

Research Article

Exact Solutions for a New Nonlinear KdV-Like Wave Equation Using Simplest Equation Method and Its Variants

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The simplest equation method presents wide applicability to the handling of nonlinear wave equations. In this paper, we focus on the exact solution of a new nonlinear KdV-like wave equation by means of the simplest equation method, the modified simplest equation method and, the extended simplest equation method. The KdV-like wave equation was derived for solitary waves propagating on an interface (liquid-air) with wave motion induced by a harmonic forcing which is more appropriate for the study of thin film mass transfer. Thus finding the exact solutions of this equation is of great importance and interest. By these three methods, many new exact solutions of this equation are obtained.

1. Introduction

Nonlinear phenomena play a crucial role in applied mathematics and physics. Calculating exact and numerical solutions, in particular the traveling wave solutions of nonlinear equations in mathematical physics, plays an important role in soliton theory. In the recent years, much effort has been spent on this task and many significant methods have been established such as inverse scattering transform [1], Bäcklund and Darboux transform [2], Hirota method [3], homogeneous balance method [4], Jacobi elliptic function method [5], tanh-function method [6], Exp-function method [7], G'/G -expansion method [8, 9], the method for construction of meromorphic solutions [10, 11], variational iteration method [12–14], and He's homotopy perturbation method [15–19].

Partially, the simplest equation method is a very powerful mathematical technique to seek more new solutions of NLEEs that can be expressed as polynomial in an elementary function which satisfies a subequation like Riccati equation, auxiliary ordinary equation, elliptic equation, and generalized Riccati equation. It has been developed by Fan [20] and Kudryashov [21] and the method optimizes the use of the auxiliary equation and effectively avoids producing some duplication solutions. There are many applications and generalizations of the method [22–25].

Recently, Bilige introduced a method called the extended simplest equation method as an extension of the simplest equation method, to look for the exact traveling wave solutions of nonlinear evolution equations (NLEEs) [26, 27]. In this method, a second order linear ordinary differential equation (ODE) is taken as the auxiliary equation. This method can construct different forms of exact traveling wave solutions which cannot be obtained by using the tanh-function method, F -expansion method, and the Exp-function method.

The nonlinear shallow water surface waves satisfy the Korteweg-de Vries (KdV) equation:

$$w_t + c_0 w_x + a_1 w w_x + a_2 w_{xxx} = 0. \quad (1)$$

This equation is only valid for long waves. Solitary waves in film flows were studied by [18, 19]. These flows also show a transition to turbulence. This process is better understood, if the dynamics of nonlinear waves are traceable.

In [28] Rees and Zimmerman derived a new nonlinear KdV-like evolution equation for surface solitary waves propagating on a liquid-air interface where the wave motion is induced by harmonic forcing:

$$w_{xxx} + k^2 w_x + \epsilon c \lambda_3 (w_x^2 - w w_{xx}) - \frac{2k^2}{c} w_t = 0. \quad (2)$$

The nonlinearity term ww_x in (1) does not appear in (2), and the reconstruction of this term from $(w_x^2 - ww_{xx})$ is not possible by order approximation equivalences; also the nonlinearity term in (2) is more steep.

It was observed by Sohail et al. [29] that wave data obtained from an online measurement technique satisfies this evolution equation, when the wave length is just a few multiples of the fluid depth. In [30], the integrability of this new nonlinear partial differential equation was discussed with a focus on the Painlevé property, the compatibility condition, and the Bäcklund transformation.

The exact traveling wave solutions of (2) are not reported in related references. We have tried many other algebraic methods to seek exact solutions of (2). However, we find that they are either not efficient or invalid. In this work, we apply the simplest equation method, the modified simplest equation method, and the extended simplest equation method for obtaining exact traveling solutions of (2). From the results we see that these methods are more effective.

The organization of the paper is as follows. In Section 2, a brief description of three simplest equation methods for finding traveling wave solutions of nonlinear equations is given. In Section 3, we will study (2) by these methods. Finally conclusions are given in Section 4.

2. Description of the Simplest Equation Methods

2.1. The Simplest Equation Method

Step 1. Consider a general nonlinear PDE in the form

$$F(u, u_x, u_t, u_{xx}, u_{xt}, \dots) = 0. \quad (3)$$

Using $u(x, t) = \phi(\xi)$, $\xi = \alpha x + \beta t$, we can rewrite (6) as the following nonlinear ODE:

$$F(\phi, \phi', \phi'', \dots) = 0, \quad (4)$$

where the prime denotes differentiation with respect to ξ .

