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Research Article

Adaptive Control of the Chaotic System via Singular System Approach

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This paper deals with the control problem of the chaotic system subject to disturbance. The sliding mode surface is designed by singular system approach, and sufficient condition for convergence is given. Then, the adaptive sliding mode controller is designed to make the state arrive at the sliding mode surface in finite time. Finally, Lorenz system is considered as an example to show the effectiveness of the proposed method.

1. Introduction

In the past decades, many studies have been devoted to the properties of nonlinear systems with applications [1–5]. One of the most important properties of nonlinear systems is the chaos system. The research of chaos system has been paid much attention, because the chaos system has very abroad application background, such as chemical reactions, power converters, biological systems, information processing, and secure communication. After Ott et al. firstly presented the approach of chaos control in 1990 [6], enormous investigations of the chaos system have been carried out in the field of control.

For different chaos systems, different methods have been employed. In order to realize chaos control or synchronization, some well-known methods have been utilized, such as back-stepping method [7], control Lyapunov function method [8], proportional-differential control method [9], neural network method [10], and sliding mode control method [11–14], among which, sliding mode control method is proved to be one of the most powerful methods because the closed-loop system has many attractive features such as fast response, good transient response, and robustness against disturbance. Reference [11] designed adaptive sliding mode controller to achieve synchronization of the chaos system subject to uncertainty, [12] proposed a new reaching law, and designed feedback law to stabilize the considered system. For a class of general form of chaos systems, [13] presented adaptive terminal sliding mode controller to make the closed-loop system stable in finite time and [14] gave chatter free sliding mode controller design method for the unknown chaos system.

It should be noted that the recent innovation [15] considered a class of Markovian jumping system and used the singular system method to design the sliding mode surface. The key step is to construct the transformation matrix $T(i) = \left[M(i)^T \quad B(i)\right]^T$. Under the transformation matrix $T(i)$, the sliding mode dynamic can be obtained. Combining with the sliding mode equation, the new system is a kind of singular system, which can simplify the analysis of the stability. Motivated by [15], this paper considers the control problem for a kind of chaos systems, where the upper bound of the disturbance is unknown. By the singular system and sliding mode control approach, an adaptive sliding mode controller is designed to make the closed-loop system stable asymptotically.

The rest of the paper is organized as follows. Section 2 presents the problem formulation and preliminaries. Section 3 designs sliding mode surface for convergence under some conditions and presents a novel adaptive sliding mode controller for the system. Section 4 simulates an example of

Lorenz system to illustrate the effectiveness of the proposed method.

2. Problem Formulation and Preliminaries

In this section, we present the formulation of the problem and preliminaries which are necessary for our further investigation.

Let us consider the following chaotic system with disturbance:

$$\dot{x} = Ax + B\left[f(x) + \omega(x) + u\right], \tag{1}$$

where $x \in R^n$ is the state and $u \in R^m$ is the control input. $f(x) : R^n \rightarrow R^m$ is known smooth matrix function. $\omega(x) \in R^m$ is the unknown disturbance of the system. $A \in R^{n\times n}$ and $B \in R^{n\times m}$ are determined matrices. It is supposed that $\operatorname{rank}(B) = m$.

When the disturbance $\omega(x) = 0$, many chaotic systems can be transformed into the form of (1). Consider Lorenz system:

$$\begin{aligned}\dot{x}_1 &= -\sigma_1 x_1 + \sigma_2 x_2,\\ \dot{x}_2 &= r x_1 - x_2 - x_1 x_3 + u_1,\\ \dot{x}_3 &= -b x_3 + x_1 x_2 + u_2,\end{aligned} \tag{2}$$

where

$$A = \begin{bmatrix} -\sigma_1 & \sigma_2 & 0 \\ r & -1 & 0 \\ 0 & 0 & -b \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$f(x) = \begin{bmatrix} -x_1 x_3 \\ x_1 x_2 \end{bmatrix}, \qquad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}. \tag{3}$$

Then, consider Chua's circuit system:

$$\begin{aligned}\dot{x}_1 &= -x_1 + x_2 + x_3,\\ \dot{x}_2 &= -\beta x_1 + u_1,\\ \dot{x}_3 &= \alpha x_1 - \alpha x_3 - \alpha l(x_3) + u_2,\end{aligned} \tag{4}$$

where $l(x_3) = n x_3 + 1/2(m-n)(|x_3 + 1| - |x_3 - 1|)$. Thus, Chua's circuit system can be written as (1) with

$$A = \begin{bmatrix} -1 & 1 & 1 \\ -\beta & 0 & 0 \\ \alpha & 0 & -\alpha \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$f(x) = \begin{bmatrix} 0 \\ -\alpha l(x_3) \end{bmatrix}, \qquad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}. \tag{5}$$

In order to establish the main result of this paper, we need the following assumption.

