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Research Article

The First Passage Time Problem for Mixed-Exponential Jump Processes with Applications in Insurance and Finance

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This paper studies the first passage times to constant boundaries for mixed-exponential jump diffusion processes. Explicit solutions of the Laplace transforms of the distribution of the first passage times, the joint distribution of the first passage times and undershoot (overshoot) are obtained. As applications, we present explicit expression of the Gerber-Shiu functions for surplus processes with two-sided jumps, present the analytical solutions for popular path-dependent options such as lookback and barrier options in terms of Laplace transforms, and give a closed-form expression on the price of the zero-coupon bond under a structural credit risk model with jumps.

1. Introduction

One-sided and two-sided exit problems for the compound Poisson processes and jump diffusion processes with two-sided jumps have been applied widely in a variety of fields. For example, in the theory of actuarial mathematics, the problem of first exit from a half-line is of fundamental interest with regard to the classical ruin problem and the expected discounted penalty function or the Gerber-Shiu function as well as the expected total discounted dividends up to ruin. See, for example, Klüppelberg et al. [1], Mordecki [2], Xing et al. [3], Cai et al. [4], Zhang et al. [5], Chi [6], and Chi and Lin [7]. In the setting of mathematical finance, the first passage time plays a crucial role for the pricing of many path-dependent options and American-type and Russian-type options; see, for example, Kou [8], Kou and Wang [9, 10], Asmussen et al. [11], Levendorskiĭ [12], Alili and Kyprianou [13], Cai et al. [14], and Cai and Kou [15], as well as certain credit risk models; see, for example, Hilberink and Rogers [16], Le Courtois and Quittard-Pinon [17], and Dong et al. [18]. Many optimal stopping strategies also turn out to boil down to the first passage problem for jump diffusion processes; see, for example, Mordecki [19]. In queueing theory one-sided and two-sided first-exit problems for the compound Poisson processes and jump diffusion processes with two-sided jumps have been playing a central role in a single-server queueing system with random workload removal; see, for example, Perry et al. [20]. Usually, when we study the first passage problem, the models with two-sided jumps are more difficult to handle than those with one-sided jumps, because the undershoot and overshoot problem could not be avoided. Despite the maturity of this field of study, it is surprising to note that, until very recently, it can only be solved for certain kinds of jump distributions, such as the Kou's double exponential jump diffusion model (see Kou [8] and Kou and Wang [9]). Recently, Cai and Kou [15] proposed a mixed-exponential jump diffusion process to model the asset return and found an expression for the joint distribution of the first passage time and the overshoot for a mixed-exponential jump diffusion process. In the most recent paper of Wen and Yin [21], two-sided first-exit problem for a jump process having jumps with rational Laplace transform was studied. However, determination of the coefficients in expressions of the above two papers still remains a mathematical and computational challenge. In this paper, we will further study the first passage problems in Cai and Kou [15] and give an explicit expression for the joint distribution of the first passage time and the overshoot for a mixed-exponential jump process with or without a diffusion. Moreover, we present several applications in insurance risk theory and in finance.

The rest of the paper is organized as follows. In Section 2, the model assumptions are formulated. In Section 3, we study the one-sided passage problem from below or above for compound Poisson process and jump diffusion process. In Section 4, we give explicit expression of the Gerber-Shiu function with two-sided jumps. In Section 5, we present the analytical solutions to the pricing problem of one barrier options and lookback options, and in the last section we derive a closed-form expression for the price of the zero-coupon bond.

2. Mathematical Model

A jump diffusion process $X = \{X(t) : t \geq 0\}$ is defined as

$$X(t) = x + \mu t + \sigma W_t + \sum_{i=1}^{N_t} Y_i, \tag{1}$$

where x is the starting point of X, $\{W_t; t \geq 0\}$ is a standard Brownian motion with $W_0 = 0$, $\{N_t; t \geq 0\}$ is a Poisson process with rate λ, constants $\mu \in \mathbb{R}$, $\sigma \geq 0$ represent the drift and the volatility of the diffusion part, respectively, and the jump sizes $\{Y_i; i \geq 1\}$ are independent and identically distributed random variables. We assume that $\{Y_i; i \geq 1\}$ are identically distributed as the canonical random variable Y with probability density function $f_Y(y)$. Moreover, it is assumed that $\{W_t\}$, $\{N_t\}$, and $\{Y_i\}$ are independent. When $\sigma = 0$, the process (1) is the so-called compound Poisson process with positive and negative jumps and linear deterministic decrease or increase between jumps according to $\mu < 0$ or $\mu > 0$. The processes cover many models appearing in the literature such as the compound Poisson risk models, the perturbed compound Poisson risk models, and their dual models. From now on, we will denote by $\{P_x : x \in \mathbb{R}\}$ the probabilities such that, under P_x, $X(0) = x$ with probability one. Moreover, E_x will be the expectation operator associated to P_x. For convenience, we will write $P = P_0$ and $E = E_0$.