Step 2. Suppose that the solution of ODE (4) can be written as follows:

$$\phi(\xi) = \sum_{i=1}^n A_i f^i(\xi), \quad (5)$$

where $f(\xi)$ are the functions that satisfy some ordinary differential equations. The simplest equation has two properties: first it is of lesser order than (4) and second we know the general solution of the simplest equation. n is a positive integer that can be determined by balancing procedure, and A_i ($i = 0, 1, 2, \dots$) are parameters to be determined.

In this paper, we will use as simplest equation the equations of Bernoulli and Riccati which are well known nonlinear ordinary differential equations and their solutions can be expressed by elementary functions.

For the Bernoulli equation

$$f'(\xi) = af(\xi) + bf^2(\xi), \quad (6)$$

equation (6) admits the following exact solutions:

$$f(\xi) = \frac{a \exp(a(\xi + \xi_0))}{1 - b \exp(a(\xi + \xi_0))} \quad (7)$$

for the case $a > 0$, $b < 0$ and

$$f(\xi) = -\frac{a \exp(a(\xi + \xi_0))}{1 + b \exp(a(\xi + \xi_0))} \quad (8)$$

for the case $a < 0$, $b > 0$, where ξ_0 is a constant.

For the Riccati equation

$$f'(\xi) = af^2(\xi) + b. \quad (9)$$

Equation (9) admits the following exact solutions:

$$f(\xi) = -\frac{\sqrt{-ab}}{a} \tanh\left(\sqrt{-ab}\xi - \frac{\varepsilon \ln \xi_0}{2}\right), \quad \varepsilon = \pm 1, \quad (10)$$

when $ab < 0$ and

$$f(\xi) = \frac{\sqrt{ab}}{a} \tan(\sqrt{ab}\xi + \xi_0) \quad (11)$$

when $ab > 0$, where ξ_0 is a constant.

Step 3. Substituting (5) along with (6) (or (9)) into (4), then the left-hand side of (4) is converted into a polynomial in $f(\xi)$; equating each coefficient of the polynomial to zero yields a set of algebraic equations for A_i, a, b .

Step 4. Solving the algebraic equations obtained in Step 3 and substituting the results into (5), we obtain the exact traveling wave solutions for (3).

2.2. The Modified Simplest Equation Method. In the modified version, one makes an ansatz for the solution $\phi(\xi)$ as

$$\phi(\xi) = \sum_{i=0}^n A_i \left(\frac{\phi'}{\phi}\right)^i, \quad (12)$$

where A_i ($i = 0, 1, 2, \dots, n$) are arbitrary constants to be determined, such that $a_n \neq 0$ and $\varphi = \varphi(\xi)$ is an unspecified function to be determined afterward.

Substitute (12) into (4) and then we account the function φ . As a result of this substitution, we get a polynomial of φ'/φ and its derivatives. In this polynomial, we equate the coefficients of the same power of φ^{-j} to zero, where $j > 0$. This procedure yields a system of equations which can be solved to find A_i ($i = 0, 1, 2, \dots, n$), φ , and φ' . Then the substitution of the results into (12) completes the determination of exact solutions of (3).

2.3. The Extended Simplest Equation Method. We suppose that the solutions for (4) can be expressed in the following form:

$$\phi(\xi) = \sum_{j_1=0}^n A_{j_1} \left(\frac{\varphi'}{\varphi}\right)^{j_1} + \sum_{j_2=0}^{n-1} B_{j_2} \left(\frac{\varphi'}{\varphi}\right)^{j_2} \frac{1}{\varphi}, \quad (13)$$

where $A_{j_1}, B_{j_2} (j_1 = 0, 1, \dots, n; j_2 = 0, 1, \dots, n - 1)$ are constants and $A_n B_{n-1} \neq 0$. The function $\varphi = \varphi(\xi)$ satisfies the second order linear ODE in the form

$$\varphi'' + \delta\varphi = \nu, \tag{14}$$

where δ and ν are constants. Equation (14) has three types of general solution with double arbitrary parameters as follows:

