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Research Article

Some Further Results on Oscillations for Neutral Delay Differential Equations with Variable Coefficients

Fatima N. Ahmed, Rokiah Rozita Ahmad, Ummul Khair Salma Din, and Mohd Salmi Md Noorani

School of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia (UKM), 43600 Bangi, Selangor, Malaysia

Correspondence should be addressed to Fatima N. Ahmed; zahra80zahra@yahoo.com

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We study the oscillatory behaviour of all solutions of first-order neutral equations with variable coefficients. The obtained results extend and improve some of the well-known results in the literature. Some examples are given to show the evidence of our new results.

1. Introduction and Preliminaries

When delays appear in additional terms involving the highest order derivative of the unknown function in a differential equation, we are then dealing with a neutral type differential equation. The oscillation theory as a part of the qualitative theory of this type of equations has been developed rapidly in the past three decades. To a large extent, the study of neutral delay differential equations is motivated by having many applications in natural science and technology. A discussion of some applications of these equations and some differences in the behaviour of their solutions and the solutions of nonneutral equations can be found in Grammatikopoulos et al. [1, 2], Agarwal et al. [3], Driver [4], Hale [5], Győri and Ladas [6], and Dong [7].

In fact, the paper of Zahariev and Baĭnov [8] seems to be the first work dealing with oscillation of neutral equations. A systematic development of oscillation theory of neutral equations was initiated by Ladas and Sficas [9].

In this paper we are concerned with the oscillation of solutions of neutral delay differential equations of the form

$$[r(t)(a(t)x(t)-p(t)x(t-\tau))]' + q(t)x(t-\sigma) = 0, \quad t \geq t_0, \tag{1}$$

where

$$r(t), a(t), p(t), q(t) \in C[[t_0,\infty), \mathbb{R}^+], \quad \tau, \sigma \in (0,\infty). \tag{2}$$

Observe that when $r(t) \equiv 1$ and $a(t) \equiv 1$, (1) reduces to the equation

$$(x(t)-p(t)x(t-\tau))' + q(t)x(t-\sigma) = 0; \quad t \geq t_0 \tag{3}$$

which has been studied by several authors including Gopalsamy and Zhang [10], Chuanxi and Ladas [11, 12], Choi and Koo [13], Greaf et al. [14], Erbe et al. [15], and Al-Amri [16].

In particular, Elabbasy and Saker [17] established some infinite sufficient conditions for the oscillation of all solutions of (3) in the case when $p(t) \equiv p$. For the oscillation of (3) in the case when p and q are constants, we refer to the papers of Ladas and Schults [18], Sficas and Stavroulakis [19], and Kulenović et al. [20].

From the literature, we note that, for the neutral delay differential equation (1), the oscillation property has been much studied under the hypothesis

$$\int_{t_0}^{\infty} q(s)\,ds = \infty, \tag{4}$$

which has been established in [9] (see, e.g., [1, 6, 12, 21].

However, condition (4) seems to be an essential condition for the oscillation. Moreover, it is also a sufficient condition for the oscillation of (3) with the critical case that $p(t) \equiv 1$, which has been established in Chuanxi and Ladas [11] and Győri and Ladas [6].

Also in [22], Yu et al. considered (3) and relaxed condition (4) to the condition

$$\int_{t_0}^{\infty} sq(s)\left(\int_{s}^{\infty} q(t)\,dt\right) ds = \infty. \tag{5}$$

Recently, Karpuz and Öcalan [23] investigated the oscillation of (3) when p and q are positive continuous functions with $0 \le p(t) \le 1$ in the case when condition (4) does not hold.

In [24], Saker and Kubiaczyk studied the nonlinear form of (1) in the case when $r(t) \equiv 1$. Other contributions related to the field of oscillations for (1) are papers of Chen et al. [25] and Kubiaczyk and Saker [26] and a recent paper of Ahmed et al. [27] which considered (1) in the case when $a(t) \equiv 1$. Known results often take the form that any solution either is oscillatory or converges to zero; see, for example, Greaf et al. [28].

Our aim in this paper is to obtain some sufficient conditions for all solutions of (1) to be oscillatory. This can be done by using several results obtained in [6, 24, 25, 27, 29]. Our results improve and extend some of the well-known results in the literature.