It is easy to see that X is a special case of Lévy processes with two-sided jumps, whose infinitesimal generator of X is given by

$$\begin{aligned}\mathscr{L}g(x) = {} & \frac{1}{2}\sigma^2 g''(x) + \mu g'(x) \\ & + \lambda \int_{-\infty}^{\infty} \left(g(x+y) - g(x)\right) f_Y(y)\, dy,\end{aligned} \tag{2}$$

for any twice continuously differentiable function g. The moment generating function of $X(t)$ is $E(e^{zX(t)}) = e^{\psi(z)t}$, $t \geq 0$, $\mathfrak{R}(z) = 0$, where $\psi(z)$, called the exponent of the Lévy process X, is defined as

$$\psi(z) = \frac{1}{2}\sigma^2 z^2 + \mu z + \lambda\left(E\left[e^{zY}\right] - 1\right). \tag{3}$$

For more about the general Lévy processes, we refer to Bertoin [22], Kyprianou [23], and Doney [24].

3. First Passage Problems

We now turn to one-sided passage problems for the Lévy process (1). For two flat barriers h and H $(h < H)$, define the first downward passage time under h and the first upward passage time over H by

$$\begin{aligned}\tau_h^- &:= \inf\{t \geq 0 : X(t) \leq h\}, \\ \tau_H^+ &:= \inf\{t \geq 0 : X(t) \geq H\},\end{aligned} \tag{4}$$

with the convention that $\inf \emptyset = \infty$. In the next two subsections we will investigate the distributions of the following quantities: first upward passage time τ_H^+ and overshoot $X(\tau_H^+) - H$; first downward passage time τ_h^- and undershoot $h - X(\tau_h^-)$.

3.1. One-Sided Exit from above. In this subsection we assume that the downward jumps have an arbitrary distribution with density f_- and Laplace transform $\widehat{f}_-$, while the upward jumps are mixed-exponential; that is,

$$f_Y(y) = p f_-(-y)\,\mathbf{1}_{\{y<0\}} + q \sum_{i=1}^{m} p_i \eta_i e^{-\eta_i y}\,\mathbf{1}_{\{y \geq 0\}}, \tag{5}$$

where constants $p, q \geq 0$, $p + q = 1$, $0 < \eta_1 < \eta_2 < \cdots < \eta_m < \infty$, and $\sum_{i=1}^{m} p_i = 1$.

The Lévy exponent of X is given by

$$\psi_1(z) = \frac{1}{2}\sigma^2 z^2 + \mu z + \lambda\left(q \sum_{i=1}^{m} \frac{p_i \eta_i}{\eta_i - z} + p\widehat{f}_-(-z) - 1\right). \tag{6}$$

Using the same argument as in Cai and Kou [15] we have the following.

Lemma 1. *(i) For sufficiently large $\alpha > 0$, if $\sigma > 0$ or $\mu > 0$ and $\sigma = 0$, then the equation $\psi_1(z) = \alpha$ has exactly $m + 1$ distinct positive roots $\beta_1, \ldots, \beta_{m+1}$ satisfying*

$$0 < \beta_1 < \beta_2 < \cdots < \beta_{m+1} < \infty. \tag{7}$$

(ii) If $\mu \leq 0$ and $\sigma = 0$, then the equation $\psi_1(z) = \alpha$ has exactly m distinct positive roots $\beta_1, \ldots, \beta_m$ satisfying

$$0 < \beta_1 < \beta_2 < \cdots < \beta_m < \infty. \tag{8}$$

Cai and Kou [15] found the joint distribution of the first passage time τ_H^+ and $X(\tau_H^+)$ in case $\sigma > 0$ under the additional assumption $f_-(y)$ is also mixed-exponential. However, for a general $f_-(y)$ in case the upward jumps are mixed-exponential (cf. Yin et al. [25]), for any sufficiently large $\alpha > 0$, $\theta < \eta_1$, and $x < H$, we have