$$\varphi(\xi) = \begin{cases} C_1 \cosh(\sqrt{-\delta}\xi) + C_2 \sinh(\sqrt{-\delta}\xi) + \frac{\nu}{\delta}, & \delta < 0, \\ C_1 \cos(\sqrt{\delta}\xi) + C_2 \sin(\sqrt{\delta}\xi) + \frac{\nu}{\delta}, & \delta > 0, \\ \frac{\nu}{2}\xi^2 + C_1\xi + C_2, & \delta = 0, \end{cases} \tag{15}$$

$$\left(\frac{\varphi'}{\varphi}\right)^2 = \begin{cases} \left(\delta C_1^2 - \delta C_2^2 - \frac{\nu^2}{\delta}\right)\left(\frac{1}{\varphi}\right)^2 - \delta + \frac{2\nu}{\varphi}, & \delta < 0, \\ \left(\delta C_1^2 + \delta C_2^2 - \frac{\nu^2}{\delta}\right)\left(\frac{1}{\varphi}\right)^2 - \delta + \frac{2\nu}{\varphi}, & \delta > 0, \\ \left(C_1^2 - 2\nu C_2\right)\left(\frac{1}{\varphi}\right)^2 + \frac{2\nu}{\varphi}, & \delta = 0, \end{cases} \tag{16}$$

where C_1, C_2 are arbitrary constants.

By substituting (13) into (4) and using the second order linear ODE (14) and (16), collecting all terms with the same order of $1/\varphi^i$ and $(1/\varphi^i)(\varphi'/\varphi)$ together, the left-hand side of (4) is converted into another polynomial in $1/\varphi^i$ and $(1/\varphi^i)(\varphi'/\varphi)$. Equating each coefficient of these different power terms to zero yields a set of algebraic equations for $A_i, B_j (i = 0, 1, \dots, n, j = 0, 1, \dots, n - 1), \delta$, and ν .

Assuming constants $A_i, B_j (i = 0, 1, \dots, n, j = 0, 1, \dots, n - 1), \delta$, and ν can be determined by solving the algebraic equations, then substituting these terms and the general solutions (15) of (14) into (13), we have many exact traveling wave solutions of the NLEE (3).

3. Exact Solutions of New KdV-Like Equation

Making a transformation $w(x, t) = \phi(\xi)$ with $\xi = \alpha x + \beta t$, (2) can be reduced to the following ODE:

$$\alpha^3 \phi''' + k^2 \alpha \phi' + \epsilon c \lambda_3 (\alpha^2 \phi'^2 - \alpha^2 \phi \phi'') - \frac{2k^2 \beta}{c} \phi' = 0, \tag{17}$$

where $\alpha, \beta, k, c, \epsilon$, and λ_3 are nonzero constants. Balancing ϕ''' and $\phi \phi''$ in (17), we obtain $n + 3 = n + n + 2$ which gives $n = 1$.

3.1. Using Simplest Equation Method. Suppose that (17) owns the solutions in the form

$$\phi(\xi) = A_0 + A_1 f(\xi). \tag{18}$$

For the Bernoulli equation, substituting (18) into (17) and making use of the Bernoulli equation (6) and then setting

all the coefficients of $f^k(\xi)$ of the resulting system to zero, we obtain an algebraic system of equations in terms of $A_i (i = 0, 1), a, b, \alpha$, and β . Solving this system of algebraic equations, we obtain

$$A_0 = \frac{3\alpha a}{\epsilon c \lambda_3}, \quad A_1 = \frac{6\alpha b}{\epsilon c \lambda_3}, \quad \alpha = \alpha, \tag{19}$$

$$\beta = -\frac{c\alpha(2\alpha^2 a^2 - k^2)}{2k^2}.$$

Therefore, using solutions (7) and (8) of (6) and ansatz (18), we obtain the following exact solution of (2):

$$w_1(x, t) = \frac{3\alpha a}{\epsilon c \lambda_3} + \frac{6\alpha b}{\epsilon c \lambda_3} \frac{a \exp(a(\xi + \xi_0))}{1 - b \exp(a(\xi + \xi_0))} \tag{20}$$

$$= \frac{3\alpha a}{\epsilon c \lambda_3} \left(1 + \frac{2b \exp(a(\xi + \xi_0))}{1 - b \exp(a(\xi + \xi_0))} \right)$$

for the case $a > 0, b < 0$ and

$$w_2(x, t) = \frac{3\alpha a}{\epsilon c \lambda_3} - \frac{6\alpha b}{\epsilon c \lambda_3} \frac{a \exp(a(\xi + \xi_0))}{1 + b \exp(a(\xi + \xi_0))} \tag{21}$$

$$= \frac{3\alpha a}{\epsilon c \lambda_3} \left(1 - \frac{2b \exp(a(\xi + \xi_0))}{1 + b \exp(a(\xi + \xi_0))} \right)$$

for the case $a < 0, b > 0$, where $\xi = \alpha x - (c\alpha(2\alpha^2 a^2 - k^2)/2k^2)t$.