As usual, a solution of (1) is said to be oscillatory if it has arbitrarily large zeros and nonoscillatory if it is either eventually positive or eventually negative. Equation (1) is said to be oscillatory if all its solutions are oscillatory.

In the sequel, unless otherwise specified, when we write a functional inequality, we assume that it holds for all sufficiently large t.

The following results are needed to specify the proofs of our main results.

Corollary 1 (see [6]). *Assume that*

$$P_i, \tau_i \in C\left[[t_0,\infty), \mathbb{R}^+\right], \quad \text{for } i = 1, 2, \dots, n. \tag{6}$$

Then the differential inequality

$$x'(t) + \sum_{i=1}^{n} P_i(t)\, x\left(t - \tau_i(t)\right) \le 0, \quad t \ge t_0, \tag{7}$$

has an eventually positive solution if and only if the equation

$$x'(t) + \sum_{i=1}^{n} P_i(t)\, x\left(t - \tau_i(t)\right) = 0, \quad t \ge t_0, \tag{8}$$

has an eventually positive solution.

Theorem 2 (see [6]). *Assume that*

$$\begin{gathered} P \in C\left[[t_0,\infty), \mathbb{R}^+\right], \quad \tau > 0, \\ \liminf_{t\to\infty} \int_{t-\tau}^{t} P(s)\, ds > \frac{1}{e}. \end{gathered} \tag{9}$$

Then,

(i) *the delay differential inequality*

$$x'(t) + P(t)\, x(t-\tau) \le 0, \quad t \ge t_0, \tag{10}$$

cannot have an eventually positive solution;

(ii) *the delay differential inequality*

$$x'(t) + P(t)\, x(t-\tau) \ge 0, \quad t \ge t_0, \tag{11}$$

cannot have an eventually negative solution.

Theorem 3 (see [6]). *Assume that* (6) *holds. Set*

$$\tau(t) = \min_{1\le i\le n}\left\{\tau_i(t)\right\}, \quad t \ge 0, \tag{12}$$

and suppose that

$$\lim_{t\to\infty} (t - \tau(t)) = \infty. \tag{13}$$

Then

$$\limsup_{t\to\infty} \int_{t-\tau(t)}^{t} \sum_{i=1}^{n} P_i(s)\, ds > 1 \tag{14}$$

is a sufficient condition for the oscillation of all solutions of (8).

Lemma 4 (see [24]). *Assume that* (2) *holds and there exists* $t^* \ge t_0 > 0$ *such that*

$$\frac{p\left(t^* + n^*\tau\right)}{a\left(t^* + (n^*-1)\tau\right)} \le 1 \quad \text{for } n^* = 0, 1, 2, \dots. \tag{15}$$

Let $x(t)$ *be an eventually positive solution of* (1). *Set*

$$z(t) = a(t)\, x(t) - p(t)\, x(t-\tau). \tag{16}$$

Then

$$z(t) > 0, \qquad z'(t) < 0. \tag{17}$$

2. Main Results

Our objective in this section is to establish the following results.

Theorem 5. *Let conditions* (2) *and* (15) *hold with* $r(t) \equiv 1$. *Assume that either*

$$\liminf_{t\to\infty} \int_{t-\sigma}^{t} \frac{q(s)}{a(s-\sigma)} ds > \frac{1}{e} \tag{18}$$

or

$$\limsup_{t\to\infty} \int_{t-\sigma}^{t} \frac{q(s)}{a(s-\sigma)} ds > 1. \tag{19}$$

Then every solution of (1) *is oscillatory.*

Proof. For the sake of obtaining a contradiction, assume that there is an eventually positive solution $x(t)$ of (1). Let $z(t)$ be defined by (16). Then by Lemma 4, we obtain

$$z(t) > 0, \quad t \geq t_1 \geq t_0. \tag{20}$$

From (1) with $r(t) \equiv 1$, we have

$$\begin{aligned} z'(t) &= -q(t)\, x(t-\sigma) \\ &= -q(t)\left[\frac{1}{a(t-\sigma)} \times \left(z(t-\sigma) + p(t-\sigma)\, x(t-\tau-\sigma)\right)\right] \\ &= -\frac{q(t)}{a(t-\sigma)} z(t-\sigma) - \frac{q(t)\, p(t-\sigma)}{a(t-\sigma)} x(t-\tau-\sigma) \\ &= -\frac{q(t)}{a(t-\sigma)} z(t-\sigma) + \frac{p(t-\sigma)}{a(t-\sigma)} \frac{q(t)}{q(t-\tau)} z'(t-\tau). \end{aligned} \tag{21}$$

