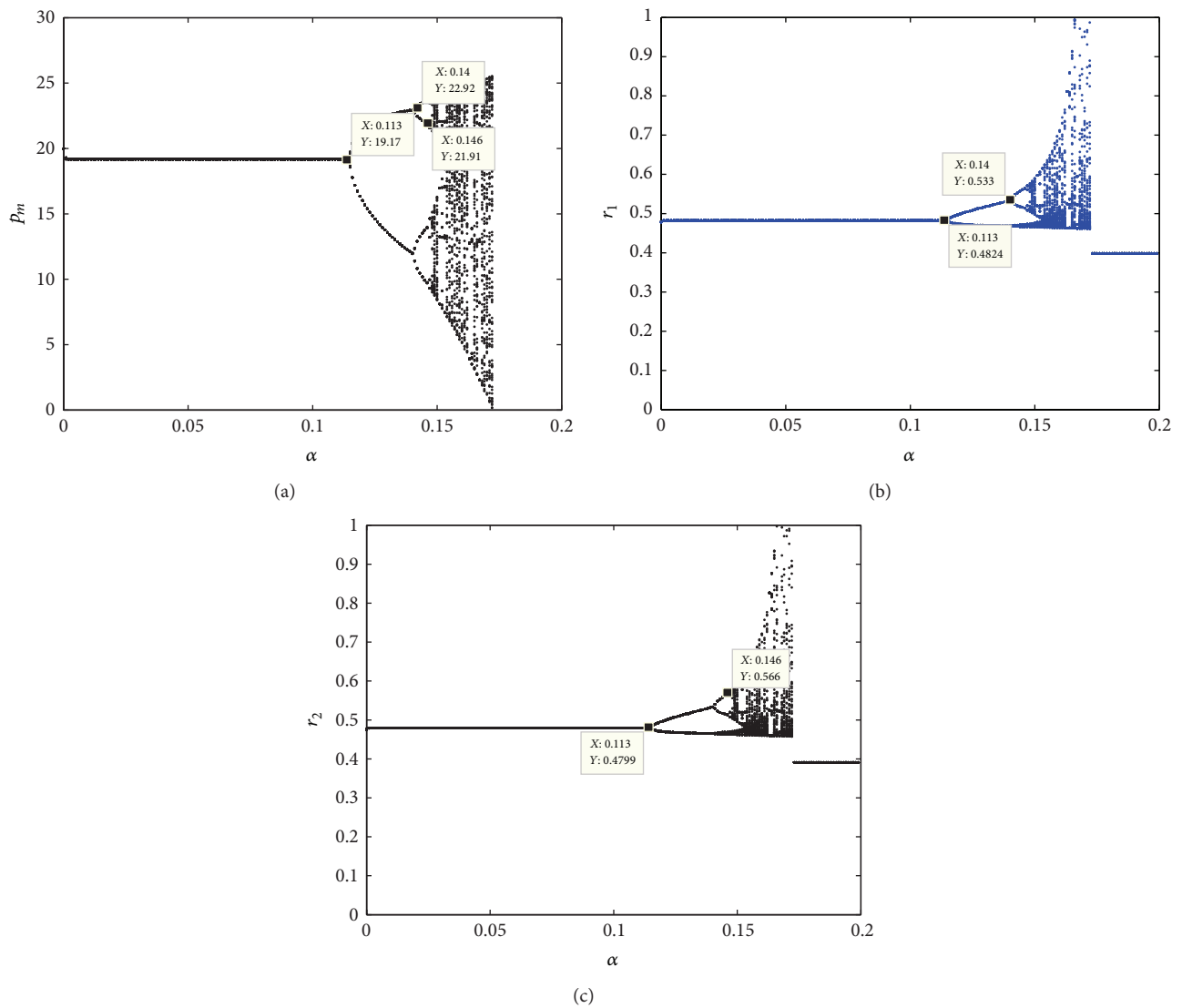


FIGURE 43: (a) $\alpha = [0, 0.05]$, p_m , (b) $\alpha = [0, 0.05]$, r_1 ; and (c) $\alpha = [0, 0.05]$, r_2 .