For the Riccati equation, substituting (18) into (17) and making use of the Riccati equation (9) and then setting all the coefficients of $f^k(\xi)$ of the resulting system to zero, we obtain an algebraic system of equations in terms of $A_i (i = 0, 1), a, b, \alpha$, and β . Solving this system of algebraic equations, we obtain

$$A_0 = 0, \quad A_1 = \frac{6\alpha a}{\epsilon c \lambda_3}, \quad \alpha = \alpha, \tag{22}$$

$$\beta = \frac{c\alpha(8\alpha^2 ab + k^2)}{2k^2}.$$

Therefore, using solutions (10) and (11) of (9) and ansatz (18), we obtain the following exact solution of (2):

$$w_3(x, t) = -\frac{6\alpha\sqrt{-ab}}{\epsilon c \lambda_3} \tanh\left(\sqrt{-ab}\xi - \frac{\epsilon \ln \xi_0}{2}\right), \tag{23}$$

for the case $ab < 0$, and

$$w_4(x, t) = \frac{6\alpha\sqrt{ab}}{\epsilon c \lambda_3} \tan(\sqrt{ab}\xi + \xi_0), \tag{24}$$

for the case $ab > 0$, where $\xi = \alpha x + (c\alpha(8\alpha^2 ab + k^2)/2k^2)t$.

3.2. Using the Modified Simplest Equation Method. Suppose that (17) owns the solutions in the form

$$\phi(\xi) = A_0 + A_1 \frac{\varphi'}{\varphi}, \tag{25}$$

where A_0 and A_1 are constants, such that $A_1 \neq 0$, and $\varphi = \varphi(\xi)$ is an unspecified function to be determined. It is simple to calculate that

$$\begin{aligned} \phi' &= A_1 \left(\frac{\phi''}{\varphi} - \left(\frac{\varphi'}{\varphi} \right)^2 \right), \\ \phi'' &= A_1 \frac{\varphi'''}{\varphi} - 3A_1 \frac{\varphi''\varphi'}{\varphi^2} + 2A_1 \left(\frac{\varphi'}{\varphi} \right)^3, \\ \phi''' &= A_1 \frac{\varphi^{(4)}}{\varphi} - A_1 \frac{4\varphi'''\varphi' + 3\varphi''^2}{\varphi^2} + 12A_1 \frac{\varphi''\varphi'^2}{\varphi^3} \\ &\quad - 6A_1 \left(\frac{\varphi'}{\varphi} \right)^4, \end{aligned} \tag{26}$$

substituting (25) and (26) into (17) and equating the coefficients of φ^j ($j = -3, -2, -1, 0$) to zero yield the following system:

$$\begin{aligned} \varphi^0 &: -A_1\alpha^2 c\varphi'^4 (6\alpha + \epsilon c\lambda_3 A_1) = 0, \\ \varphi^{-1} &: A_1 (k^2\alpha c\varphi'' - 2k^2\beta\varphi'' - \epsilon c^2\lambda_3\alpha^2 A_0\varphi''' + \alpha^3 c\varphi^{(4)}) = 0, \\ \varphi^{-2} &: A_1 (-3\alpha^3 c\varphi''^2 - 4\alpha^3 c\varphi'''\varphi' - k^2\alpha c\varphi'^2 + \epsilon c^2\lambda_3\alpha^2 A_1\varphi''^2 \\ &\quad + 3\epsilon c^2\lambda_3\alpha^2 A_0\varphi''\varphi' - \epsilon c^2\lambda_3\alpha^2 A_1\varphi'''\varphi' \\ &\quad + 2k^2\beta\varphi'^2) = 0, \\ \varphi^{-3} &: A_1\alpha^2 c\varphi'^2 (12\alpha\varphi'' + \epsilon c\lambda_3 A_1\varphi'' - 2\epsilon c\lambda_3 A_0\varphi') = 0. \end{aligned} \tag{27}$$