Hence,

$$z'(t) - \frac{p(t-\sigma)}{a(t-\sigma)} \frac{q(t)}{q(t-\tau)} z'(t-\tau) + \frac{q(t)}{a(t-\sigma)} z(t-\sigma) = 0. \tag{22}$$

Set

$$\lambda(t) = -\frac{z'(t)}{z(t)}. \tag{23}$$

Then

$$\lambda(t) > 0; \quad t \geq t_1. \tag{24}$$

Substituting (23) in (22), we get

$$-\lambda(t)\, z(t) - \frac{p(t-\sigma)}{a(t-\sigma)} \frac{q(t)}{q(t-\tau)} \times \left(-\lambda(t-\tau)\, z(t-\tau)\right) + \frac{q(t)}{a(t-\sigma)} z(t-\sigma) = 0, \tag{25}$$

which reduces to equation

$$\lambda(t) = \lambda(t-\tau) \frac{p(t-\sigma)}{a(t-\sigma)} \frac{q(t)}{q(t-\tau)} \times \exp\left(\int_{t-\tau}^{t} \lambda(s)\, ds\right) + \frac{q(t)}{a(t-\sigma)} \exp\left(\int_{t-\sigma}^{t} \lambda(s)\, ds\right). \tag{26}$$

In view of our hypotheses, it is clear that

$$\lambda(t) \geq \frac{q(t)}{a(t-\sigma)} \exp\left(\int_{t-\sigma}^{t} \lambda(s)\, ds\right). \tag{27}$$

Then

$$-\frac{z'(t)}{z(t)} \geq \frac{q(t)}{a(t-\sigma)} \frac{z(t-\sigma)}{z(t)}. \tag{28}$$

Hence, from (23) and (27), we see that $z(t)$ is positive solution of the delay differential inequality

$$z'(t) + \frac{q(t)}{a(t-\sigma)} z(t-\sigma) \leq 0. \tag{29}$$

Then, by Corollary 1, the delay differential equation

$$z'(t) + \frac{q(t)}{a(t-\sigma)} z(t-\sigma) = 0 \tag{30}$$

has an eventually positive solution as well. On the other hand, from Theorems 2 and 3, we have that (18) or (19) implies that (30) cannot have an eventually positive solution. This contradicts the fact that $z(t) > 0$. The proof is complete. □

Example 6. Consider the equation

$$\begin{aligned} &\left[x(t) - \left(1 + \frac{1}{2}\sin t\right) x\left(t - \frac{\pi}{2}\right)\right]' \\ &\quad + \left[\left(2\sqrt{2}+1\right)\frac{4}{\pi} + 2\cos 2t\right] \\ &\quad \times x\left(t - \frac{\pi}{4}\right) = 0; \quad t \geq 0. \end{aligned} \tag{31}$$

Note that all the hypotheses of Theorem 5 are satisfied;

$$\begin{aligned} &\liminf_{t\to\infty} \int_{t-\sigma}^{t} \frac{q(s)}{a(s-\sigma)} ds \\ &\quad = \liminf_{t\to\infty} \int_{t-\pi/4}^{t} \left[\left(2\sqrt{2}+1\right)\frac{4}{\pi} + 2\cos 2s\right] ds \\ &\quad = \liminf_{t\to\infty} \left[2\sqrt{2} + 1 + \sin 2t + \cos 2t\right] > \frac{1}{e}. \end{aligned} \tag{32}$$

Then every solution of (31) oscillates.

