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Research Article

Reliable l_2–l_∞ and H_∞ Control for Nonlinear Singular Systems via Dynamic Output Feedback

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The reliable l_2–l_∞ and H_∞ control for a class of Lipschitz nonlinear discrete-time singular systems with time delay is investigated via dynamic feedback control. The main goal of this paper is to design a generalized nonlinear controller such that, for possible actuator failures, the closed-loop system is regular, casual, and stable with a given l_2–l_∞ and H_∞ disturbance attenuation level being satisfied. Some sufficient conditions are obtained in terms of linear matrix inequalities (LMIs), and the controller design method is also proposed. Finally, a numerical example is included to illustrate the effectiveness of our proposed results.

1. Introduction

Singular systems [1] are also referred to as generalized systems or descriptor systems, differential-algebraic systems, or implicit systems, which arise in many practical physical systems such as electric systems, robotic systems, power systems, networked control systems, and space navigation systems. Considerable efforts have been done to the system analysis [2, 3], engineering applications [4], and control synthesis [5–14] for singular systems. Compared with the stability analysis of normal systems,that of singular systems is much more complicated since regularity and absence of impulse (or casual) are necessary to be considered simultaneously. Meanwhile, nonlinearity is an universal phenomenon existing in practical control systems, which can not be ignored. And It is always a source of instability and poor performance. For nonlinear singular systems, the solutions may not exist even if the systems' linear parts are regular. Currently, for a class of Lipschitz continuous nonlinear singular systems, the robust nonlinearH_∞filtering was investigated in [15]. And the authors in [16] considered a class of nonlinear continuous plant presented by a Takagi-Sugeno fuzzy model and designed a nonfragile H_∞ filter. However, up to date, there are few papers considering the control problem of discrete-time singular systems, especially for the nonlinear discrete-time singular ones. On the other hand, reliable control is an interesting problem in control theory, and has gained considerable attention, and a number of results have been reported in the literature [17, 18]. Up to date, to the best of our knowledge, the research on discrete-time singular nonlinear systems with time delays is still an open problem that deserves further investigation.

Performance analysis has grown, in the past few decades, as one of the most important problems in control theory in addition to stability analysis. Many control problems, to a certain extent, can be equal to designing proper controller such that the closed-loop system is asymptotical stable and its performance satisfies some requirements. The general adopted performance indexes include H_2 index, H_∞ index, L_1 index, and L_2–L_∞ index. In particular, in recent years, there are many important results on the problem of stabilization based on H_∞ and L_2–L_∞ control which have been reported in literature [9, 15, 19–26].

Motivated by the above discussion, in this paper, we focus on a generalized framework for reliable nonlinear l_2–l_∞ and H_∞ control of a discrete-time Lipschitz singular system subject to time-delay and disturbance uncertainties. The main contributions of this paper are (1) a new criterion for nonlinear discrete-time singular systems is derived. The obtained criterion can ensure the regularity, casuality, and stability of the considered system. With introducing some slack matrices in the derivation, the solution space of the controller parameters is expanded. (2) A novel reliable nonlinear l_2–l_∞ and H_∞ controller is proposed, which is of more general dynamically framework. (3) In the proposed controller design method, we only solve one strict LMI, but no any semidefinite positive matrix inequality is needed, which causes a simpler design method.

Notation 1. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n \times m$ real matrices. Sym(A) denotes $A + A^T$ for simplicity. The superscripts -1 and T indicate the inverse and the transpose of a matrix, respectively. The symbol $*$ denotes the symmetric part of a symmetric matrix. I is an identity matrix of appropriate dimensions, while I_r is an $r \times r$ identity matrix and diag$\{\cdots\}$ denotes a block-diagonal matrix. The symmetric matrix $P > 0$ (or $P \geq 0$) means that P is positive definite (or positive semidefinite).

2. Preliminaries and Problem Formulation

Consider the following nonlinear discrete-time singular time-delay system described by

$$\begin{aligned} Ex(k+1) &= Ax(k) + A_1 x(k-d) \\ &\quad + \Phi(k, x_k) + Bu(k) + D\omega(k), \\ y(k) &= Cx(k) + C_1 x(k-d) + D_1\omega(k), \\ z(k) &= Lx(k) + B_1 u(k) + D_2\omega(k), \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $y(k) \in \mathbb{R}^l$ is the measured output, $z(k) \in \mathbb{R}^q$ is the controlled output signal, $u(k) \in \mathbb{R}^m$ is the control input, $\omega(k) \in \mathbb{R}^p$ is the nonzero exogenous disturbance input that belongs to $l_2[0,\infty)$, and $\Phi(k, x_k)$ is nonlinear function about the state $x(k)$. Here, we denote $x(k) = x_k$. d denotes a constant time delay. E, A, A_1, B, D, C, C_1, D_1, L, B_1, and D_2 are known real constant matrices of appropriate dimensions. Note that if the matrix E is nonsingular, the singular system (1) could be reduced to a conventional state-space system.