Solving the first equation of system (27), one has $A_1 = 0$ and $A_1 = -6\alpha/\epsilon c\lambda_3$. When $A_1 = 0$, we obtain trivial solution; therefore, the case is rejected. Substituting $A_1 = -6\alpha/\epsilon c\lambda_3$ into the other equations of system (27), we can obtain the following results:

$$A_0 = 0, \quad \alpha = \frac{2\beta}{c}, \quad \beta = \beta, \quad \varphi = k_1\xi + k_2, \tag{28}$$

where k_1 and k_2 are arbitrary constants, and

$$\begin{aligned} A_0 &= \pm \frac{3k\sqrt{2\alpha c(\alpha c - 2\beta)}}{2\alpha c^2\epsilon\lambda_3}, \quad \alpha = \alpha, \\ \beta &= \beta, \quad \varphi = k_1 + k_2 \exp\left(\frac{k\sqrt{2\alpha c(\alpha c - 2\beta)}}{2\alpha^2 c}\xi\right), \end{aligned} \tag{29}$$

where k_1 and k_2 are arbitrary constants, $2\alpha c(\alpha c - 2\beta) > 0$.

Substituting (28) into (25), we can get the exact solution of (2):

$$w_5(x, t) = A_1 \frac{\varphi'}{\varphi} = -\frac{12\beta k_1}{c^2\epsilon\lambda_3(k_1\xi + k_2)}, \tag{30}$$

where $\xi = (2\beta/c)x + \beta t$.

Substituting (29) into (25), we can get the exact solution of (2):

$$\begin{aligned} w_6(x, t) &= A_0 + A_1 \frac{\varphi'}{\varphi} \\ &= \frac{3k\epsilon\sqrt{2\alpha c(\alpha c - 2\beta)}}{2\epsilon c^2\lambda_3} \\ &\quad \times \left(\frac{-k_1 + k_2 \exp\left(\left(\frac{-k\epsilon\sqrt{2\alpha c(\alpha c - 2\beta)}/2\alpha^2 c}{\xi}\right)\right)}{k_1 + k_2 \exp\left(\left(\frac{-k\epsilon\sqrt{2\alpha c(\alpha c - 2\beta)}/2\alpha^2 c}{\xi}\right)\right)} \right), \end{aligned} \tag{31}$$

where $\xi = \alpha x + \beta t$, $\epsilon = \pm 1$.

We can arbitrarily choose the parameters k_1 and k_2 . Therefore, if we set $k_1 = \pm k_2$, (31) reduces to

$$\begin{aligned} w_7(x, t) &= -\frac{3k\sqrt{2\alpha c(\alpha c - 2\beta)}}{2\epsilon c^2\lambda_3} \\ &\quad \times \tanh\left(\frac{k\sqrt{2\alpha c(\alpha c - 2\beta)}}{4\alpha^2 c}\xi\right), \\ w_8(x, t) &= -\frac{3k\sqrt{2\alpha c(\alpha c - 2\beta)}}{2\epsilon c^2\lambda_3} \\ &\quad \times \coth\left(\frac{k\sqrt{2\alpha c(\alpha c - 2\beta)}}{4\alpha^2 c}\xi\right), \end{aligned} \tag{32}$$

where $2\alpha c(\alpha c - 2\beta) > 0$.

Using hyperbolic function identities, from (32), we obtain the following periodic solutions:

$$\begin{aligned} w_9(x, t) &= \frac{3k\sqrt{-2\alpha c(\alpha c - 2\beta)}}{2\epsilon c^2\lambda_3} \\ &\quad \times \tan\left(\frac{k\sqrt{-2\alpha c(\alpha c - 2\beta)}}{4\alpha^2 c}\xi\right), \\ w_{10}(x, t) &= -\frac{3k\sqrt{-2\alpha c(\alpha c - 2\beta)}}{2\epsilon c^2\lambda_3} \\ &\quad \times \cot\left(\frac{k\sqrt{-2\alpha c(\alpha c - 2\beta)}}{4\alpha^2 c}\xi\right). \end{aligned} \tag{33}$$

3.3. Using the Extended Simplest Equation Method. Suppose that (17) owns the solutions in the form

$$\phi(\xi) = A_0 + A_1 \frac{\phi'}{\phi} + B_1 \frac{1}{\phi}, \quad (34)$$

where $A_0, A_1,$ and B_1 are constants to be determined later, $A_1 A_3 \neq 0,$ and function $\varphi = \varphi(\xi)$ satisfies the second order linear ODE (14).