Assumption 1. The disturbance $\omega(x)$ is Lipschitz with Lipschitz constant γ; that is,

$$\|\omega(x)\| \le \gamma \|x\|, \tag{6}$$

where $\gamma > 0$ is unknown parameter.

Remark 2. In many physical systems, the parameter γ is usually unknown. The treatment of the disturbance is similar to that in the former work such as [16, 17]; thus, it is a more general assumption.

Since this paper employs the singular system to deal with the control of chaotic system, in what follows, we introduce some basics about the singular system.

Let us consider the following singular system:

$$E\dot{x} = Ax, \tag{7}$$

where $x \in R^n$ is the state and $E \in R^{n\times n}$ and $A \in R^{n\times n}$ are determined matrices. E is singular with $\operatorname{rank}(E) < n$.

Definition 3 (see [18]). The system (7) is said to be

(1) regular if $\det(sE - A)$ is not identically zero,

(2) impulse-free if $\deg(\det(sE - A)) = \operatorname{rank}(E)$,

(3) stable if all the roots of $\det(sE - A) = 0$ have negative real parts,

(4) admissible if it is regular, impulse-free, and stable.

Lemma 4 (see [19]). *The system (7) is admissible if and only if there exists nonsingular matrix P such that*

(1) $E^T P = P^T E \ge 0$,

(2) $A^T P + P^T A < 0$.

3. The Design of Sliding Mode Surface

In this section, we use the singular system approach to design the sliding mode surface for the system (1). The sliding mode surface S in this paper is chosen as $S = Cx$, where $C \in R^{(n-m)\times n}$ is designed later. Since $\operatorname{rank}(B) = m$, there exists $N \in R^{(n-m)\times n}$ such that $NB = 0$. Let $M = \left[\begin{smallmatrix} N \\ B^T \end{smallmatrix}\right]$; then, M is nonsingular matrix and $NM^{-1} = [I_{n-m} \ \ 0]$. Left multiplying both sides of (1), we have

$$M\dot{x} = MAx + MB\left[f(x) + \omega(x) + u\right]; \tag{8}$$

that is,

$$\begin{aligned} N\dot{x} &= NAx + NB\left[f(x) + \omega(x) + u\right],\\ B^T\dot{x} &= B^T Ax + B^T B\left[f(x) + \omega(x) + u\right].\end{aligned} \tag{9}$$

Since $NB = 0$, (9) becomes

$$\begin{aligned} N\dot{x} &= NAx,\\ B^T\dot{x} &= B^T Ax + B^T B\left[f(x) + \omega(x) + u\right].\end{aligned} \tag{10}$$

By the theory of sliding mode control, the first equation of (10) means the sliding mode dynamic. Since $S = Cx = 0$, then we have

$$\begin{aligned} N\dot{x} &= NAx,\\ 0 &= Cx.\end{aligned} \tag{11}$$

Theorem 5. *Let P be a nonsingular matrix, and denote that $PM^{-1} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$, where $P_{11} \in R^{(n-m)\times(n-m)}$, $P_{22} \in R^{m\times m}$. If the following conditions hold*

$$P_{11} = P_{11}^T \geq 0, \tag{12}$$

$$P_{12} = 0, \tag{13}$$

$$\begin{aligned} &A^T N^T P_{11} N + N^T P_{11} N A + C^T P_{21} N + N^T P_{21}^T C \\ &\quad + C^T P_{22} B^T + B P_{22}^T C < 0, \end{aligned} \tag{14}$$

then the state of the system (11) *will converge to 0 asymptotically.*

Proof. The system (11) is admissible if and only if the conditions of Lemma 4 are satisfied; that is,

$$\begin{bmatrix} N^T & 0 \end{bmatrix} P = P^T \begin{bmatrix} N \\ 0 \end{bmatrix} \geq 0, \tag{15}$$

$$\begin{bmatrix} A^T N^T & C^T \end{bmatrix} P + P^T \begin{bmatrix} NA \\ C \end{bmatrix} < 0. \tag{16}$$

Firstly, let us consider (15), which means that

$$M^{-T} \begin{bmatrix} N^T & 0 \end{bmatrix} P M^{-1} = M^{-T} P^T \begin{bmatrix} N \\ 0 \end{bmatrix} M^{-1} \geq 0. \tag{17}$$