In this paper, we assume that E is a singular matrix with $0 < \operatorname{rank} E = r < n$; then, there exist nonsingular matrices $P \in \mathbb{R}^{n\times n}$ and $Q \in \mathbb{R}^{n\times n}$, such that

$$PEQ = \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}. \tag{2}$$

Therefore, without loss of generality, we take

$$E = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \qquad A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}. \tag{3}$$

We also assume that the system (1) is locally Lipschitz with respect to $x(k)$ in a region $\mathscr{D}$ containing the origin; that is,

$$\begin{aligned} &\Phi(0, x_0) = 0, \\ &\|\Phi(k, x_{1k}) - \Phi(k, x_{2k})\| \\ &\quad \leqslant \|H(x_{1k} - x_{2k})\|, \quad \forall x_{1k}, x_{2k} \in \mathscr{D}, \end{aligned} \tag{4}$$

where $\|\cdot\|$ is the induced 2-norm and H is the Lipschitz real matrix of $\Phi(k, x_k)$ of appropriate dimensions.

Firstly, we consider the autonomous discrete-time singular time-delay system of (1)

$$Ex(k+1) = Ax(k) + A_1 x(k-d) + \Phi(k, x_k) \tag{5}$$

and introduce some elementary definitions that will be adopted throughout this paper.

Definition 1 (see [10]). The pair (E, A) is said to be regular if there exists a scalar $s \in \mathscr{C}$ such that $\det(sE - A) \neq 0$, the pair (E, A) is said to be causal if $\deg(\det(sE - A)) = \operatorname{rank}(E)$, and the pair (E, A) is said to be stable if all the roots of $\det(sE - A)$ lie in the interior of unit disk. We call the pair (E, A) admissible if it is regular, casual, and stable, simultaneously. Furthermore, the system (5) is said to be regular, casual, and stable (asymptotically stable) if the pair (E, A) is admissible.

For system (1), we propose the following dynamic output feedback controller:

$$\begin{aligned} E_f x_f(k+1) &= A_f x_f(k) + B_f y(k) + W_f \Phi(k, x_f(k)), \\ u(k) &= C_f x_f(k) + D_f y(k), \end{aligned} \tag{6}$$

where $x_f(k) \in \mathbb{R}^n$ and $u_f(k) \in \mathbb{R}^m$ are the state and the output of the controller, respectively. Hence, the matrices A_f, B_f, C_f, D_f, and W_f are the controller gain matrices to be determined.

Remark 2. If set $W_f = 0$, the controller (6) will yield the following normal dynamic structure:

$$\begin{aligned} E_f x_f(k+1) &= A_f x_f(k) + B_f y(k), \\ u(k) &= C_f x_f(k) + D_f y(k). \end{aligned} \tag{7}$$

When the actuators experience failures, we use $u^F(k)$ to describe the control input signal sent from the actuator. In general, we consider the actuator failure model [17] with failure parameter F

$$u^F(k) = Fu(k), \tag{8}$$

where

$$F = \operatorname{diag}\{f_1, f_2, \ldots, f_m\} \tag{9}$$

with $|f_i| \leqslant \delta_i$, for any $i = 1, 2, \ldots, m$. It is easy to get

$$F \leqslant \Delta_F = \operatorname{diag}\{\delta_1, \delta_2, \ldots, \delta_m\}. \tag{10}$$

Considering the controller physical implementation convenience in the practical engineering, here, we are interested in a normal controller (6) but not in a singular one. In addition, the design of a controller (6) is simpler than that of a singular controller viewed from the theoretical analysis. Therefore, in this paper, without loss of generality, we assume that the controller is in regular state-space system; that is $E_f = I$. Just for convenience, we set $D_f = 0$.

Define the augmented vector

$$\xi(k) = \begin{pmatrix} x(k) \\ x_f(k) \end{pmatrix}. \quad (11)$$

With the aforementioned operator and system (1), controllers (6) and (8), the closed-loop system is immediate

$$\widetilde{E}\xi(k+1) = \widetilde{A}\xi(k) + \widetilde{A}_1\xi(k-d) + G\Lambda(k,\xi(k)) + \widetilde{B}\omega(k),$$
$$z(k) = \widetilde{L}\xi(k) + D_2\omega(k), \quad (12)$$

where

$$\widetilde{E} = \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix}, \qquad \widetilde{A} = \begin{bmatrix} A & BFC_f \\ B_fC & A_f \end{bmatrix},$$
$$\widetilde{A}_1 = \begin{bmatrix} A_1 & 0 \\ B_fC_1 & 0 \end{bmatrix}, \qquad G = \begin{bmatrix} I & 0 \\ 0 & W_f \end{bmatrix},$$
$$\widetilde{B} = \begin{bmatrix} D \\ B_fD_1 \end{bmatrix}, \qquad \widetilde{L} = \begin{bmatrix} L & B_1FC_f \end{bmatrix}, \quad (13)$$
$$\Lambda(k,\xi(k)) = \left[\phi(k,x_k)^T \quad \phi(k,x_f(k))^T\right]^T.$$

The objective of this paper is to design a general nonlinear dynamic output feedback controller of the form (6) such that the resulting closed-loop system (12) is regular, casual, and stable, with a prescribed l_2–l_∞ and H_∞ performance level γ being satisfied. More specially, we are dedicated to find the controller gain matrices A_f, B_fC_f, and W_f such that

(i) the closed-loop system (12) with $\omega(k) = 0$ is admissible;

(ii) under the zero-valued initial state condition, for any nonzero disturbance input $\omega(k) \in l_2[0,\infty)$, we have

$$\|z(k)\|_\infty < \gamma\|\omega\|_2, \qquad \|z(k)\|_2 < \gamma\|\omega\|_2, \quad (14)$$

where γ is a known positive scalar and

$$\|z(k)\|_\infty = \left(\sup z^T(k)\,z(k)\right)^{1/2},$$
$$\|z(k)\|_2 = \left(\sum_{k=1}^{\infty} z^T(k)\,z(k)\right)^{1/2}, \quad (15)$$
$$\|\omega(k)\|_2 = \left(\sum_{k=1}^{\infty} \omega^T(k)\omega(k)\right)^{1/2}.$$

3. Main Results

Firstly, a generalized stability criterion for discrete-time nonlinear singular system is proposed in this section. And then, based on this obtained result, a sufficient condition for the existence of a desired full-order l_2–l_∞ and H_∞ controller (6) for system (1) is obtained, which can guarantee that the resulting closed-loop system (12) is admissible (regular, casual, and stable) while satisfying a prescribed l_2–l_∞ and H_∞ performance γ. Also, the controller design method is derived.