By substituting (34) into (17), using the second order linear ODE expressions (14) and (16), collecting all terms with the same order of $1/\varphi^i$ and $(1/\varphi^i)(\varphi'/\varphi)$ together, the left-hand side of (17) is converted into another polynomial in $1/\varphi^i$ and $(1/\varphi^i)(\varphi'/\varphi).$ Equating each coefficient of these different power terms to zero yields a set of algebraic equations for $A_0, A_1, B_1, \alpha, \beta, \delta,$ and $\nu.$ Solving these equations, we obtain the following results.

If $\delta > 0,$ one has

$$A_0 = 0, \quad A_1 = \pm \sqrt{\frac{\delta}{\Delta}} B_1, \quad B_1 = B_1,$$

$$\alpha = \pm \frac{1}{3} c \epsilon \lambda_3 \sqrt{\frac{\delta}{\Delta}} B_1, \quad \delta = \delta, \quad \nu = \nu,$$

$$\beta = \pm \left(\left((9\delta^2 k^2 C_2^2 - 9k^2 \nu^2 + 9\delta^2 k^2 C_1^2 + 2B_1^2 \delta^2 c^2 \epsilon^2 \lambda_3^2) c^2 \epsilon \lambda_3 \sqrt{\frac{\delta}{\Delta}} (54\Delta k^2)^{-1} \right) B_1, \right) \quad (35)$$

where $\Delta = \delta^2 C_1^2 + \delta^2 C_2^2 - \nu^2 > 0,$ and

$$A_0 = 0, \quad A_1 = A_1, \quad B_1 = 0,$$

$$\alpha = -\frac{1}{3} c \epsilon \lambda_3 A_1, \quad \delta = \pm \frac{\nu}{\sqrt{C_1^2 + C_2^2}}, \quad \nu = \nu, \quad (36)$$

$$\beta = -\frac{c^2 \epsilon \lambda_3}{54k^2} \left(\pm \frac{2c^2 \epsilon^2 \lambda_3^2 A_1^2 \nu}{\sqrt{C_1^2 + C_2^2}} + 9k^2 \right) A_1,$$

$$A_0 = 0, \quad A_1 = A_1, \quad B_1 = 0, \quad \alpha = -\frac{1}{6} c \epsilon \lambda_3 A_1,$$

$$\beta = -\frac{c^2 \epsilon \lambda_3 (2c^2 \epsilon^2 \lambda_3^2 A_1^2 \delta + 9k^2)}{108k^2} A_1, \quad \delta = \delta, \quad \nu = 0, \quad (37)$$

$$A_0 = 0, \quad A_1 = A_1, \quad B_1 = B_1,$$

$$\alpha = -\frac{1}{3} c \epsilon \lambda_3 A_1, \quad \delta = \frac{B_1^2}{A_1^2 (C_1^2 + C_2^2)}, \quad \nu = 0,$$

$$\beta = -\frac{c^2 \epsilon \lambda_3 (2c^2 \epsilon^2 \lambda_3^2 B_1^2 + 9k^2 C_1^2 + 9k^2 C_2^2)}{54k^2 (C_1^2 + C_2^2)} A_1. \quad (38)$$

Substituting (35)–(38) into (34), making use of solutions (15) of (14), we can obtain, respectively, exact traveling wave solutions expressed by trigonometric functions of (2).

Consider

$$w_{11}(x, t) = \pm \frac{\delta B_1 (-C_1 \sin(\sqrt{\delta}\xi) + C_2 \cos(\sqrt{\delta}\xi))}{\sqrt{\Delta} (C_1 \cos(\sqrt{\delta}\xi) + C_2 \sin(\sqrt{\delta}\xi) + (\nu/\delta))} + \frac{B_1}{C_1 \cos(\sqrt{\delta}\xi) + C_2 \sin(\sqrt{\delta}\xi) + (\nu/\delta)}, \quad (39)$$

where $\xi = \alpha x + \beta t;$ α, β are determined in (35); $B_1, C_1, C_2, \delta,$ and ν are arbitrary constants.

Consider

$$w_{12}(x, t) = \frac{A_1 \sqrt{\delta} (-C_1 \sin(\sqrt{\delta}\xi) + C_2 \cos(\sqrt{\delta}\xi))}{C_1 \cos(\sqrt{\delta}\xi) + C_2 \sin(\sqrt{\delta}\xi) + (\nu/\delta)}, \quad (40)$$

where $\xi = \alpha x + \beta t;$ α, β, δ are determined in (36); $A_1, C_1,$ and $C_2,$ and ν are arbitrary constants.