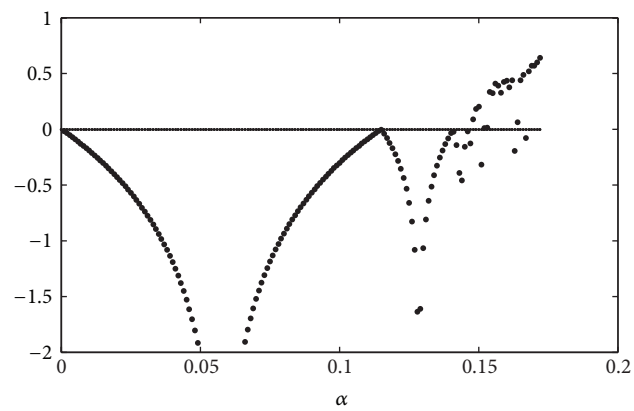


FIGURE 44: $\alpha = [0, 0.05]$, max Lyapunov spectrum.

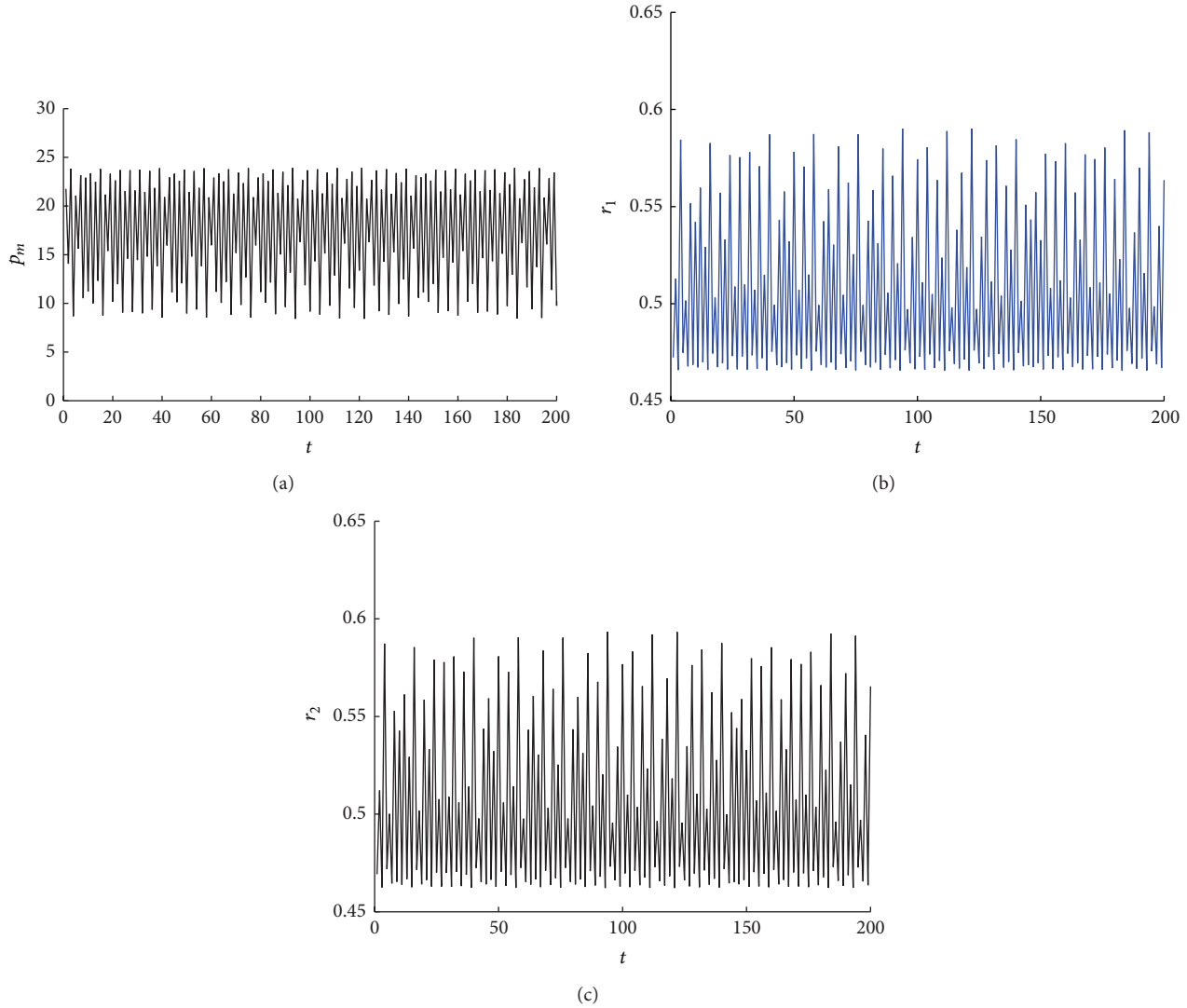


FIGURE 45: (a) $\alpha = 0.15$, the power spectrum of p_m , (b) $\alpha = 0.15$, the spectrum of r_1 ; and (c) $\alpha = 0.15$, the power spectrum of r_2 .