$$E_x\left(e^{-\alpha\tau_H^+ + \theta X(\tau_H^+)}\right) = \sum_{k=1}^{m+1} w_k e^{\beta_k x}, \tag{9}$$

where $w := (w_1, \dots, w_{m+1})'$ is a vector uniquely determined by the following system $ABw = J$, where A is an $(m+1) \times (m+1)$ matrix

$$A = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \dfrac{\eta_1}{\eta_1 - \beta_1} & \dfrac{\eta_1}{\eta_1 - \beta_2} & \cdots & \dfrac{\eta_1}{\eta_1 - \beta_{m+1}} \\ \vdots & \vdots & \vdots & \vdots \\ \dfrac{\eta_m}{\eta_m - \beta_1} & \dfrac{\eta_m}{\eta_m - \beta_2} & \cdots & \dfrac{\eta_m}{\eta_m - \beta_{m+1}} \end{bmatrix}, \quad (10)$$

B is an $(m+1) \times (m+1)$ diagonal matrix, and J is an $(m+1)$-dimensional vector

$$B = \mathrm{Diag}\left\{e^{\beta_1 H}, \dots, e^{\beta_{m+1} H}\right\},$$
$$J = e^{\theta H}\left(1, \frac{\eta_1}{\eta_1 - \theta}, \dots, \frac{\eta_m}{\eta_m - \theta}\right)'. \quad (11)$$

In this paper we will determine the coefficients w_l's explicitly. Moreover, we also consider the cases $\mu > 0, \sigma = 0$ and $\mu \le 0, \sigma = 0$.

Theorem 2. *For any sufficiently large $\alpha > 0$, one has,*

(i) *for $\theta < \eta_1$ and $x < H$,*

$$E_x\left(e^{-\alpha\tau_H^+ + \theta X(\tau_H^+)} \mathbf{1}_{\{\tau_H^+ < \infty\}}\right) = e^{\theta H} \sum_{k=1}^{N} B_k \frac{\prod_{i=1, i\neq k}^{N} (1 - \theta/\beta_i)}{\prod_{i=1}^{m} (1 - \theta/\eta_i)} e^{-\beta_k (H - x)}, \quad (12)$$

(ii) *for $y \ge 0$, $x < H$,*

$$E_x\left(e^{-\alpha\tau_H^+} \mathbf{1}_{\{X(\tau_H^+) - H \in dy\}}\right) = \sum_{k=1}^{N} B_k \left(A_{k0}\delta_0(y) + \sum_{l=1}^{m} A_{kl}\eta_l e^{-\eta_l y}\right) e^{-\beta_k (H-x)} dy, \quad (13)$$

(iii) *for $x < H$,*

$$E_x\left(e^{-\alpha\tau_H^+} \mathbf{1}_{\{X(\tau_H^+) = H\}}\right) = \sum_{k=1}^{N} B_k A_{k0} e^{-\beta_k (H-x)}, \quad (14)$$

(iv) *for $x < H$, $y \ge 0$,*

$$E_x\left(e^{-\alpha\tau_H^+} \mathbf{1}_{\{X(\tau_H^+) - H > y\}}\right) = \sum_{k=1}^{N} B_k \left(\sum_{l=1}^{m} A_{kl} e^{-\eta_l y}\right) e^{-\beta_k (H-x)}, \quad (15)$$

(v) *for $x < H$,*

$$E_x\left(e^{-\alpha\tau_H^+}\right) = \sum_{k=1}^{N} B_k e^{-\beta_k (H-x)}, \quad (16)$$

where $\beta_1, \dots, \beta_N$ are the positive roots of the equation $\psi_1(\beta) = \alpha$, $\delta_0(x)$ is the Dirac delta at $x = 0$, and

$$N = \begin{cases} m+1, & \text{if } \sigma > 0, \text{ or } \sigma = 0, \ \mu > 0, \\ m, & \text{if } \sigma = 0, \ \mu \le 0, \end{cases}$$
$$B_j = \frac{\prod_{k=1}^{m} \left(1 - \beta_j/\eta_k\right)}{\prod_{k=1, k\neq j}^{N} \left(1 - \beta_j/\beta_k\right)}, \quad j = 1, \dots, N,$$
$$A_{k0} = \begin{cases} \dfrac{\prod_{i=1}^{m} \eta_i}{\prod_{i=1, i\neq k}^{N} \beta_i}, & \text{if } \sigma > 0, \text{ or } \sigma = 0, \ \mu > 0, \\ 0, & \text{if } \sigma = 0, \ \mu \le 0, \end{cases}$$
$$A_{kl} = \frac{\prod_{i=1, i\neq k}^{N} (1 - \eta_l/\beta_i)}{\prod_{i=1, i\neq l}^{m} (1 - \eta_l/\eta_i)}, \quad l = 1, 2, \dots, m. \quad (17)$$