Consider

$$w_{13}(x, t) = \frac{A_1 \sqrt{\delta} (-C_1 \sin(\sqrt{\delta}\xi) + C_2 \cos(\sqrt{\delta}\xi))}{C_1 \cos(\sqrt{\delta}\xi) + C_2 \sin(\sqrt{\delta}\xi)}, \quad (41)$$

where $\xi = \alpha x + \beta t;$ α, β are determined in (37); $A_1, C_1, C_2,$ and δ are arbitrary constants.

Consider

$$w_{14}(x, t) = \frac{A_1 \sqrt{\delta} (-C_1 \sin(\sqrt{\delta}\xi) + C_2 \cos(\sqrt{\delta}\xi))}{C_1 \cos(\sqrt{\delta}\xi) + C_2 \sin(\sqrt{\delta}\xi)} + \frac{B_1}{C_1 \cos(\sqrt{\delta}\xi) + C_2 \sin(\sqrt{\delta}\xi)}, \quad (42)$$

where $\xi = \alpha x + \beta t;$ α, β, δ are determined in (38); $A_1, B_1,$ and $C_1,$ and C_2 are arbitrary constants.

If $\delta < 0,$ one has

$$A_0 = 0, \quad A_1 = \pm \sqrt{\frac{-\delta}{\Delta}} B_1, \quad B_1 = B_1,$$

$$\alpha = \pm \frac{1}{3} c \epsilon \lambda_3 \sqrt{\frac{-\delta}{\Delta}} B_1, \quad \delta = \delta, \quad \nu = \nu,$$

$$\beta = \pm \left(\left((-9\delta^2 k^2 C_2^2 - 9k^2 \nu^2 + 9\delta^2 k^2 C_1^2 + 2B_1^2 \delta^2 c^2 \epsilon^2 \lambda_3^2) c^2 \epsilon \lambda_3 \sqrt{\frac{-\delta}{\Delta}} (54\Delta k^2)^{-1} \right) B_1, \right) \quad (43)$$

where $\Delta = -\delta^2 C_1^2 + \delta^2 C_2^2 + \nu^2 > 0.$

Consider

$$A_0 = 0, \quad A_1 = A_1, \quad B_1 = 0, \\ \alpha = -\frac{1}{3}c\epsilon\lambda_3 A_1, \quad \delta = \pm \frac{\nu}{\sqrt{C_2^2 - C_1^2}}, \quad \nu = \nu, \tag{44}$$

$$\beta = -\frac{c^2\epsilon\lambda_3}{54k^2} \left(\pm \frac{2c^2\epsilon^2\lambda_3^2 A_1^2 \nu}{\sqrt{C_2^2 - C_1^2}} + 9k^2 \right) A_1,$$

$$A_0 = 0, \quad A_1 = A_1, \quad B_1 = 0, \quad \alpha = -\frac{1}{6}c\epsilon\lambda_3 A_1,$$

$$\beta = -\frac{c^2\epsilon\lambda_3 (2c^2\epsilon^2\lambda_3^2 A_1^2 \delta + 9k^2)}{108k^2} A_1, \\ \delta = \delta, \quad \nu = 0, \tag{45}$$

$$A_0 = 0, \quad A_1 = A_1, \quad B_1 = B_1,$$

$$\alpha = -\frac{1}{3}c\epsilon\lambda_3 A_1, \quad \delta = -\frac{B_1^2}{A_1^2 (C_2^2 - C_1^2)}, \quad \nu = 0, \tag{46}$$

$$\beta = \frac{c^2\epsilon\lambda_3 (2c^2\epsilon^2\lambda_3^2 B_1^2 + 9k^2 C_1^2 - 9k^2 C_2^2)}{54k^2 (C_2^2 - C_1^2)} A_1.$$

Substituting (43)–(46) into (34), making use of solutions (15) of (14), we can obtain, respectively, exact traveling wave solutions expressed by hyperbolic functions of (2).

Consider

$$w_{15}(x, t) \\ = \pm \frac{\delta B_1 (C_1 \sinh(\sqrt{-\delta}\xi) + C_2 \cosh(\sqrt{-\delta}\xi))}{\sqrt{\Delta} (C_1 \cosh(\sqrt{-\delta}\xi) + C_2 \sinh(\sqrt{-\delta}\xi) + (\nu/\delta))} \\ + \frac{B_1}{C_1 \cosh(\sqrt{-\delta}\xi) + C_2 \sinh(\sqrt{-\delta}\xi) + (\nu/\delta)}, \tag{47}$$

where $\xi = \alpha x + \beta t$; α, β are determined in (43); B_1, C_1, C_2, δ , and ν are arbitrary constants.