Since $NM^{-1} = \begin{bmatrix} I_{n-m} & 0 \end{bmatrix}$ and $PM^{-1} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$, (17) is equivalent to

$$\begin{bmatrix} I_{n-m} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} P_{11}^T & P_{21}^T \\ P_{12}^T & P_{22}^T \end{bmatrix} \begin{bmatrix} I_{n-m} & 0 \\ 0 & 0 \end{bmatrix} \geq 0; \tag{18}$$

then,

$$\begin{bmatrix} P_{11} & P_{12} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} P_{11}^T & 0 \\ P_{12}^T & 0 \end{bmatrix} \geq 0. \tag{19}$$

Thus, (19) is equivalent to (12) and (13). Then, we consider (16), and it is

$$\begin{bmatrix} A^T N^T & C^T \end{bmatrix} \begin{bmatrix} P_{11} & 0 \\ P_{21} & P_{22} \end{bmatrix} M + M^T \begin{bmatrix} P_{11}^T & P_{21}^T \\ 0 & P_{22}^T \end{bmatrix} \begin{bmatrix} NA \\ C \end{bmatrix} < 0; \tag{20}$$

that is,

$$\begin{aligned} &\begin{bmatrix} A^T N^T & C^T \end{bmatrix} \begin{bmatrix} P_{11} & 0 \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} N \\ B^T \end{bmatrix} \\ &\quad + \begin{bmatrix} N^T & B \end{bmatrix} \begin{bmatrix} P_{11}^T & P_{21}^T \\ 0 & P_{22}^T \end{bmatrix} \begin{bmatrix} NA \\ C \end{bmatrix} < 0. \end{aligned} \tag{21}$$

By direct computation, (21) is equivalent to (14). Thus, we can conclude that if (12), (13), and (14) hold, then the system (11) is admissible; that is, the state of the system (11) will converge to 0 asymptotically. □

Remark 6. The main task of this section is to design C; it is obvious that (12) and (14) are needed, and (13) is easy to be satisfied.

It should be noted that (14) is not LMI; an equivalent condition of (14) is given as the following theorem.

Theorem 7. *Let $C^T P_{21} = C_1$, $C^T P_{22} = C_2$. Inequality* (14) *holds if and only if the following conditions hold:*

$$\begin{aligned} &A^T N^T P_{11} N + N^T P_{11} N A + C_1 N \\ &\quad + N^T C_1^T + C_2 B^T + B C_2^T < 0, \end{aligned} \tag{22}$$

$$\operatorname{rank}(C_2) = \operatorname{rank}(\begin{bmatrix} C_1 & C_2 \end{bmatrix}). \tag{23}$$

Proof. $C^T P_{21} = C_1, C^T P_{22} = C_2$; then,

$$\begin{aligned} &A^T N^T P_{11} N + N^T P_{11} N A + C_1^T N \\ &\quad + N^T C_1^T + C_2 B^T + B C_2^T < 0. \end{aligned} \tag{24}$$

It is known that the matrices C_1 and C_2 are not independent; they should satisfy $C^T P_{21} = C_1$, $C^T P_{22} = C_2$. Since P is nonsingular, then P_{22} is nonsingular; $C^T P_{22} = C_2$ means that $\operatorname{rank}(C^T) = \operatorname{rank}(C_2)$. To guarantee the solvability of $C^T P_{21} = C_1$, the sufficient and necessary condition is that $\operatorname{rank}(C_2) = \operatorname{rank}([C_1 \ \ C_2])$. Hence, we complete the proof. □

Remark 8. In order to make the computation more tractable, here, we set $P_{21} = 0$; then, $C_1 = 0$. Equation (23) is $\operatorname{rank}(C_2) = \operatorname{rank}([0 \ \ C_2])$, which holds naturally. Equation (22) becomes

$$A^T N^T P_{11} N + N^T P_{11} N A + C_2 B^T + B C_2^T < 0. \tag{25}$$

Then, we solve (12) and (25) by LMI toolbox in Matlab; thus, C can be computed by $C = P_{22}^{-T} C_2^T$, where P_{22} is any nonsingular matrix.

Remark 9. It is known that (14) is equivalent to

$$\begin{aligned} &B^T \left[A^T N^T P_{11} N + N^T P_{11} N A + C^T P_{21} N + N^T P_{21}^T C \right. \\ &\quad \left. + C^T P_{22} B^T + B P_{22}^T C \right] B < 0. \end{aligned} \tag{26}$$

In view of the fact that $NB = 0$, then

$$B^T C^T P_{22} B^T B + B^T B P_{22}^T C B < 0, \tag{27}$$

which implies that CB is nonsingular.