Proof. We prove the result for the case $\sigma > 0$ only; the rest of the cases can be proved similarly. To prove Theorem 2, the most difficult part is to find the inverse of matrix A. For simplicity, we write

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad (18)$$

where

$$A_{11} = (1), \qquad A_{12} = (1, \dots, 1)_{1\times m},$$
$$A_{21} = \left(\frac{\eta_1}{\eta_1 - \beta_1}, \dots, \frac{\eta_m}{\eta_m - \beta_1}\right)',$$
$$A_{22} = \begin{bmatrix} \dfrac{\eta_1}{\eta_1 - \beta_2} & \cdots & \dfrac{\eta_1}{\eta_1 - \beta_{m+1}} \\ \vdots & \vdots & \vdots \\ \dfrac{\eta_m}{\eta_m - \beta_2} & \cdots & \dfrac{\eta_m}{\eta_m - \beta_{m+1}} \end{bmatrix}. \quad (19)$$

Note that A_{22} can be written as $A_{22} = J_1 C_1$, where $J_1 = \mathrm{Diag}\{\eta_1, \dots, \eta_m\}$ is a diagonal matrix, $C_1 = \{1/(\eta_i - \beta_{j+1})\}_{1\le i,j\le m}$ is a Cauchy matrix of order m which is invertible, and the inverse is given by $C_1^{-1} = \{d_{ij}\}_{m\times m}$, where

$$d_{ij} = \left(\eta_j - \beta_{i+1}\right) \frac{A_1\left(\beta_{i+1}\right)}{A_1'\left(\eta_j\right)\left(\beta_{i+1} - \eta_j\right)} \frac{B_1\left(\eta_j\right)}{B_1'\left(\beta_{i+1}\right)\left(\eta_j - \beta_{i+1}\right)}. \quad (20)$$

Here,

$$A_1(x) = \prod_{i=1}^{m} \left(x - \eta_i\right), \qquad B_1(x) = \prod_{i=1}^{m} \left(x - \beta_{i+1}\right). \quad (21)$$

Then the inverse of A_{22} is given by

$$A_{22}^{-1} = \begin{bmatrix} \frac{1}{\eta_1} d_{11} & \cdots & \frac{1}{\eta_m} d_{1m} \\ \frac{1}{\eta_1} d_{21} & \cdots & \frac{1}{\eta_m} d_{2m} \\ \vdots & \vdots & \vdots \\ \frac{1}{\eta_1} d_{m1} & \cdots & \frac{1}{\eta_m} d_{mm}. \end{bmatrix}. \quad (22)$$

The determinant of C_1 is given by (see Calvetti and Reichel [26])

$$\det(C_1) = \frac{\prod_{1\le i<j\le m} (\eta_i - \eta_j)(\beta_{j+1} - \beta_{i+1})}{\prod_{i,j=1}^{m} (\eta_i - \beta_{j+1})}. \quad (23)$$

After some algebra,

$$\frac{A}{A_{22}} = \left(\frac{\prod_{i=1}^{m} (\beta_{i+1} - \beta_1)}{\prod_{i=1}^{m} (\eta_i - \beta_1)} \right)_{1\times 1}, \quad (24)$$

where

$$\frac{A}{A_{22}} := A_{11} - A_{12} A_{22}^{-1} A_{21} \quad (25)$$

is the Schur complement of the block A_{22} in A, which is a matrix of order 1. By Schur's formula (see Zhang [27]),

$$\det(A) = \det(A_{22}) \cdot \det\left(\frac{A}{A_{22}}\right) \neq 0. \quad (26)$$

Moreover, by Banachiewicz inversion formula (see Zhang [27]), the inverse of A is given by

$$A^{-1} = \left(\frac{A}{A_{22}}\right)^{-1} \begin{bmatrix} 1 & -A_{12}A_{22}^{-1} \\ -A_{22}^{-1}A_{21} & A_{22}^{-1}A_{21}A_{12}A_{22}^{-1} + A_{22}^{-1}\left(\frac{A}{A_{22}}\right) \end{bmatrix}. \quad (27)$$

