

## Editorial

# $L^p$ Theory of Differential Forms and Related Operators with Applications

Shusen Ding,<sup>1</sup> Yong Wang,<sup>2</sup> Peilin Shi,<sup>3</sup> and Yuming Xing<sup>2</sup>

<sup>1</sup> Department of Mathematics, Seattle University, Seattle, WA 98122, USA

<sup>2</sup> Department of Mathematics, Harbin Institute of Technology, Harbin, China

<sup>3</sup> Department of Epidemiology, Harvard University, Boston, MA 02115, USA

Correspondence should be addressed to Shusen Ding; [sding@seattleu.edu](mailto:sding@seattleu.edu)

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In order to stimulate further investigation on  $L^p$  theory of differential forms and related operators, to provide readers with a good research resource, and to build up a timely communication stage for our colleagues in these areas, we decided to edit and publish this special issue for *Abstract and Applied Analysis*. Differential forms are extensions of functions and have become important tools for many fields of sciences and engineering, such as theoretical physics, general relativity, and potential theory. Operators are important tools in analysis and PDEs. Much progress has been made in the study of differential forms and related operators in recent years. This special issue provided some of the latest and significant results in this field.

The papers appearing in the special issue highlight recent advances in the areas. Specifically, three papers in this special issue deal with Poincaré-type inequalities for differential forms and related operators applied to these forms. Both the local and global Poincaré-type inequalities with  $L^p$ -norms for some operators, such as homotopy and Green's operator, were established in different domains in these papers. A class of variational integrals whose Euler equations are nonhomogeneous  $A$ -harmonic equations was introduced and the relationship between the minimization problem and the Euler equation was investigated in one of the papers published in this special issue. The existence and uniqueness of weak solution for the obstacle problem of the nonhomogeneous  $A$ -harmonic equation with variable exponent were discussed, and the existence of the solutions of the equation in the weighted variable exponent Sobolev

space was proved in one of the papers. We also contributed a survey paper on  $L^p$  theory of the homotopy operator applied to differential forms, in which an up-to-date account of the recent advances made in the study of this field was provided. One paper is concerned with the abstract Cauchy problems that depend on parameters. The situation considered in the paper is when the operator of the Cauchy problem is not densely defined. By applying integrated semigroup theory and the results on continuity in the parameters of  $C_0$ -semigroup and integrated semigroup, the author obtains the results on the existence and continuity in parameters of the classical solutions of the Cauchy problems. In one of the papers, the inverse eigenvalue problem for irreducible doubly stochastic matrices of small orders was studied. The guest editors of this special issue did not expect such a variety of the papers which has largely enriched this special issue when they first proposed this issue. For instance, readers will find a paper focused an efficient variational method for image restoration. Image restoration is one of the most fundamental issues in imaging science. Total variation regularization is widely used in image restoration problems for its capability to preserve edges. A constrained minimization problem with double total variation regularization terms was solved. Also, in another paper appearing in this issue, new regularization models for image denoising with a spatially dependent regularization parameter were discussed.

We believe that the new results and methods appearing in this special issue papers will develop broad and rigorous conceptual understanding of the  $L^p$  theory of differential

forms and related operators as well as the related topics and will explore the further applications in different fields of mathematics and related areas.

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Finally, the guest editors would like to express their appreciation to the authors and reviewers who have spent so much time and effort on this special issue.

*Shusen Ding*  
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