3.1. l_2–l_∞ and H_∞ Performance Analysis. In this subsection, we concentrate our attention on the problems of system admissibility containing regularity, casuality, and stability and l_2–l_∞ and H_∞ performance analysis for the considered system (1). Initially, considering the autonomous nonlinear singular system (5), we have the following.

Theorem 3. *For any nonlinear function $\Phi(k, x_k)$ satisfying (4), the system (5) is admissible, if there exist positive definite matrices Q, R, matrices M, N, and a symmetry matrix P of the following form:*

$$M = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}, \qquad P = \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix} \quad (16)$$

with $P_1 = P_1^T > 0$, $P_3 = P_3^T$, such that

$$\mathscr{M} = \begin{bmatrix} (1,1) & E^TRE + MA_1 & M & A^TP + A^TN^T - d^2E^TR - M \\ * & -Q - E^TRE & 0 & A_1^TP + A_1^TN^T \\ * & * & -I & P + N^T \\ * & * & * & -Sym(N) + P + d^2R \end{bmatrix}, \quad (17)$$

where $(1,1) = -3E^TPE + Q + (d^2-1)E^TRE + H^TH + Sym(MA)$.

Proof. Firstly, we will prove the regularity and casuality of system (5). From (17), we have

$$-3E^TPE + \mathrm{Sym}(MA) < 0. \quad (18)$$

Then, if follows from (3) that

$$\begin{bmatrix} -3P_1 + \mathrm{Sym}(M_1A_{11}) & M_1A_{11} + A_{21}^TM_2^T \\ * & Sym(M_2A_{22}) \end{bmatrix} < 0. \quad (19)$$

It is obvious that

$$Sym(M_2A_{22}) < 0 \quad (20)$$

which implies that the matrix A_{22} is singular. In this case, we can get

$$\det(sE - A) = \det\left(sI_r - A_{11} + A_{12}A_{22}^{-1}A_{21}\right) \cdot \det(-A_{22}) \quad (21)$$

when choosing the scalar s to be of some value which is not equal to any eigenvalue of the matrix $A_{11} - A_{12}A_{22}^{-1}A_{21}$; then, we have

$$\det\left(sI_r - A_{11} + A_{12}A_{22}^{-1}A_{21}\right) \neq 0 \quad (22)$$

and therefore

$$\det(sE - A) \neq 0. \quad (23)$$

Hence, by Definition 1, the pair (E, A) is regular; that is, the system (5) is regular. Further, it follows from (21) that

$$\deg\ \det(sE - A) = r = \mathrm{rank}\,E \quad (24)$$

which yields that the pair (E, A) is casual; that is, the system (5) is casual. Thus, condition (17) in Theorem 3 can guarantee the regularity and casuality of system (5).

Next, in order to show the stability of system (5), we define the following Lyapunov-Krasovskii functional candidate as:

$$\begin{aligned} V(k) &= 3x^T(k)E^TPEx(k) + \sum_{i=-d}^{-1} x^T(k+i)Qx(k+i) \\ &\quad + d\sum_{j=-d}^{-1}\sum_{i=k+j}^{k-1}(x(i+1)-x(i))^T \\ &\quad \times E^TRE(x(i+1)-x(i)), \end{aligned} \tag{25}$$

where P has been defined in (16). Taking the difference of the Lyapunov functional $V(k)$ leads to

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k) \\ &= 3x^T(k+1)E^TPEx(k+1) - 3x^T(k)E^TPEx(k) \\ &\quad + x^T(k)Qx(k) - x^T(k-d)Qx(k-d) \\ &\quad + d^2(x(k+1)-x(k))^TE^TRE(x(k+1)-x(k)) \\ &\quad - d\sum_{i=k-d}^{k-1}(x(i+1)-x(i))^T \\ &\quad \times E^TRE(x(i+1)-x(i)) \\ &= \left(Ax(k) + A_1x(k-d) + \phi(k,x_k)\right)^T \\ &\quad \times PEx(k+1) + x^T(k+1)E^T \\ &\quad \times P\left(Ax(k) + A_1x(k-d) + \phi(k,x_k)\right) \\ &\quad + x^T(k+1)E^TPEx(k+1) - 3x^T(k)E^TPEx(k) \\ &\quad + x^T(k)Qx(k) - x^T(k-d)Qx(k-d) \\ &\quad + d^2(x(k+1)-x(k))^TE^TRE(x(k+1)-x(k)) \\ &\quad - d\sum_{i=k-d}^{k-1}(x(i+1)-x(i))^TE^TRE(x(i+1)-x(i)) \\ &\leqslant \left(Ax(k) + A_1x(k-d) + \phi(k,x_k)\right)^T \\ &\quad \times PEx(k+1) + x^T(k+1) \\ &\quad \times E^TP\left(Ax(k) + A_1x(k-d) + \phi(k,x_k)\right) \\ &\quad + x^T(k+1)E^TPEx(k+1) - 3x^T(k)E^TPEx(k) \\ &\quad + x^T(k)Qx(k) - x^T(k-d)Qx(k-d) \\ &\quad + d^2(x(k+1)-x(k))^TE^TRE(x(k+1)-x(k)) \\ &\quad - d\sum_{i=k-d}^{k-1}(x(i+1)-x(i))^TE^TRE(x(i+1)-x(i)) \\ &\quad + x^T(k)H^THx(k) - \phi^T(k,x_k)\phi(k,x_k). \end{aligned} \tag{26}$$