After some algebra, we have

$$\begin{aligned} A_{12}A_{22}^{-1} &= \left(\frac{B_1(\eta_1)}{\eta_1 A_1'(\eta_1)}, \ldots, \frac{B_1(\eta_m)}{\eta_m A_1'(\eta_m)} \right), \\ A_{22}^{-1}A_{21} &= \left(\sum_{j=1}^{m} \frac{d_{1j}}{\eta_j - \beta_1}, \ldots, \sum_{j=1}^{m} \frac{d_{mj}}{\eta_j - \beta_1} \right)', \\ A_{22}^{-1}A_{21}A_{12}A_{22}^{-1} &+ A_{22}^{-1}\left(\frac{A}{A_{22}}\right) \\ &= \left(\frac{B_1(\eta_j)}{\eta_j A_1'(\eta_j)} \sum_{l=1}^{m} \frac{d_{il}}{\eta_l - \beta_1} + \frac{\prod_{k=1}^{m}(\beta_{k+1} - \beta_1)}{\eta_j \prod_{u=1}^{m}(\eta_u - \beta_1)} d_{ij} \right)_{1\le i,\, j\le m}. \end{aligned} \quad (28)$$

Now by solving $ABw = J$ we find that

$$\begin{aligned} w &= B^{-1}A^{-1}J \\ &= e^{\theta H} \left(B_1 \frac{\prod_{i=1, i\neq 1}^{m+1} (1 - \theta/\beta_i)}{\prod_{i=1}^{m} (1 - \theta/\eta_i)} e^{-\beta_1 H}, \ldots, \right. \\ &\quad \left. B_{m+1} \frac{\prod_{i=1, i\neq m+1}^{m+1} (1 - \theta/\beta_i)}{\prod_{i=1}^{m} (1 - \theta/\eta_i)} e^{-\beta_{m+1} H} \right)', \end{aligned} \quad (29)$$

from which and from (9) we get (12).

By the fractional expansion,

$$\begin{aligned} &\frac{\prod_{i=1, i\neq k}^{m+1} (1 - \theta/\beta_i)}{\prod_{i=1}^{m} (1 - \theta/\eta_i)} \\ &\quad = A_{k0} + A_{k1} \frac{\eta_1}{\eta_1 - \theta} + \cdots + A_{km} \frac{\eta_m}{\eta_m - \theta}, \end{aligned} \quad (30)$$

where the coefficients A_{kl}'s are defined in the theorem. Substituting (30) into (12) and inverting it on θ immediately lead to (13). Equations (14)–(16) are direct consequence of (13). This ends the proof of Theorem 2. □

Example 3. Let $m = 1$; several expressions are obtained by Theorem 2. When $\sigma > 0$ or $\sigma = 0$ and $\mu > 0$, for $x < H$, $\theta < \eta_1$, and $y \geq 0$, we recover the following three formulae which are obtained by Kou and Wang [10]:

$$\begin{aligned} &E_x\left(e^{-\alpha\tau_H^+ + \theta X(\tau_H^+)}\right) \\ &\quad = e^{\theta H} \left(\frac{(\beta_2 - \theta)(\eta_1 - \beta_1)}{(\eta_1 - \theta)(\beta_2 - \beta_1)} e^{-\beta_1 (H-x)} \right. \\ &\qquad \left. + \frac{(\beta_1 - \theta)(\beta_2 - \eta_1)}{(\eta_1 - \theta)(\beta_2 - \beta_1)} e^{-\beta_2 (H-x)} \right), \\ &E_x\left(e^{-\delta\tau_H^+} \mathbf{1}_{\{X(\tau_H^+) - H > y\}}\right) \\ &\quad = e^{-\eta_1 y} \frac{(\beta_2 - \eta_1)(\eta_1 - \beta_1)}{\eta_1 (\beta_2 - \beta_1)} \left(e^{-\beta_1 (H-x)} - e^{-\beta_2 (H-x)}\right), \\ &E_x\left(e^{-\delta\tau_H^+}\right) = \frac{\beta_2(\eta_1 - \beta_1)}{\eta_1(\beta_2 - \beta_1)} e^{-\beta_1 (H-x)} + \frac{\beta_1(\beta_2 - \eta_1)}{\eta_1(\beta_2 - \beta_1)} e^{-\beta_2 (H-x)}. \end{aligned} \quad (31)$$