Theorem 7. *Let conditions* (2) *and* (15) *hold with* $r(t) \equiv 1$. *Assume that either*

$$\liminf_{t\to\infty} \int_{t-\tau}^{t} \frac{q(s)\, p(s-\sigma)}{a(s-\tau-\sigma)\, a(s-\sigma)} ds > \frac{1}{e} \tag{33}$$

or

$$\limsup_{t\to\infty} \int_{t-\tau}^{t} \frac{q(s)\, p(s-\sigma)}{a(s-\tau-\sigma)\, a(s-\sigma)} ds > 1. \tag{34}$$

Then every solution of (1) *is oscillatory.*

Proof. For the sake of obtaining a contradiction, assume that there is an eventually positive solution $x(t)$ of (1). Let $z(t)$ be defined by (16). Proceeding as in the proof of Theorem 5, we again obtain (26), from which one can easily see that

$$\lambda(t) \geq \frac{q(t)}{a(t-\sigma)}. \tag{35}$$

Then

$$\lambda(t-\tau) \geq \frac{q(t-\tau)}{a(t-\tau-\sigma)}. \tag{36}$$

Substituting (36) in (26), we find.

$$\begin{aligned}\lambda(t) \geq{}& \frac{q(t)\,p(t-\sigma)}{a(t-\tau-\sigma)\,a(t-\sigma)}\exp\left(\int_{t-\tau}^{t}\lambda(s)\,ds\right)\\ &+\frac{q(t)}{a(t-\sigma)}\exp\left(\int_{t-\sigma}^{t}\lambda(s)\,ds\right).\end{aligned} \tag{37}$$

This implies that

$$\lambda(t) \geq \frac{q(t)\,p(t-\sigma)}{a(t-\tau-\sigma)\,a(t-\sigma)}\exp\left(\int_{t-\tau}^{t}\lambda(s)\,ds\right). \tag{38}$$

Then from (23) and (37), we can see that $z(t)$ is positive solution of the delay differential inequality

$$z'(t) + \frac{q(t)\,p(t-\sigma)}{a(t-\tau-\sigma)\,a(t-\sigma)}z(t-\tau) \leq 0. \tag{39}$$

Then, by Corollary 1, the delay differential equation

$$z'(t) + \frac{q(t)\,p(t-\sigma)}{a(t-\tau-\sigma)\,a(t-\sigma)}z(t-\tau) = 0 \tag{40}$$

has an eventually positive solution as well. On the other hand, by Theorems 2 and 3, we have that (33) or (34) implies that (40) cannot have an eventually positive solution. This contradicts the fact that $z(t) > 0$. The proof is complete. □

Example 8. Consider

$$\begin{aligned}&\left(x(t)-\left(\frac{3}{2}+\sin t\right)x\left(t-\frac{\pi}{2}\right)\right)'\\ &\quad+\frac{\left(\sqrt{5}+1/5\right)(2/\pi)+\cos t}{3/2-\sin t}x(t-\pi)=0;\\ &\qquad t\geq 0.\end{aligned} \tag{41}$$

Here, we have

$$\begin{aligned}q(t) &= \frac{\left(\sqrt{5}+1/5\right)(2/\pi)+\cos t}{3/2-\sin t} > 0,\\ \limsup_{t\to\infty}&\int_{t-\tau}^{t}\frac{q(s)\,p(s-\sigma)}{a(s-\tau-\sigma)\,a(s-\sigma)}ds\\ &=\limsup_{t\to\infty}\int_{t-\pi/2}^{t}\left(\frac{3}{2}+\sin(s-\pi)\right)\\ &\qquad\times\left[\frac{\left(\left(\sqrt{5}+1/5\right)(2/\pi)+\cos s\right)}{(3/2-\sin s)}\right]ds\\ &=\limsup_{t\to\infty}\int_{t-\pi/2}^{t}\left(\left(\sqrt{5}+\frac{1}{5}\right)\frac{2}{\pi}+\cos s\right)ds\\ &=\limsup_{t\to\infty}\left(\sqrt{5}+\frac{1}{5}+\sin t+\cos t\right) > 1.\end{aligned} \tag{42}$$

Therefore, every solution of (41) is oscillatory.