4. The Design of Adaptive Sliding Mode Controller

In this section, the adaptive controller is designed to drive the state of the system (1) into the sliding mode surface $S = 0$ in finite time. And it is given as the following theorem.

Theorem 10. *Let* $\gamma_1 = \|CB\|\gamma$ *and let* ε, η *be both positive constants. If the adaptive controller is designed as*

$$u = \begin{cases} -(CB)^{-1}\dfrac{S}{\|S\|}\left[\|CAx\| + \|CBf(x)\| + \hat{\gamma}_1\|x\| + \varepsilon\right], & S \neq 0, \\ 0, & S = 0, \end{cases} \tag{28}$$

where

$$\dot{\hat{\gamma}}_1 = \begin{cases} \eta\|x\|, & S \neq 0, \\ 0, & S = 0, \end{cases} \tag{29}$$

then the state of the system (1) *reaches the sliding mode surface* $S = 0$ *in finite time.*

Proof. Consider the Lyapunov function as

$$V = \|S\| + \frac{1}{2}\eta^{-1}\tilde{\gamma}_1^2, \tag{30}$$

where $\tilde{\gamma}_1 = \gamma_1 - \hat{\gamma}_1$. It is obvious that $\dot{\tilde{\gamma}}_1 = -\dot{\hat{\gamma}}_1$. Calculate the derivative of V along the trajectory of the closed-loop system (1); that is,

$$\begin{aligned} \dot{V} &= \frac{S^T\dot{S}}{\|S\|} + \eta^{-1}\tilde{\gamma}_1\dot{\tilde{\gamma}}_1 \\ &= \frac{S^T}{\|S\|}\left[CAx + CB\left(f(x) + \omega(x) + u\right)\right] - \tilde{\gamma}_1\|x\|. \end{aligned} \tag{31}$$

It can be direct to verify that

$$\begin{aligned} \frac{S^T}{\|S\|}(CAx) &\leq \|CAx\|, \\ \frac{S^T}{\|S\|}\left[CBf(x)\right] &\leq \|CBf(x)\|, \\ \frac{S^T}{\|S\|}\left[CB\omega(x)\right] &\leq \|CB\|\gamma\|x\| = \gamma_1\|x\|, \\ \frac{S^T}{\|S\|}CBu &\leq \left[-\|CAx\| - \|CBf(x)\| - \hat{\gamma}_1\|x\| - \varepsilon\right]. \end{aligned} \tag{32}$$

Substituting (32) into (31), we can get that

$$\dot{V} \leq -\varepsilon, \tag{33}$$

which means that $\dot{V} \leq -\varepsilon < 0$ if $S \neq 0$. It is also the fact that $\dot{V} = 0$ if $S = 0$. Thus, we can conclude that S will approach 0 in finite time. Thus, we have completed the proof of Theorem 10. □

Remark 11. It is known that the equilibrium $\tilde{\gamma}_1 = 0$ is only Lyapunov stable, which means that the estimation of γ_1 can only go to a small neighborhood of the real values; hence, $\hat{\gamma}_1$ may not necessarily converge to the real value γ_1.

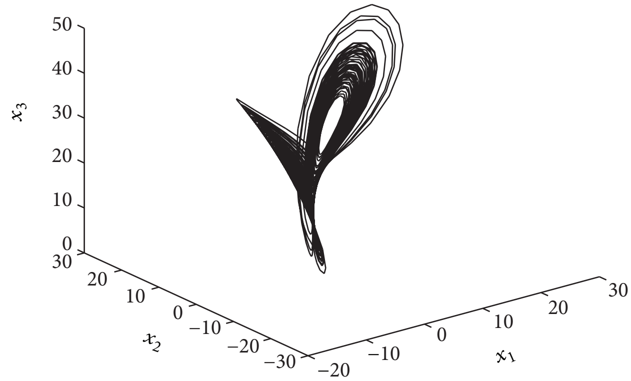

Figure 1: The attractors of Lorenz system.

Remark 12. We show that (33) implies that S is finite time stable. Let the initial time and the settling time be t_1 and t_2. From (33), we can compute that

$$V(t_2) - V(t_1) \leq -\varepsilon(t_2 - t_1). \tag{34}$$

In view of $V(t_2) \geq 0$, t_2 can be estimated as

$$t_2 \leq t_1 + \frac{V(t_1)}{\varepsilon}, \tag{35}$$

which means that S is finite time stable.