When $\sigma = 0$ and $\mu \leq 0$, then for $x < H$, $\theta < \eta_1$, and $y \geq 0$,

$$\begin{aligned} E_x\left(e^{-\delta\tau_H^+ + \theta X(\tau_H^+)}\right) &= e^{\theta H} \frac{\eta_1 - \beta_1}{\eta_1 - \theta} e^{-\beta_1 (H-x)}, \\ E_x\left(e^{-\delta\tau_H^+} \mathbf{1}_{\{X(\tau_H^+) - H > y\}}\right) &= e^{-\eta_1 y} \frac{\eta_1 - \beta_1}{\eta_1} e^{-\beta_1 (H-x)}. \end{aligned} \quad (32)$$

3.2. One-Sided Exit from below. In this subsection we assume that the upward jumps have an arbitrary distribution with

Laplace transform $\widehat{f}_+$, while the downward jumps are mixed-exponential; that is,

$$f_Y(y) = pf_+(y) + q\sum_{j=1}^{m} p_j\eta_j e^{\eta_j y}\mathbf{1}_{\{y<0\}}, \tag{33}$$

where constants $p, q \geq 0$, $p + q = 1$, $0 < \eta_1 < \eta_2 < \cdots < \eta_m < \infty$, and $\sum_{j=1}^{m} p_j = 1$. By (3), the Lévy exponent of X is given by

$$\psi_2(z) = \frac{1}{2}\sigma^2 z^2 + \mu z + \lambda\left(p\widehat{f}_+(-z) + q\sum_{j=1}^{m}\frac{p_j\eta_j}{\eta_j + z} - 1\right). \tag{34}$$

By replacing X by $-X$ in the previous section, we get the main finding in this section.

Theorem 4. *For any sufficiently large $\alpha > 0$, one has,*

(i) *for $\theta > 0$, $x > h$,*

$$E_x\left(e^{-\alpha\tau_h^- + \theta X(\tau_h^-)}\mathbf{1}_{\{\tau_h^- < \infty\}}\right) = e^{-\theta h}\sum_{k=1}^{J} B_k \frac{\prod_{i=1, i\neq k}^{J}(1 + \theta/r_i)}{\prod_{i=1}^{m}(1 + \theta/\eta_i)} e^{-r_k(x-h)}, \tag{35}$$

(ii) *for $x > h$, $y \geq 0$,*

$$E\left(e^{-\alpha\tau_h^-}\mathbf{1}_{\{h - X(\tau_h^-)\in dy\}}\right) = \sum_{k=1}^{J} B_k\left(A_{k0}\delta_0(y) + \sum_{l=1}^{m} A_{kl}\eta_l e^{-\eta_l y}\right)e^{-r_k(x-h)}dy, \tag{36}$$

(iii) *for $x > h$,*

$$E_x\left(e^{-\alpha\tau_h^-}\mathbf{1}_{\{X(\tau_h^-) = h\}}\right) = \sum_{k=1}^{J} B_k A_{k0} e^{-r_k(x-h)}, \tag{37}$$

(iv) *for $x > h$,*

$$\begin{aligned} E_x\left(e^{-\alpha\tau_h^-}\mathbf{1}_{\{X(\tau_h^-) < h\}}\right) &= \sum_{k=1}^{J} B_k\left(\sum_{l=1}^{m} A_{kl}\right)e^{-r_k(x-h)} \\ &= \sum_{k=1}^{J} B_k(1 - A_{k0})e^{-r_k(x-h)}, \end{aligned} \tag{38}$$

(v) *for $x > h$,*

$$E_x\left(e^{-\alpha\tau_h^-}\right) = \sum_{k=1}^{J} B_k e^{-r_k(x-h)}, \tag{39}$$

where $-r_1, \ldots, -r_J$ are the negative roots of the equation $\psi_2(r) = \alpha$ and

$$\begin{aligned} J &= \begin{cases} m+1, & \sigma > 0, \text{ or } \sigma = 0, \ \mu < 0, \\ m, & \sigma = 0, \ \mu \geq 0, \end{cases} \\ B_j &= \frac{\prod_{k=1}^{m}(1 - r_j/\eta_k)}{\prod_{k=1, k\neq j}^{J}(1 - r_j/r_k)}, \quad j = 1, \ldots, J, \\ A_{k0} &= \begin{cases} \dfrac{\prod_{i=1}^{m}\eta_i}{\prod_{i=1, i\neq k}^{J} r_i}, & \sigma > 0, \text{ or } \sigma = 0, \ \mu > 0, \\ 0, & \sigma = 0, \ \mu \leq 0, \end{cases} \\ A_{kl} &= \frac{\prod_{i=1, i\neq k}^{J}(1 - \eta_l/r_i)}{\prod_{i=1, i\neq l}^{m}(1 - \eta_l/\eta_i)}, \quad l = 1, 2, \ldots, m. \end{aligned} \tag{40}$$

Remark 5. The result (39) agrees with the result of Theorem 1.1 in Mordecki [2], where only the case of $\sigma > 0$ and $p_i \geq 0$ $(i = 1, \ldots, m)$ is considered.