the t period marginal profits are negative, in the $t + 1$ period the collection price will be lowered. The repeated game model of dynamic adjustment is

$$\begin{aligned}
 p_m(t+1) &= p_m(t) + \alpha p_m(t) \\
 &\times (((1-r_1)(\beta_1 - \delta_2) + (1-r_2)(\beta_2 - \delta_1)) \\
 &\times (p_0 - 2p_m(t) - c_r) - k_1 - k_2). \quad (27)
 \end{aligned}$$

In formula (27), α is adjustment coefficient of collection price and r_1, r_2 are as follows:

$$\begin{aligned}
 r_1(t) &= (p_m(t)(2\beta_1\beta_2 - 2\beta_1\delta_1 + \delta_1\beta_2 - \delta_1\delta_2) \\
 &+ 2\beta_2k_1 + 2\beta_1\beta_2A_1 + k_2\delta_1 + \beta_2A_2\delta_1) \\
 &\times (p_m(t)(4\beta_1\beta_2 - \delta_1\delta_2))^{-1}, \quad (28)
 \end{aligned}$$

$$\begin{aligned}
 r_2(t) &= (p_m(t)(2\beta_1\beta_2 - 2\beta_1\delta_2 + \delta_2\beta_1 - \delta_1\delta_2) \\
 &+ 2\beta_1k_2 + 2\beta_1\beta_2A_2 + k_1\delta_2 + \beta_1A_2\delta_2) \\
 &\times (p_m(t)(4\beta_1\beta_2 - \delta_1\delta_2))^{-1}. \quad (29)
 \end{aligned}$$

Substitution parameters value into system (17) can be concluded which is as follows:

$$\begin{aligned}
 p_m(t+1) &= p_m(t) + \alpha p_m(t)(17.403 - 0.908p_m(t)), \\
 r_1(t) &= 0.398 + \frac{1.617}{p_m(t)}, \\
 r_2(t) &= 0.391 + \frac{1.703}{p_m(t)}. \quad (30)
 \end{aligned}$$

From system (30) it can be concluded that the manufacture firstly makes decision and the decision variables directly relate to α . But collectors' decision variables directly relate to p_m . The following simulates complex dynamics characteristics of system (30) through numerical simulation.