5. Numerical Example

We consider the Lorenz system (1) with disturbance where

$$A = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & 0 \\ 0 & 0 & -8/3 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$f(x) = \begin{bmatrix} -x_1x_3 \\ x_1x_2 \end{bmatrix}, \qquad \omega(x) = \begin{bmatrix} 0.5\sin x_1 \\ 0 \end{bmatrix}. \tag{36}$$

N is chosen as $N = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$; then M is identity matrix. Solving the LMIs (12) and (25) yields

$$P_{11} = 0.0562, \qquad C = \begin{bmatrix} -0.5619 & -0.5365 & 0 \\ 0 & 0 & -0.5365 \end{bmatrix}. \tag{37}$$

Since $\|\omega(x)\| \leq 0.5\|x\|$, then $\gamma_1 = 0.3793$. The parameter ε is chosen as 0.5. The sliding mode surface is $S = Cx$, and the adaptive controller is designed by (28) and (29), respectively.

We now complete the simulation by Simulink in Matlab. The initial state of the system (1) is $\begin{bmatrix} -0.2 & 0.2 & 0.1 \end{bmatrix}^T$, and the initial value of $\hat{\gamma}_1$ is chosen as 0.3. Figure 1 shows the attractors of the Lorenze system (open-loop system (1)). Figure 2 shows the time response of the states of the closed-loop system (1). Figure 3 is about the estimated parameters γ_1, and $\hat{\gamma}_1$ converges to 0.2807 but does not converge to the nominal value $\gamma_1 = 0.3793$; that is, it is Lyapunov stable. From the result of simulation, we can conclude that the method proposed is effective in this paper.

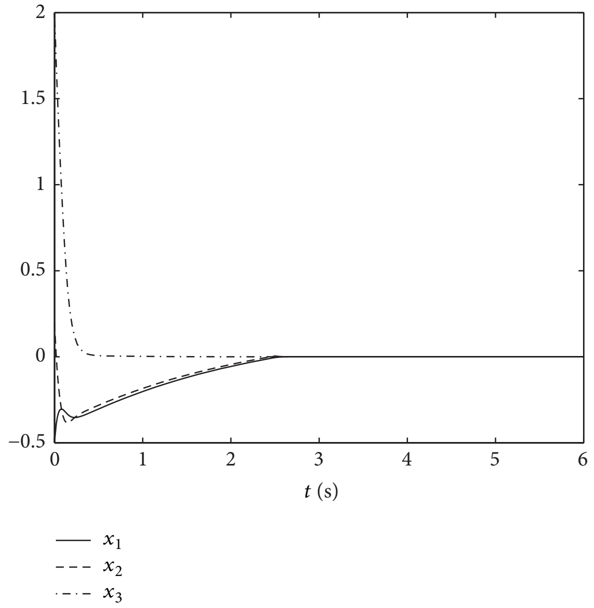

Figure 2: Response of the states of the closed-loop system (1).

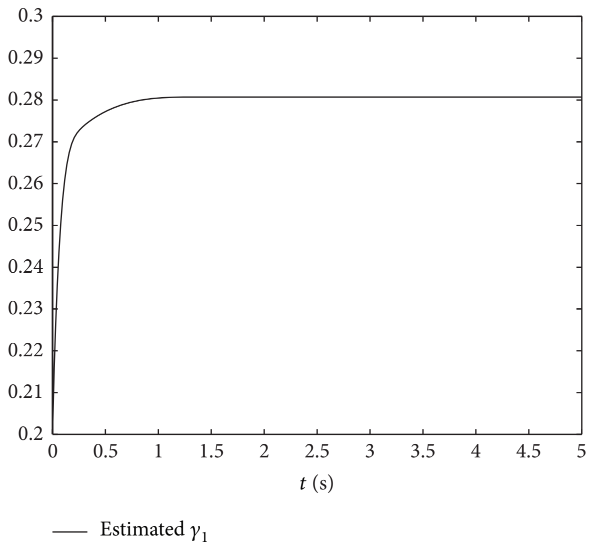

Figure 3: The estimation of unknown parameter γ_1.

6. Conclusions

We consider the control problem for the chaotic system, where the upper bound of the disturbance is unknown. We give sliding mode surface to guarantee the convergence of the sliding mode dynamic by singular system method and then design adaptive controller to make the closed-loop system reach the sliding mode in finite time. We also present a numerical example to show the validation of the proposed method. In the future work, we will extend the results of the paper to the hyper-chaotic system.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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