Theorem 9. *Let conditions* (2) *and* (15) *hold with* $r(t)\equiv 1$ *and* $\tau \geq \sigma$. *Assume that either*

$$\liminf_{t\to\infty}\int_{t-\sigma}^{t}\left[\frac{q(s)\,p(s-\sigma)}{a(s-\tau-\sigma)\,a(s-\sigma)}+\frac{q(s)}{a(s-\sigma)}\right]ds > \frac{1}{e} \tag{43}$$

or

$$\limsup_{t\to\infty}\int_{t-\sigma}^{t}\left[\frac{q(s)\,p(s-\sigma)}{a(s-\tau-\sigma)\,a(s-\sigma)}+\frac{q(s)}{a(s-\sigma)}\right]ds > 1. \tag{44}$$

Then every solution of (1) *is oscillatory.*

Proof. For the sake of obtaining a contradiction, assume that there is an eventually positive solution $x(t)$ of (1). Let $z(t)$ be defined by (16). Proceeding as in the proof of Theorem 5, we again obtain (26) and (37). From (23) in (37), we see that $z(t)$ is positive solution of the delay differential inequality

$$\begin{aligned}z'(t) &+ \frac{q(t)\,p(t-\sigma)}{a(t-\tau-\sigma)\,a(t-\sigma)}z(t-\tau)\\ &+\frac{q(t)}{a(t-\sigma)}z(t-\sigma) \leq 0.\end{aligned} \tag{45}$$

Since $z'(t) < 0$ and $\tau \geq \sigma$, (45) yields

$$z'(t) + \left[\frac{q(s)\,p(s-\sigma)}{a(s-\tau-\sigma)\,a(s-\sigma)}+\frac{q(s)}{a(s-\sigma)}\right]z(t-\sigma) \leq 0. \tag{46}$$

Then, by Corollary 1, the delay differential equation

$$z'(t) + \left[\frac{q(s)\,p(s-\sigma)}{a(s-\tau-\sigma)\,a(s-\sigma)}+\frac{q(s)}{a(s-\sigma)}\right]z(t-\sigma) = 0 \tag{47}$$

has an eventually positive solution as well. On the other hand, by Theorems 2 and 3, we have that (43) or (44) implies that (47) cannot have an eventually positive solution. This contradicts the fact that $z(t) > 0$. The proof is complete. □

Theorem 10. *Let conditions* (2) *and* (15) *hold with* $r(t) \equiv 1$ *and* $\tau \geq \sigma$. *Assume that either*

$$\liminf_{t\to\infty} \int_{t-\sigma}^{t} \left[\frac{q(s)\,p(s-\sigma)\,p(s-\tau-\sigma)}{a(s-\sigma)\,a(s-\tau-\sigma)} + \frac{q(s)}{a(s-\sigma)} \right] ds > \frac{1}{e} \tag{48}$$

or

$$\limsup_{t\to\infty} \int_{t-\sigma}^{t} \left[\frac{q(s)\,p(s-\sigma)\,p(s-\tau-\sigma)}{a(s-\sigma)\,a(s-\tau-\sigma)} + \frac{q(s)}{a(s-\sigma)} \right] ds > 1. \tag{49}$$

Then every solution of (1) *is oscillatory.*

Proof. For the sake of obtaining a contradiction, assume that there is an eventually positive solution $x(t)$ of (1). Let $z(t)$ be defined by (16). Proceeding as in the proof of Theorem 7, we again obtain (37), which guarantees that eventually

$$\lambda(t) \geq \frac{q(t)\,p(t-\sigma)}{a(t-\tau-\sigma)\,a(t-\sigma)}. \tag{50}$$

Then

$$\lambda(t-\tau) \geq \frac{q(t-\tau)\,p(t-\tau-\sigma)}{a(t-\tau-\sigma)\,a(t-2\tau-\sigma)}. \tag{51}$$

Substituting (51) in (26), we obtain.

$$\lambda(t) \geq \frac{q(t)\,p(t-\sigma)\,p(t-\tau-\sigma)}{a(t-\tau-\sigma)\,a(t-\sigma)} \exp\left(\int_{t-\tau}^{t} \lambda(s)\,ds \right) + \frac{q(t)}{a(t-\sigma)} \exp\left(\int_{t-\sigma}^{t} \lambda(s)\,ds \right). \tag{52}$$

From (23) and (52), we see that $z(t)$ is positive solution of the delay differential inequality

$$z'(t) + \frac{q(t)\,p(t-\sigma)\,p(s-\tau-\sigma)}{a(t-\tau-\sigma)\,a(t-\sigma)} z(t-\tau) + \frac{q(t)}{a(t-\sigma)} z(t-\sigma) \leq 0. \tag{53}$$