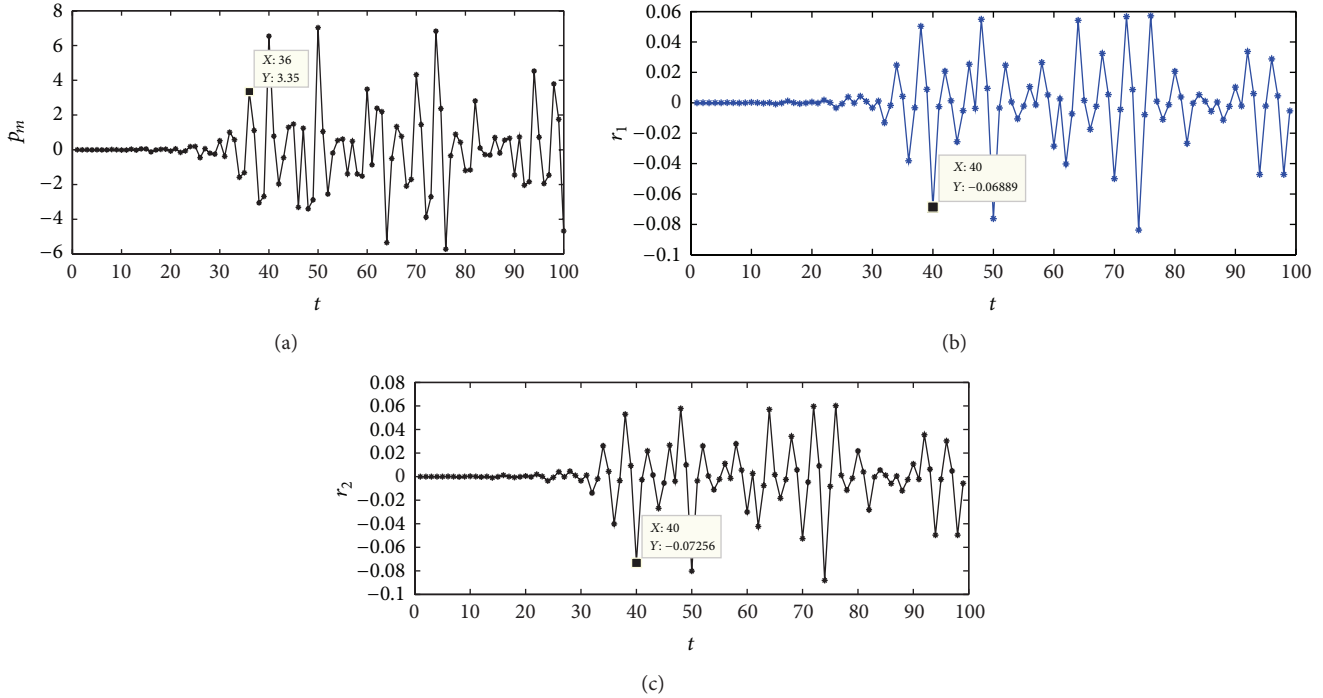


FIGURE 46: (a) $\alpha = 0.15$, the initial value sensitivity of p_m , (b) $\alpha = 0.15$, the initial value sensitivity of r_1 , and (c) $\alpha = 0.15$, the initial value sensitivity of r_2 .

3.2.2. Numerical Simulation

(1) *α Influence on the Collecting Market.* When $\alpha = [0, 0.05]$, the initial values of p_m , r_1 , and r_2 are 20, 0.4, and 0.5; p_m , r_1 , and r_2 are as shown in Figures 43(a), 43(b), and 43(c). From the figures, we can conclude that the system is in stable state when $0 < \alpha < 0.113$, and the stable values of p_m , r_1 , and r_2 are 19.17, 0.4824, and 0.4799. When $\alpha = 0.113$, the system occurs the first bifurcation, then after cycle 2, 4 cycle, and so forth, the system is gradually into the chaotic state.

(2) *System Variables Power Spectrum.* According to the numerical simulation, not only system variables graph in the phase space along with time can be drew, but also power spectrum graph of system variables can be estimated by period chart method. When $\alpha = 0.15$, power spectrum graph of the manufacturer and two collectors is shown in Figures 45(a), 45(b), and 45(c). According to the numerical results, no matter how large the collection price adjustment parameters are, the collection price traverses the whole value area over time, but p_m , r_1 , and r_2 in the system (19) always limit in a certain range, which also verifies boundedness and ergodicity of chaos.

(3) *The Influence of Initial Setup.* When $\alpha = 0.15$, sensitive dependence on initial value of p_m , r_1 , and r_2 is as shown in Figures 46(a), 46(b), and 46(c). p_m values are, respectively, taken to be 20 and 20.001, after 36 cycles iteration, p_m difference value is 3.35 which is 3350 times the initial difference value. r_1 and r_2 difference values are 69 times and 73 times the initial difference values. Furthermore, from Figure 46 we can conclude that system variables values are approximate

in the first period; with increase of iterations the difference increases obviously. The influence to the manufacturer is far greater than collectors because of the collectors' decision-making later than the manufacturer's. After the collectors observe action of the manufacturer, they make decision. The collectors' decision-making is superior to the manufacturer.

3.3. Model Comparison and Analysis

(1) *Profits Analysis of Nash Equilibrium.* When $\alpha = [0, 0.05]$, profits of the manufacturer and two collectors are as shown in Figures 47(a), 47(b), 47(c), and 47(d). Figure 47(a) shows that profits of the manufacturer in the stable state are not the best; it means Nash equilibrium is not optimal solution; the optimal solution of single cycle may not bring enterprise maximum returns. Furthermore, from Figure 47(a) it can be concluded that, with collecting price increasing, profits state instead into chaos state, the optimal price is 19.64 in the chaotic state, and the corresponding profits are 208.6. Compared to Figure 47(b), profits of the manufacture before chaotic state are 216.4, so the internal randomness of chaotic system cannot react to the actual situation of the market.