It is clear to see that

$$\begin{aligned} &- d\sum_{i=k-d}^{k-1}(x(i+1)-x(i))^TE^TRE(x(i+1)-x(i)) \\ &\quad \leqslant -\sum_{i=k-d}^{k-1}(x(i+1)-x(i))^TE^TRE \\ &\qquad \times \sum_{i=k-d}^{k-1}(x(i+1)-x(i)) \\ &\quad = -\left(x(k)^T - x(k-d)^T\right)E^TRE(x(k)-x(k-d)). \end{aligned} \tag{27}$$

Then, combining (27) into (26) yields

$$\Delta V(k) \leqslant \varsigma(k)^T\Theta\varsigma(k), \tag{28}$$

where

$$\begin{aligned} \varsigma(k) &= \begin{bmatrix} x(k) \\ x(k-d) \\ \phi(k,x_k) \\ Ex(k+1) \end{bmatrix}, \\ \Theta &= \begin{bmatrix} -3E^TPE + Q + \left(d^2-1\right)E^TRE + H^TH & E^TRE & 0 & A^TP - d^2E^TR \\ * & -Q - E^TRE & 0 & A_1^TP \\ * & 0 & -I & P \\ * & * & P & P + d^2R \end{bmatrix}. \end{aligned} \tag{29}$$

Recalling the state-space model of system (5), it is easy to see that

$$\begin{bmatrix} A & A_1 & B & -I \end{bmatrix} \varsigma(k) = 0. \tag{30}$$

For any matrices M, N of appropriate dimension, the following can be obtained:

$$2\varsigma(k)^T \Gamma \begin{bmatrix} A & A_1 & B & -I \end{bmatrix} \varsigma(k) = 0, \tag{31}$$

where

$$\Gamma = \begin{bmatrix} M^T & 0 & 0 & N^T \end{bmatrix}^T. \tag{32}$$

Now, if condition (17) is satisfied, we have

$$\mathscr{M} = \Theta + \text{Sym}\left(\Gamma \begin{bmatrix} A & A_1 & B & -I \end{bmatrix}\right) < 0. \tag{33}$$

Then,

$$\varsigma^T(k)\mathscr{M}\varsigma(k) = \varsigma^T(k)\Theta\varsigma^T(k) < 0. \tag{34}$$

From (34) and (26), we get $\Delta V(k) < 0$; thus, the system (5) is stable. Summing up the above, we conclude that if condition (17) holds, then, system (5) is regular, casual, and asymptotically stable. Thus, this completes the proof. □

In the sequel, we will focus on the l_2–l_∞ and H_∞ performance analysis of system (12). Based on Theorem 3, we obtain the following.

Theorem 4. *Given a positive scalar γ, for any consistent initial condition x_0 and the nonlinear function $\phi(k, x_k)$ in (4), the system (12) is admissible, while satisfying a prescribed l_2–l_∞ and H_∞ performance γ, if there exist positive definite matrices $\widetilde{Q}$, $\widetilde{R}$, a symmetry matrix $\widetilde{P}$, and matrices $\widetilde{N}$, $\widetilde{M}$ such that the following inequality holds:*

$$\mathscr{M}_1 = \begin{bmatrix} \widetilde{(1,1)} & \widetilde{M}\widetilde{A}_1 + \widetilde{E}^T\widetilde{R}\widetilde{E} & \widetilde{M}G & \widetilde{M}\widetilde{B} + \widetilde{L}^T D_2 & -\widetilde{M} + \widetilde{A}^T\widetilde{P} + \widetilde{A}^T\widetilde{N}^T - d^2\widetilde{E}^T\widetilde{R} \\ 0 & -\widetilde{Q} - \widetilde{E}^T\widetilde{R}\widetilde{E} & 0 & 0 & \widetilde{A}_1^T\widetilde{P} + \widetilde{A}_1^T\widetilde{N}^T \\ 0 & 0 & -I & 0 & G^T\widetilde{P} + G^T\widetilde{N}^T \\ * & * & * & -\gamma^2 I + D_2^T D_2 & \widetilde{B}^T\widetilde{P} + \widetilde{B}^T\widetilde{N}^T \\ * & * & * & * & -Sym\left(\widetilde{N}\right) + \widetilde{P} + d^2\widetilde{R} \end{bmatrix} < 0, \tag{35}$$

where

$$\begin{aligned} \widetilde{(1,1)} &= -3\widetilde{E}^T\widetilde{P}\widetilde{E} + \widetilde{Q} + \left(d^2 - 1\right)\widetilde{E}^T\widetilde{R}\widetilde{E} \\ &\quad + \widetilde{H}^T\widetilde{H} + \widetilde{L}^T\widetilde{L} + Sym\left\{\widetilde{M}\widetilde{A}\right\}, \\ \widetilde{H} &= \text{diag}\{H, H\}, \end{aligned} \tag{36}$$

$$\begin{aligned} &\widetilde{P} = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}, \qquad P_1 = \begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix}, \\ &P_{11} = P_{11}^T > 0, \qquad P_{22} = P_{22}^T, \\ &P_2 = P_2^T > 0, \qquad \widetilde{M} = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}, \\ &M_1 = \text{diag}\{M_{11}, M_{22}\}. \end{aligned} \tag{37}$$