Example 6. Let $m = 1$ in Theorem 4. When $\sigma > 0$ or $\sigma = 0$ and $\mu < 0$, for $\theta < \eta_1$ and $y \geq 0$,

$$\begin{aligned} E_x\left(e^{-\alpha\tau_h^- + \theta X(\tau_h^-)}\right) &= e^{\theta h}\left(\frac{(r_2 + \theta)(\eta_1 - r_1)}{(\theta + \eta_1)(r_2 - r_1)}e^{-r_1(x-h)} + \frac{(r_1 + \theta)(r_2 - \eta_1)}{(\theta + \eta_1)(r_2 - r_1)}e^{-r_2(x-h)}\right), \\ E_x\left(e^{-\alpha\tau_h^-}\mathbf{1}_{\{h - X(\tau_h^-) > y\}}\right) &= e^{-\eta_1 l}\frac{(r_2 - \eta_1)(\eta_1 - r_1)}{\eta_1(r_2 - r_1)}\left(e^{-r_1(x-h)} - e^{-r_2(x-h)}\right), \\ E_x\left(e^{-\alpha\tau_h^-}\right) &= \frac{r_2(\eta_1 - r_1)}{\eta_1(r_2 - r_1)}e^{-r_1(x-h)} + \frac{r_1(r_2 - \eta_1)}{\eta_1(r_2 - r_1)}e^{-r_2(x-h)}. \end{aligned} \tag{41}$$

When $\sigma = 0$ and $\mu \geq 0$, then for $\theta < \eta_1$ and $y \geq 0$,

$$\begin{aligned} E_x\left(e^{-\alpha\tau_h^- + \theta X(\tau_h^-)}\right) &= e^{\theta h}\frac{\eta_1 - r_1}{\theta + \eta_1}e^{-r_1(x-h)}, \\ E_x\left(e^{-\alpha\tau_h^-}\mathbf{1}_{\{h - X(\tau_h^-) > y\}}\right) &= e^{-\eta_1 y}\frac{\eta_1 - r_1}{\eta_1}e^{-r_1(x-h)}. \end{aligned} \tag{42}$$

4. Applications to Gerber-Shiu Functions

We consider an insurance risk model in which the insurer's surplus process is defined as

$$U(t) = u + \mu t + \sigma W_t + \sum_{i=1}^{N_t} Y_i \equiv u + X(t) - x, \quad t \geq 0, \tag{43}$$

where $X(t)$ is defined by (1) with jump density (33). The time of (ultimate) ruin is defined as $\tau = \inf\{t \geq 0 : U(t) \leq 0\}$, where $\tau = \infty$ if ruin does not occur in finite time. As applications, we obtain the following special case of the Gerber-Shiu functions for surplus processes with two-sided jumps:

$$\phi(u) = E\left(e^{-\alpha\tau} w(|U(\tau)|) 1(\tau < \infty) \mid U(0) = u\right),$$
$$\phi_d(u) = E\left(e^{-\alpha\tau} w(|U(\tau)|) 1(\tau < \infty, U(\tau) = 0) \mid U(0) = u\right),$$
$$\phi_s(u) = E\left(e^{-\alpha\tau} w(|U(\tau)|) 1(\tau < \infty, U(\tau) < 0) \mid U(0) = u\right), \tag{44}$$

where $\alpha > 0$ is interpreted as the force of interest and w is a nonnegative function defined on $[0, \infty)$. Note that a more general form of Gerber-Shiu function was originally introduced in Gerber and Shiu [28] for the classical risk model.

From Theorem 4(ii) we get the following result.