(2) *Profits Analysis of Stackelberg Equilibrium.* When $\alpha = [0, 0.05]$, profits of the manufacturer and two collectors are as shown in Figures 48(a), 48(b), 48(c), and 48(d). Figure 48 shows that profits of the manufacturer in the stable state are the best; it means Stackelberg equilibrium is the optimal solution, and the optimal solution of the closed-loop supply chain is optimal. Figures 48(a), 48(b), 48(c), and 48(d) are consistent.

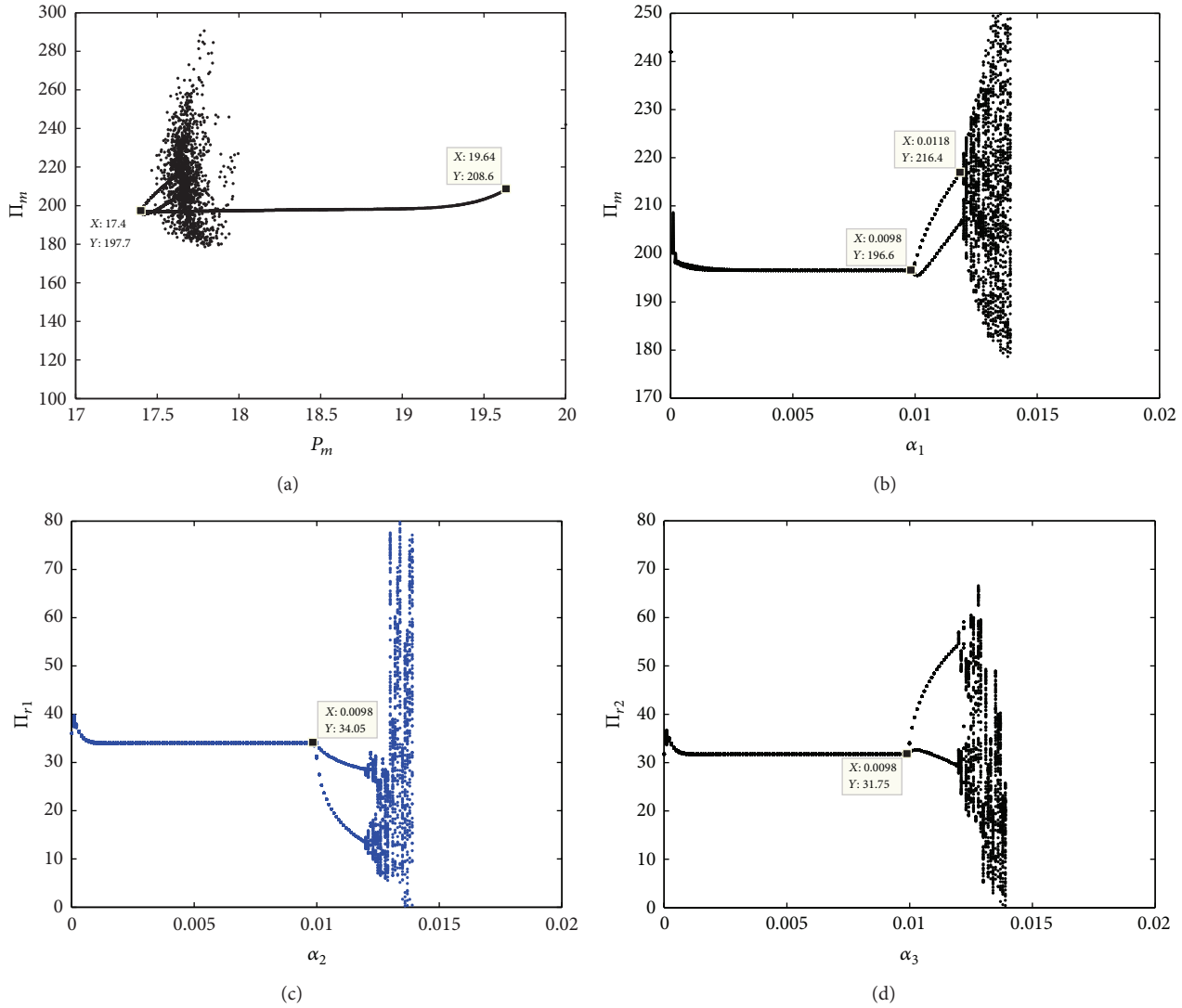


FIGURE 47: (a) Π_m graph with p_m , (b) Π_m graph with α_1 , (c) Π_{r1} graph with α_2 , and (d) Π_{r2} graph with α_3 .