Proof. Firstly, it is easy to see that $\mathscr{M}_1 < 0$ implies that $\mathscr{M} < 0$; in other words, condition (35) can guarantee condition (17). Therefore, according to Theorem 3, if condition (35) is satisfied, the system (1) with $\omega(k) = 0$ is admissible. We denote the following performance index:

$$J \triangleq \sum_{k=0}^{\infty} \left[z^T(k) z(k) - \gamma^2 \omega^T(k)\omega(k)\right]. \tag{38}$$

Under zero-initial condition, $V(k) = 0$, we have

$$\begin{aligned} J &\leqslant \sum_{k=0}^{\infty} \left[z^T(k) z(k) - \gamma^2 \omega^T(k)\omega(k) + \Delta V(k)\right] \\ &= \sum_{k=0}^{\infty} \sigma^T(k)\Theta_1\sigma(k), \end{aligned} \tag{39}$$

where

$$\sigma(k) = \begin{bmatrix} \xi^T(k) & \xi^T(k-d) & \Lambda^T(k, \xi_k) & \omega^T(k) & \xi^T(k+1)\widetilde{E}^T \end{bmatrix}^T, \tag{40}$$

$$\Theta_1 = \begin{bmatrix} -3\widetilde{E}^T\widetilde{P}\widetilde{E} + \widetilde{Q} + \left(d^2 - 1\right)\widetilde{E}^T\widetilde{R}\widetilde{E} + \widetilde{H}^T\widetilde{H} + \widetilde{L}^T\widetilde{L} & \widetilde{E}^T\widetilde{R}\widetilde{E} & 0 & \widetilde{L}^T D_2 & \widetilde{A}^T\widetilde{P} - d^2\widetilde{E}^T\widetilde{R} \\ * & -\widetilde{Q} - \widetilde{E}^T\widetilde{R}\widetilde{E} & 0 & 0 & \widetilde{A}_1^T\widetilde{P} \\ * & * & -I & 0 & G^T\widetilde{P} \\ * & * & * & -\gamma^2 I + D_2^T D_2 & \widetilde{B}^T\widetilde{P} \\ * & * & * & * & \widetilde{P} + d^2\widetilde{R} \end{bmatrix}. \tag{41}$$

Similar to the prove process in Theorem 3, we can easily get

$$\left[\widetilde{A} \;\; \widetilde{A}_1 \;\; G \;\; \widetilde{B} \;\; -I\right] \sigma(k) = 0. \tag{42}$$

Setting

$$\begin{aligned} \overline{\mathscr{B}} &= \left[\widetilde{A} \;\; \widetilde{A}_1 \;\; G \;\; \widetilde{B} \;\; -I\right], \\ \overline{\mathscr{X}}^T &= \left[\widetilde{M}^T \;\; 0 \;\; 0 \;\; 0 \;\; \widetilde{N}^T\right] \end{aligned} \tag{43}$$

yields

$$\mathscr{M}_1 = \Theta_1 + \overline{\mathscr{X}}\overline{\mathscr{B}} + \overline{\mathscr{B}}^T \overline{\mathscr{X}}^T. \tag{44}$$

Therefore, if condition (35) in Theorem 4 is satisfied, then $\Theta_1 + \overline{\mathscr{X}}\overline{\mathscr{B}} + \overline{\mathscr{B}}^T \overline{\mathscr{X}}^T < 0$. We have

$$\sigma^T(k)\,\Theta_1 \sigma^T(k) = \sigma^T(k)\,\mathscr{M}_1 \sigma^T(k) < 0. \tag{45}$$

Hence, it follows from (39) that

$$\begin{aligned} J &\triangleq \sum_{k=0}^{\infty} \left[z^T(k)\,z(k) - \gamma^2 \omega^T(k)\,\omega(k)\right] \\ &\leqslant \sum_{k=0}^{\infty} \sigma^T(k)\,\Theta_1 \sigma(k) < 0 \end{aligned} \tag{46}$$

which means

$$\sum_{k=0}^{\infty} z^T(k)\,z(k) < \gamma^2 \sum_{k=0}^{\infty} \omega^T(k)\,\omega(k). \tag{47}$$

Hence, condition (35) can guarantee $\|z(k)\|_2 < \gamma\|\omega\|_2$; that is, the H_∞ performance index is satisfied. It follows from (47) that

$$\sup z^T(k)\,z(k) \leqslant \sum_{k=0}^{\infty} z^T(k)\,z(k) < \gamma^2 \sum_{k=0}^{\infty} \omega^T(k)\,\omega(k). \tag{48}$$

Hence, condition (35) can also guarantee $\|z(k)\|_\infty < \gamma\|\omega\|_2$; that is, the l_2–l_∞ performance index is also satisfied. Thus, this completes the proof. □

Remark 5. By introducing two slack matrix variables M and N, Theorem 3 presents a novel stability criterion for nonlinear discrete-time singular system, in which the slack matrix variable N could provide more free dimensions in the solution space. The condition obtained in Theorem 4 can guarantee not only the l_2–l_∞ disturbance attenuation level but also the H_∞ disturbance attenuation level. Note that, for the sake of simplicity, we assume that the l_2–l_∞ disturbance attenuation level and the H_∞ disturbance attenuation level are the same γ. In practice, if the required two levels are different, we only need to solve two inequalities.