Since $z'(t) < 0$ and $\tau \geq \sigma$, then (53) gives

$$z'(t) + \left[\frac{q(s)\,p(s-\sigma)\,p(s-\tau-\sigma)}{a(s-\tau-\sigma)\,a(s-\sigma)} + \frac{q(s)}{a(s-\sigma)} \right] z(t-\sigma) \leq 0. \tag{54}$$

Then, by Corollary 1, the delay differential equation

$$z'(t) + \left[\frac{q(s)\,p(s-\sigma)\,p(s-\tau-\sigma)}{a(s-\tau-\sigma)\,a(s-\sigma)} + \frac{q(s)}{a(s-\sigma)} \right] z(t-\sigma) = 0 \tag{55}$$

has an eventually positive solution as well. On the other hand, by Theorems 2 and 3, we have that (48) or (49) implies that (55) cannot have an eventually positive solution. This contradicts the fact that $z(t) > 0$. The proof is complete. □

Theorem 11. *Let conditions* (2) *and* (15) *hold with* $\tau \geq \sigma$. *Assume that either*

$$\liminf_{t\to\infty} \int_{t-\sigma}^{t} \frac{q(t)}{r(t-\sigma)\,a(t-\sigma)}\,ds > \frac{1}{e}. \tag{56}$$

or

$$\limsup_{t\to\infty} \int_{t-\sigma}^{t} \frac{q(t)}{r(t-\sigma)\,a(t-\sigma)}\,ds > 1 \tag{57}$$

Then every solution of NDDE (1) *oscillates.*

Proof. For the sake of obtaining a contradiction, assume that there is an eventually positive solution $x(t)$ of (1). Let $z(t)$ be defined by (16). Then by Lemma 4, it follows that

$$z(t) > 0. \tag{58}$$

As $z(t) \leq a(t)x(t) \Rightarrow x(t) \geq z(t)/a(t)$, $a(t) > 0$, then from (1), we get

$$(r(t)z(t))' + \frac{q(t)}{a(t-\sigma)} z(t-\sigma) \leq 0. \tag{59}$$

Dividing the last inequality by $r(t) > 0$, we obtain

$$z'(t) + \frac{r'(t)}{r(t)} z(t) + \frac{q(t)}{r(t)\,a(t-\sigma)} z(t-\sigma) \leq 0. \tag{60}$$

Let

$$z(t) = e^{-\int_{t_0}^{t} (r'(s)/r(s))ds}\, y(t). \tag{61}$$

This implies that $y(t) > 0$. Substituting (61) in (27) yields for all $t \geq t_0$

$$y'(t) + \frac{q(t)}{r(t-\sigma)\,a(t-\sigma)} y(t-\sigma) \leq 0. \tag{62}$$

Then, by Corollary 1, the delay differential equation

$$y'(t) + \frac{q(t)}{r(t-\sigma)\,a(t-\sigma)} y(t-\sigma) = 0, \quad t \geq t_0, \tag{63}$$

has an eventually positive solution as well. On the other hand, by Theorems 2 and 3, we have that (56) or (57) implies that (63) cannot have an eventually positive solution. This contradicts the fact that $y(t) > 0$. The proof is complete. □

Example 12. Consider the NDDE

$$\left[\frac{1}{t}\left(x(t)-\left(\frac{3}{2}+\sin t\right)x(t-\pi)\right)\right]' + \frac{1}{t-(\pi/2)}x\left(t-\frac{\pi}{2}\right)=0, \quad t>\frac{\pi}{2}. \tag{64}$$

Note that Theorem 2.1 [26] cannot be applied on (64), but one can see that all the conditions needed in Theorem 11 are satisfied, where

$$\lim_{t\to\infty}\inf\int_{t-\sigma}^{t}\frac{q(s)}{r(s-\sigma)a(s-\sigma)}ds = \lim_{t\to\infty}\inf\int_{t-\pi/2}^{t}ds=\frac{\pi}{2}>\frac{1}{e}. \tag{65}$$

Therefore, every solution of (64) is oscillatory.

Remark 13. Theorem 11 improves and extends Theorem 2.1 [26], Theorem 2.2 [27], Theorem 6.4.2 [6], Theorem 3 [11], Theorem 7 [1], and Theorem 7 [9].

Conflict of Interests

The authors declare that is no conflict of interests regarding the publication of this paper.

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