Through numerical simulation and comparison analysis, it can be concluded that Stackelberg game equilibrium is better than Nash equilibrium from the point of view of the profits and collecting price. Three conclusions can be summarized as follows. First, when the manufacturer and the two collectors make decision by static game, the collection price and profits of the three parties are lower than the optimal of dynamic game equilibrium. Second, as the manufacturer and the two collectors independently make decisions, the three parties will try to lower the collecting price in order to obtain the maximum profits. Third, as the manufacturer and the two collectors independently make decisions, the system more easily reaches chaos, and the cycle of stable state is shorter.

4. Centralized Control Decision-Making Model

Centralized control is that the manufacturers and the collectors codetermine to realize profits maximization of the supply

chain system. We get formulas (2) and (3) into the formula (4) and attain the profits function of supply chain as follows:

$$\begin{aligned}
 \Pi &= \Pi_m + \Pi_{r1} + \Pi_{r2} \\
 &= k_1 (p_0 - c_r - A_1) + k_2 (p_0 - c_r - A_2) \\
 &\quad + p_m(t) (1 - r_1(t)) \\
 &\quad \times ((p_0 - c_r) (1 - \delta_2) - k_1 - A_1 - A_2 \delta_2) \\
 &\quad - p_m(t)^2 (\beta_1 (1 - r_1(t))^2 + \beta_2 (1 - r_2(t))^2) \\
 &\quad + p_m(t) (1 - r_2(t)) \\
 &\quad \times ((p_0 - c_r) (1 - \delta_1) - k_2 - A_2 - A_1 \delta_1) \\
 &\quad + p_m(t)^2 (\delta_1 + \delta_2) (1 - r_1(t)) (1 - r_2(t)).
 \end{aligned} \tag{31}$$