Remark 6. Theorem 4 gives a sufficient condition for the existence of a full-order l_2–l_∞ and H_∞ controller of the forms (6) and (8) for system (1). According to the expression of closed-loop system (12), it can be seen that the condition obtained in Theorem 4 is nonlinear matrix inequality (nLMI) with respect to the parameter matrices $\widetilde{P}$, $\widetilde{N}$, and $\widetilde{M}$ and the controller gain matrices A_f, B_f, C_f, and W_f since some crosses of these determined parameters are appearing in (35) in nonlinear fashion. In order to facilitate solving, in the following subsection, some matrix transformations are needed to transform the nLIM (35) to a strict LMI.

3.2. l_2–l_∞ and H_∞ Controller Design

Theorem 7. *Given scalars $\gamma > 0$, α, and β, for the discrete-time nonlinear singular system (1), there exists a full-order dynamic feedback controller (6) such that the closed-loop system (12) is admissible with a prescribed l_2–l_∞ and H_∞ performance γ, if there exist positive definite matrices $\widetilde{Q}$, $\widetilde{R}$ and matrices $\widetilde{P}$, V_1, V_2, V_3, and J of appropriate form, such that*

$$\begin{bmatrix} \mathscr{M}_{20} & \left[B^T M_1^T \;\; 0 \;\; \cdots \;\; B^T\left(N_1^T + P_1\right) \;\; 0 \;\; B_1^T\right]^T & \left[0 \;\; \epsilon J \;\; 0 \;\; \cdots \;\; 0\right]^T \Delta_F^T \\ * & -\epsilon I & 0 \\ * & * & -\epsilon I \end{bmatrix} < 0, \tag{49}$$

where

$$\mathscr{M}_{20} = \begin{bmatrix} (1,1) & (1,2) & M_1 A_1 + E^T R_1 E & E^T R_2 & M_1 & 0 & M_1 D & (1,8) & (1,9) & L^T \\ * & (2,2) & \alpha V_2 C_1 + R_2^T E & R_3 & 0 & \alpha V_3 & \alpha V_2 D_1 & (2,8) & (2,9) & 0 \\ * & * & (3,3) & -Q_2 - E^T R_2 & 0 & 0 & 0 & (3,8) & (3,9) & 0 \\ * & * & * & -Q_3 - R_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & P_1 + N_1^T & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 & (1+\beta) V_3^T & 0 \\ * & * & * & * & * & * & -\gamma^2 I & D^T\left(P_1 + N_1^T\right) & (1+\beta) D_1^T V_2^T & D_2^T \\ * & * & * & * & * & * & * & (8,8) & d^2 R_2 & 0 \\ * & * & * & * & * & * & * & * & (9,9) & 0 \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix}, \tag{50}$$

$$
\begin{aligned}
(1,1) &= -3E^T P_1 E + Q_1 + \left(d^2-1\right) E^T R_1 E \\
&\quad + Sym\left(M_1 A\right) + H^T H, \\
(1,2) &= Q_2 + \left(d^2-1\right) E^T R_2 + \alpha C^T V_2^T, \\
(1,8) &= -M_1 + A^T P_1 + A^T N_1^T - d^2 E^T R_1, \\
(1,9) &= (\beta+1) C^T V_2^T - d^2 E^T R_2, \\
(2,2) &= -3P_2 + Q_3 + \left(d^2-1\right) R_3 + \alpha Sym\left(V_1\right) + H^T H, \\
(2,8) &= -d^2 R_2^T, \\
(2,9) &= (\beta+1) V_1^T - \alpha P_2 - d^2 R_3, \\
(3,3) &= -Q_1 - E^T R_1 E, \\
(3,8) &= A_1^T\left(P_1 + N_1^T\right), \\
(3,9) &= (1+\beta) C_1^T V_2^T, \\
(8,8) &= -Sym\left(N_1\right) + P_1 + d^2 R_1, \\
(9,9) &= (1-2\beta) P_2 + d^2 R_3.
\end{aligned} \tag{51}
$$

Then, the parameters of the desired l_2–l_∞ and H_∞ dynamic output feedback controller can be taken as

$$
\begin{aligned}
A_f &= \left(P_2\right)^{-1} V_1, \qquad B_f = \left(P_2\right)^{-1} V_2, \\
W_f &= \left(P_2\right)^{-1} V_3, \qquad C_f = J.
\end{aligned} \tag{52}
$$

Proof. The matrices $\widetilde{P}$ and $\widetilde{M}$ in Theorem 4 have been defined in (37). We assume that the matrices $\widetilde{Q}$, $\widetilde{R}$, and $\widetilde{N}$ in Theorem 4 are of the following form:

$$
\begin{aligned}
\widetilde{Q} &= \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix}, \qquad \widetilde{R} = \begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix}, \\
\widetilde{N} &= \begin{bmatrix} N_1 & 0 \\ 0 & \beta P_2 \end{bmatrix},
\end{aligned} \tag{53}
$$

where $P_{11} = P_{11}^T \in \mathbb{R}^{r\times r} > 0$, $P_{13} = P_{13}^T$, $P_2 = P_2^T > 0$, $M_{11} \in \mathbb{R}^{r\times r}$, $M_{22} \in \mathbb{R}^{(n-r)\times(n-r)}$, and $N_1 \in \mathbb{R}^{n\times n}$.