Consider

$$w_{16}(x, t) \\ = \frac{A_1 \sqrt{-\delta} (C_1 \sinh(\sqrt{-\delta}\xi) + C_2 \cosh(\sqrt{-\delta}\xi))}{C_1 \cosh(\sqrt{-\delta}\xi) + C_2 \sinh(\sqrt{-\delta}\xi) + (\nu/\delta)}, \tag{48}$$

where $\xi = \alpha x + \beta t$; α, β, δ are determined in (44); A_1, C_1 , and C_2 , and ν are arbitrary constants.

Consider

$$w_{17}(x, t) \\ = \frac{A_1 \sqrt{-\delta} (C_1 \sinh(\sqrt{-\delta}\xi) + C_2 \cosh(\sqrt{-\delta}\xi))}{C_1 \cosh(\sqrt{-\delta}\xi) + C_2 \sinh(\sqrt{-\delta}\xi)}, \tag{49}$$

where $\xi = \alpha x + \beta t$; α, β are determined in (45); A_1, C_1, C_2 , and δ are arbitrary constants.

Consider

$$w_{18}(x, t) \\ = \frac{A_1 \sqrt{-\delta} (C_1 \sinh(\sqrt{-\delta}\xi) + C_2 \cosh(\sqrt{-\delta}\xi))}{C_1 \cosh(\sqrt{-\delta}\xi) + C_2 \sinh(\sqrt{-\delta}\xi)} \\ + \frac{B_1}{C_1 \cosh(\sqrt{-\delta}\xi) + C_2 \sinh(\sqrt{-\delta}\xi)}, \tag{50}$$

where $\xi = \alpha x + \beta t$; α, β, δ are determined in (46); A_1, B_1 , and C_1 , and C_2 are arbitrary constants.

If $\delta = 0$, one has

$$A_0 = 0, \quad A_1 = A_1, \quad B_1 = B_1, \quad \alpha = -\frac{1}{3}c\epsilon\lambda_3 A_1, \\ \beta = -\frac{1}{6}c^2\epsilon\lambda_3 A_1, \quad \nu = \frac{A_1^2 C_1^2 - B_1^2}{2A_1^2 C_2}, \tag{51}$$

$$A_0 = 0, \quad A_1 = A_1, \quad B_1 = 0, \quad \alpha = -\frac{1}{6}c\epsilon\lambda_3 A_1, \\ \beta = -\frac{1}{12}c^2\epsilon\lambda_3 A_1, \quad \nu = 0. \tag{52}$$

Substituting (51) and (52) into (34), making use of solutions (15) of (14), we can obtain, respectively, exact traveling wave solutions expressed by rational functions of (2).

Consider

$$w_{19}(x, t) = \frac{2A_1 (\nu\xi + C_1)}{\nu\xi^2 + 2(C_1\xi + C_2)} + \frac{2B_1}{\nu\xi^2 + 2(C_1\xi + C_2)}, \tag{53}$$

where $\xi = \alpha x + \beta t$; α, β , and ν are determined in (51); B_1, C_1 , and C_2 are arbitrary constants.

Consider

$$w_{20}(x, t) = \frac{A_1 C_1}{C_1 \xi + C_2}, \tag{54}$$

where $\xi = \alpha x + \beta t$; α and β are determined in (52); A_1, C_1 , and C_2 are arbitrary constants.

4. Conclusions

In [28] Rees derived a new nonlinear KdV-like evolution equation (2) for surface solitary waves propagating on a liquid-air interface where the wave motion is induced by harmonic forcing. The study on the exact solutions for (2) is meaningful and important. However, exact traveling wave solutions of (2) are not reported in related references. In this work, we apply the simplest equation method, the modified simplest equation method, and the extended simplest equation method for obtaining exact traveling solutions of (2). And some new exact solutions expressed by hyperbolic

function, trigonometric function, and rational functions are obtained. The correctness of all the solutions is verified by substituting them into original equation (2). It is easy to see that the simplest equation methods are direct, concise, effective, and reliable, which can be used for many other NLEEs in mathematical physics.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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