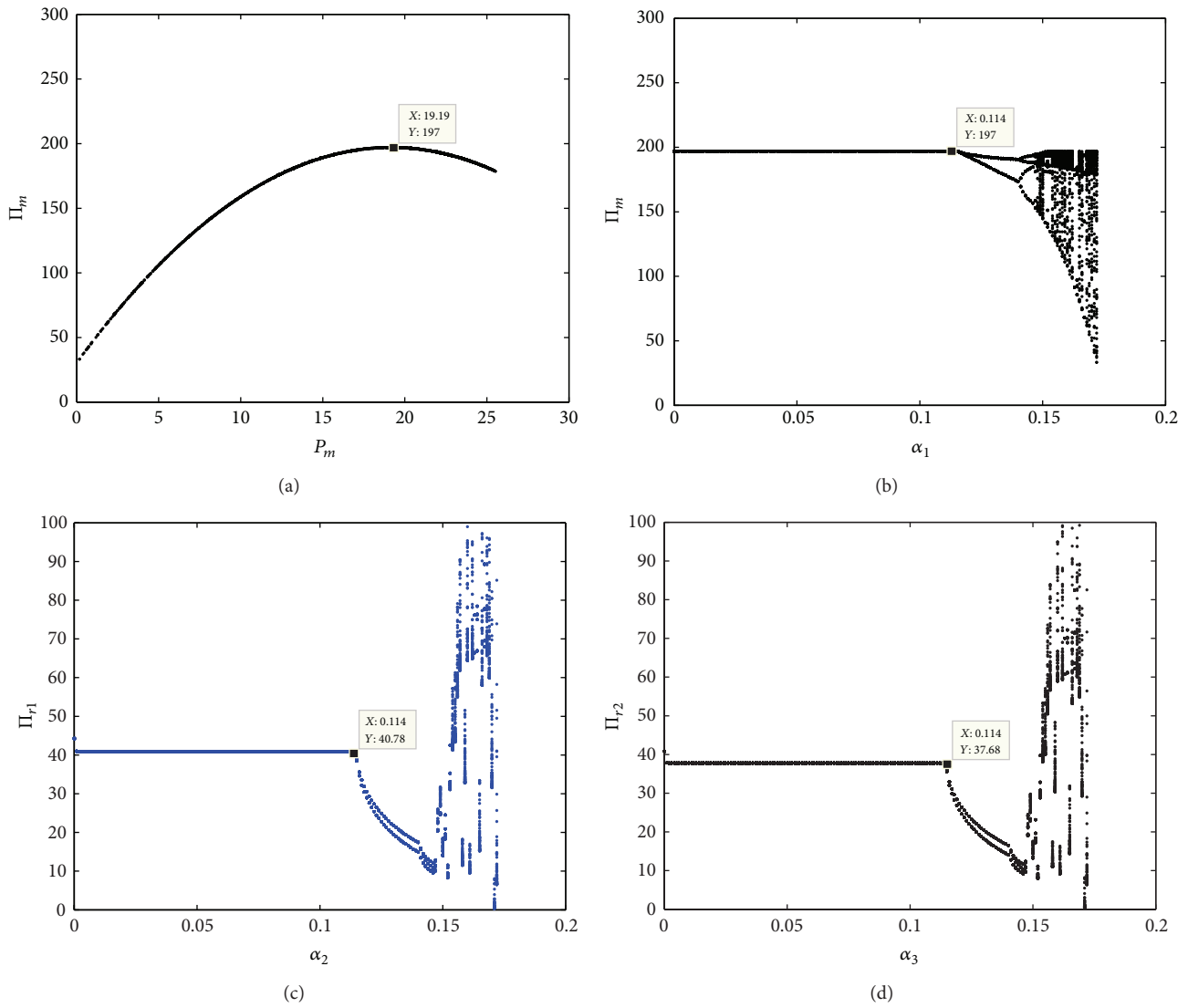


FIGURE 48: (a) Π_m graph with p_m , (b) Π_m graph with α_1 , (c) Π_{r1} graph with α_2 , and (d) Π_{r2} graph with α_3 .

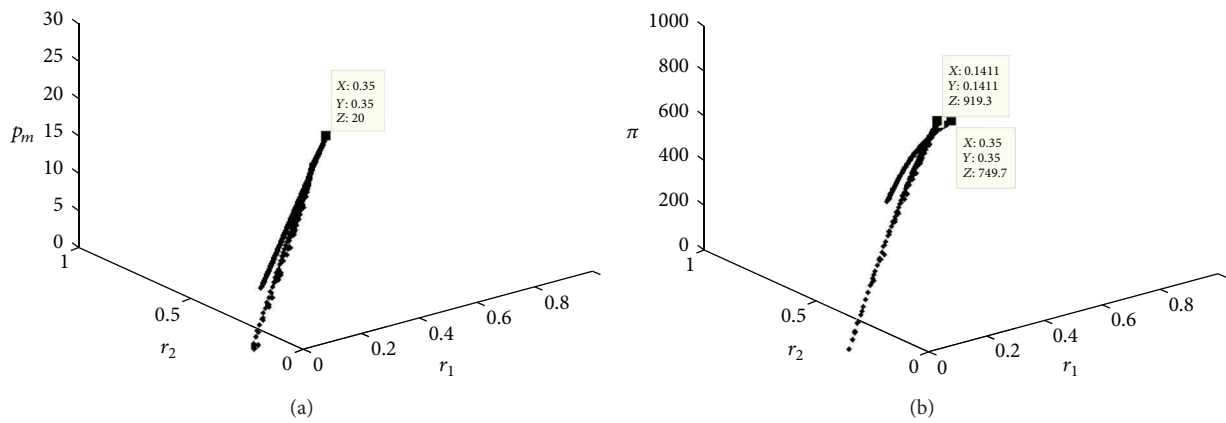


FIGURE 49: (a) p_m , r_1 , and r_2 conditions with time and (b) Π , r_1 , r_2 condition with time.