Setting

$$
\begin{aligned}
P_2 A_f &= V_1, \qquad P_2 B_f = V_2, \\
P_2 W_f &= V_3, \qquad C_f = J
\end{aligned} \tag{54}
$$

yields

$$
\begin{aligned}
\widetilde{M}\widetilde{A} &= \begin{bmatrix} M_1 & 0 \\ 0 & \alpha P_2 \end{bmatrix} \begin{bmatrix} A & BFC_f \\ B_f C & A_f \end{bmatrix} \\
&= \begin{bmatrix} M_1 A & M_1 BFJ \\ \alpha P_2 B_f C & \alpha P_2 A_f \end{bmatrix} = \begin{bmatrix} M_1 A & M_1 BFC_f \\ \alpha V_2 C & \alpha V_1 \end{bmatrix}, \\
\widetilde{M}\widetilde{A}_1 &= \begin{bmatrix} M_1 & 0 \\ 0 & \alpha P_2 \end{bmatrix} \begin{bmatrix} A_1 & 0 \\ B_f C_1 & 0 \end{bmatrix} \\
&= \begin{bmatrix} M_1 A_1 & 0 \\ \alpha P_2 B_f C_1 & 0 \end{bmatrix} = \begin{bmatrix} M_1 A_1 & 0 \\ \alpha V_2 C_1 & 0 \end{bmatrix}, \\
\widetilde{M} G &= \begin{bmatrix} M_1 & 0 \\ 0 & \alpha P_2 \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & W \end{bmatrix} = \begin{bmatrix} M_1 & 0 \\ 0 & \alpha P_2 W \end{bmatrix}, \\
\widetilde{M}\widetilde{B} &= \begin{bmatrix} M_1 & 0 \\ 0 & \alpha P_2 \end{bmatrix} \begin{bmatrix} D \\ B_f D_1 \end{bmatrix} \\
&= \begin{bmatrix} M_1 D \\ \alpha P_2 B_f D_1 \end{bmatrix} = \begin{bmatrix} M_1 D \\ \alpha V_2 D_1 \end{bmatrix}, \\
\widetilde{A}^T\left(\widetilde{N}^T + P\right) &= \begin{bmatrix} A & BFC_f \\ B_f C & A_f \end{bmatrix}^T \begin{bmatrix} N_1^T + P_1 & 0 \\ 0 & (1+\beta) P_2 \end{bmatrix} \\
&= \begin{bmatrix} A^T\left(N_1^T + P_1\right) & (1+\beta) C^T V_2^T \\ J^T F^T B^T \left(N_1^T + P_1\right) & (1+\beta) V_1^T \end{bmatrix}, \\
\widetilde{A}_1^T\left(\widetilde{N}^T + P\right) &= \begin{bmatrix} A_1 & 0 \\ B_f C_1 & 0 \end{bmatrix}^T \begin{bmatrix} N_1^T + P_1 & 0 \\ 0 & (1+\beta) P_2 \end{bmatrix} \\
&= \begin{bmatrix} A_1^T\left(N_1^T + P_1\right) & (1+\beta) C_1^T V_2^T \\ 0 & 0 \end{bmatrix}, \\
G^T\left(\widetilde{N}^T + P\right) &= \begin{bmatrix} I & 0 \\ 0 & W \end{bmatrix} \begin{bmatrix} N_1^T + P_1 & 0 \\ 0 & (1+\beta) P_2 \end{bmatrix} \\
&= \begin{bmatrix} N_1^T + P_1 & 0 \\ 0 & (1+\beta) V_3^T \end{bmatrix}, \\
\widetilde{B}^T\left(\widetilde{N}^T + P\right) &= \begin{bmatrix} D \\ B_f D_1 \end{bmatrix}^T \begin{bmatrix} N_1^T + P_1 & 0 \\ 0 & (1+\beta) P_2 \end{bmatrix} \\
&= \begin{bmatrix} D^T\left(N_1^T + P_1\right) & (1+\beta) D_1^T V_2^T \end{bmatrix}.
\end{aligned} \tag{55}
$$

Then, it follows from the above and Schur complement that the inequality (35) is equivalent to

$$
\begin{aligned}
\mathscr{M}_2 &= \mathscr{M}_{20} + \begin{bmatrix} B^T M_1^T & 0 & \cdots & B^T\left(N_1^T + P_1\right) & 0 & B_1^T \end{bmatrix}^T \\
&\quad \times F \begin{bmatrix} 0 & J & 0 & \cdots & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & J & 0 & \cdots & 0 & 0 \end{bmatrix}^T
\end{aligned}
$$

$$\times F^T \left[B^T M_1^T \ \ 0 \ \cdots \ B^T \left(N_1^T + P_1 \right) \ \ 0 \ \ B_1^T \right]^T < 0, \tag{56}$$

where $\mathscr{M}_{20}$ is defined in (50).