The second-order derivatives about p_m , r_1 , and r_2 in formula (31) are $\partial^2 \Pi / \partial p_m^2 = -2(\beta_1(1-r_1(t))^2 + \beta_2(1-r_2(t))^2) + 2(\delta_1 + \delta_2)(1-r_1(t))(1-r_2(t))$, $\partial^2 \Pi / \partial r_1^2 = -2\beta_1 p_m(t)^2 < 0$, $\partial^2 \Pi / \partial r_2^2 = -2\beta_2 p_m(t)^2 < 0$. When meeting the condition of $(\delta_1 + \delta_2)(1-r_1(t))(1-r_2(t)) < \beta_1(1-r_1(t))^2 + \beta_2(1-r_2(t))^2$, Π is strictly concave function about p_m , r_1 , and r_2 . Therefore, the optimal solutions of p_m , r_1 , and r_2 can be obtained by $\partial \Pi / \partial p_m(t) = 0$, $\partial \Pi / \partial r_1 = 0$, and $\partial \Pi / \partial r_2 = 0$. The process is as follows:

$$\begin{aligned} \frac{\partial \Pi}{\partial p_m} &= (1-r_1(t))((p_0-c_r)(1-\delta_2)-k_1-A_1-A_2\delta_2) \\ &\quad + (1-r_2(t))((p_0-c_r)(1-\delta_1)-k_2-A_2-A_1\delta_1) \\ &\quad - 2p_m(t)(\beta_1(1-r_1(t))^2 + \beta_2(1-r_2(t))^2) \\ &\quad + 2p_m(t)(\delta_1 + \delta_2)(1-r_1(t))(1-r_2(t)) = 0, \\ \frac{\partial \Pi}{\partial r_1} &= -p_m(t)((p_0-c_r)(1-\delta_2)-k_1-A_1-A_2\delta_2) \\ &\quad + 2p_m(t)^2\beta_1(1-r_1(t)) \\ &\quad - p_m(t)^2(\delta_1 + \delta_2)(1-r_2(t)) = 0, \\ \frac{\partial \Pi}{\partial r_2} &= -p_m(t)((p_0-c_r)(1-\delta_1)-k_2-A_2-A_1\delta_1) \\ &\quad + 2p_m(t)^2\beta_2(1-r_2(t)) \\ &\quad - p_m(t)^2(\delta_1 + \delta_2)(1-r_1(t)) = 0. \end{aligned} \quad (32)$$

Value with above the top: $p_0 = 90$, $c_r = 30$, $A_1 = 1$, $A_2 = 0.8$, $k_1 = 1$, $k_2 = 1$, $\beta_1 = 0.6$, $\beta_2 = 0.55$, $\delta_1 = 0.2$, $\delta_2 = 0.2$. $p_m(1) = 20$, $r_1(1) = 0.35$, and $r_2(1) = 0.3$. The equilibrium solution of formula (22) is $(\bar{p}_m, \bar{r}_1, \bar{r}_2) = (20, 0.35, 0.35)$.

Manufacturers and collecting business make decision based on rational expectations, and the decision-making basis is marginal profits. Figure 16 indicates that p_m , r_1 , and r_2 three-dimensional changes with time. X, Y, and Z, respectively, show r_1 , r_2 , and p_m . Figure 49 describes system profits Π ; r_1 and r_2 change with time. Maximum system profits are 919.3. In this condition, r_1 , r_2 are, respectively, 0.1411 and 0.1411. In the centralized control decision-making model, we can conclude that the collecting business profits are suffered, but the manufacturer profits are superior to decentralized control decision-making model. Therefore, not only the system profits maximization but also business profits are considered in reality.

5. Conclusions

This paper researches on that used products collecting pricing game model and complexity analysis in a closed-loop supply chain which consist of a manufacturer and two collectors. Corresponding to the discrete system, we have established and analyzed in detail the corresponding continuity system.

Through quantitative analysis of collecting price, time sequence diagram, and Lyapunov index, the paper describes game evolution rule in the closed-loop supply chain. We describe a series of chaotic system characteristics, based on which, we have a detailed analysis of the Lyapunov index which is under the condition of different parameter combination, and draw the figure under different conditions. The analysis found that, first, when the collection price is to a critical value, the system into chaos state. Second, the sale price of remanufacturing products are more than a critical value, the system into chaos state. Last, the collection price system of the initial value of which is sensitive, small changes in the initial value may cause market price fluctuations.

It also draws conclusions that business profits, supply chain system profits, collecting price in Stackelberg game model are superior to Nash game model. In Nash equilibrium, the manufacturer and the collectors all lower the collecting price to gain, respectively, maximum profits. The system is easier into chaos state than Stackelberg, and stable state period is shorter.

Through comparing centralized decision-making with decentralized decision-making, we have concluded that the collecting price and system profits of the latter are lower. This conclusion for the supply chain guide enterprises sure each phase of the best provide decision basis for the collection price, in order to achieve maximize returns.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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