Then, from (10) and (56) and the inequality $x^T y + y^T x \leqslant \epsilon x^T x + \epsilon^{-1} y^T y$, for any $\epsilon > 0$, we have

$$\begin{aligned} \mathscr{M}_2 &\leqslant \mathscr{M}_{20} + \epsilon^{-1} \left[B^T M_1^T \ \ 0 \ \cdots \ B^T \left(N_1^T + P_1 \right) \ \ 0 \ \ B^T \right] \\ &\quad \times \left[B^T M_1^T \ \ 0 \ \cdots \ B^T \left(N_1^T + P_1 \right) \ \ 0 \ \ B^T \right]^T \\ &\quad + \epsilon [0 \ \ J \ \ 0 \ \cdots \ 0 \ \ 0]^T F^T F [0 \ \ J \ \ 0 \ \cdots \ 0 \ \ 0] \\ &\leqslant \mathscr{M}_{20} + \epsilon^{-1} \left[B^T M_1^T \ \ 0 \ \cdots \ B^T \left(N_1^T + P_1 \right) \ \ 0 \ \ B^T \right] \\ &\quad \times \left[B^T M_1^T \ \ 0 \ \cdots \ B^T \left(N_1^T + P_1 \right) \ \ 0 \ \ B^T \right]^T \\ &\quad + \epsilon [0 \ \ J \ \ 0 \ \cdots \ 0 \ \ 0]^T \Delta_F^T \Delta_F [0 \ \ J \ \ 0 \ \cdots \ 0 \ \ 0], \end{aligned} \tag{57}$$

where ϵ is any unknown arbitrarily positive scalar.

By Schur complement again, we have that $\mathscr{M}_2 < 0$ in (56) is equivalent to (49). That is to say, the LMI (49) in Theorem 7 can guarantee the inequality (35) in Theorem 4. The matrix variables P_2, V_1, V_2, V_3, and J are to be designed. Then, from Theorem 4, if (49) holds, the closed-loop system (12) is admissible with a prescribed l_2–l_∞ and H_∞ performance γ. And from (54), the controller gain solution (52) is immediate. Thus, this completes the proof. □

Remark 8. In this paper, the reliable l_2–l_∞ and H_∞ dynamic output feedback control problem for discrete-time nonlinear singular systems are studied. Considering the controller physical implementation convenience in the practical engineering, we are interested in a normal state-space controller (6) in this paper. In Theorem 7, the desired full-order dynamic feedback controller (6) can be obtained by solving a strict LMI (49) efficiently. The performance γ can be obtained and described as

$$\underset{P_2, V_1, V_2, V_3, J}{\text{Minimize}} \ \gamma^2 \tag{58}$$

subject to linear matrix inequality (49).

4. Numerical Example

In this section, we give a numerical example to illustrate the effectiveness of the obtained controller design method. Consider the nonlinear singular system (1) with

$$\begin{gathered} E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \qquad A = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}, \\ A_1 = \begin{bmatrix} -0.1 & 0 \\ 0.2 & 0.5 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \qquad D = \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix}, \\ C = [-0.1 \ \ 0.2], \qquad C_1 = [0.2 \ \ -0.1], \\ D_1 = -1, \qquad L = [0.2 \ \ -0.1], \\ D_2 = 0.5, \qquad H = \operatorname{diag}\{01, 0.1\}, \qquad \Delta_F = 0.5. \end{gathered} \tag{59}$$

Given the scalars $\epsilon = 10$, $\alpha = \beta = 2$, the time-delay $d = 5$. We also assume the nonlinear functions $\Phi(k, x_k)$ as

$$\Phi(k, x_k) = \frac{1}{\sqrt{10}} \begin{bmatrix} \sin(x_1(k)) \\ \sin(x_2(k)) \end{bmatrix}. \tag{60}$$

From (4), we can get

$$H = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}. \tag{61}$$

By applying the minimization problem in (58), the minimal value of the l_2–l_∞ and H_∞ performance γ is 0.8619. And for a special $\gamma = 1$, the corresponding solutions of the determined matrices are given as follows:

$$\begin{gathered} V_1 = \begin{bmatrix} 337.2315 & 8.2266 \\ 8.2266 & 337.2315 \end{bmatrix}, \qquad V_2 = \begin{bmatrix} 1.4780 \\ 1.4780 \end{bmatrix}, \\ V_3 = \begin{bmatrix} 9.4326 & 0.0083 \\ 0.0083 & 9.4326 \end{bmatrix}, \\ J = [6.2550 \ \ 6.2550], \qquad P_2 = \begin{bmatrix} 1140.4 & 85 \\ 85 & 1140.4 \end{bmatrix}. \end{gathered} \tag{62}$$

Then, from (52), our obtained l_2–l_∞ and H_∞ dynamic feedback controller parameters can be obtained

$$\begin{gathered} A_f = \begin{bmatrix} 0.2968 & -0.0149 \\ -0.0149 & 0.2968 \end{bmatrix}, \\ B_f = \begin{bmatrix} 0.0012 \\ 0.0012 \end{bmatrix}, \\ W_f = \begin{bmatrix} 0.0083 & -0.0006 \\ -0.0006 & 0.0083 \end{bmatrix}, \\ C_f = [6.2550 \ \ 6.2550]. \end{gathered} \tag{63}$$

5. Conclusion

A generalized l_2–l_∞ and H_∞ dynamic feedback control problem for nonlinear discrete-time singular systems has been investigated in this paper. By introducing some slack matrix variables, a less conservative condition is obtained, which can ensure that the studied nonlinear discrete-time singular system is regular, casual, and stable. Based on this obtained condition, a sufficient LMI-based condition is obtained such that the resulting closed-loop system is regular, casual, and stable while satisfying a prescribed l_2–l_∞ and H_∞ performance level γ. The desired controller parameters can be computed only by solving a strict LMI. Finally, a numerical example shows the validity of our proposed method.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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