Corollary 7. *Suppose that $U(t)$ drifts to $+\infty$; then one has*

$$\phi(u) = \int_0^\infty w(y) K_u^{(\alpha)}(y)\, dy, \tag{45}$$

$$\phi_d(u) = w(0) \sum_{k=1}^{J} B_k A_{k0} e^{-r_k u}, \tag{46}$$

$$\phi_s(u) = \sum_{k=1}^{J} B_k \left(\sum_{l=1}^{m} A_{kl} \eta_l \int_0^\infty w(y) e^{-\eta_l y} dy \right) e^{-r_k u}, \tag{47}$$

where B_k's, A_{kl}'s, and r_k's are defined as in Theorem 4 and

$$K_u^{(\alpha)}(y) = \sum_{k=1}^{J} B_k \left(A_{k0} \delta_0(y) + \sum_{l=1}^{m} A_{kl} \eta_l e^{-\eta_l y} \right) e^{-r_k u}. \tag{48}$$

Remark 8. We compare our results with the existing literature. In case $\sigma = 0$ and Y has a double exponential distribution, the result (45) was found by Cai et al. [4]. For $\sigma = 0$ and $\mu = 0$, the result (45) was found by Albrecher et al. [29, (3.2)]. For $\mu = 0$, the result (45) was found by Albrecher et al. [29, (9.3)]. For $\sigma = 0$ and $\mu < 0$, the results (45)–(47) were found by Cheung (see Albrecher et al. [29, PP. 443-444]).

5. Applications to Pricing Path-Dependent Options

As applications of our model in finance, we will study the risk-neutral price of barrier and lookback options. These options have a fixed maturity T and a payoff that depends on the maximum (or minimum) of the asset price on $[0, T]$. The asset price process $\{S(t) : t \geq 0\}$ under a risk-neutral probability measure $\mathbb{P}$ is assumed to be $S(t) = e^{X(t)}$, where $X(t)$ is given by (1), $S(0) = e^{X(0)} := S_0$. We are going to derive pricing formulae for standard single barrier options and lookback options, based on the results obtained in Section 3.

5.1. Lookback Options. The value of a lookback option depends on the maximum or minimum of the stock price over the entire life span of the option. Let the risk-free interest rate be $r > 0$. Given a strike price K and the maturity T, it is well known that (see, e.g., Schoutens [30]) using risk-neutral valuation and after choosing an equivalent martingale measure $\mathbb{P}$ the initial (i.e., $t = 0$) price of a fixed-strike lookback put option is given by

$$L_{\text{fix}}^{P}(K, T) = e^{-rT} \mathbb{E}\left(\sup_{0 \leq t \leq T} S(t) - K \right)^{+}. \tag{49}$$

The initial price of a fixed-strike lookback call option is given by

$$L_{\text{fix}}^{C}(K, T) = e^{-rT} \mathbb{E}\left(K - \inf_{0 \leq t \leq T} S(t) \right)^{+}. \tag{50}$$

The initial price of a floating-strike lookback put option is given by

$$L_{\text{floating}}^{P}(T) = e^{-rT} \mathbb{E}\left(\sup_{0 \leq t \leq T} S(t) - S(T) \right)^{+}. \tag{51}$$

The initial price of a floating-strike lookback call option is given by

$$L_{\text{floating}}^{C}(T) = e^{-rT} \mathbb{E}\left(S(T) - \inf_{0 \leq t \leq T} S(t) \right)^{+}. \tag{52}$$

In the standard Black-Scholes setting, closed-form solutions for lookback options have been derived by Merton [31] and Goldman et al. [32]. For the double mixed-exponential jump diffusion model, Cai and Kou [15] derived the Laplace transforms of the lookback put option price with respect to the maturity T; however, the coefficients do not determinate explicitly.

We will only consider lookback put options because lookback call options can be obtained similarly. For jump diffusion process (1) with jump size density (5), the condition $\eta_1 > 1$ is imposed to ensure that the expectation of $e^{-rt}S(t)$ is well defined.

Theorem 9. *For all sufficiently large $\delta > 0$, one has,*

(i) *for $K \geq S_0$,*

$$\int_0^\infty e^{-\delta T} L_{\mathit{fix}}^{P}(K, T)\, dT = \frac{S_0}{r+\delta} \sum_{i=1}^{N} \frac{\prod_{l=1}^{m} (1 - \beta_{i,r+\delta}/\eta_l)}{\prod_{k=1, k\neq i}^{N} (1 - \beta_{i,r+\delta}/\beta_{k,r+\delta})} \frac{1}{\beta_{i,r+\delta} - 1} \left(\frac{S_0}{K} \right)^{\beta_{i,r+\delta} - 1}; \tag{53}$$

(ii) *then*

$$\int_0^\infty e^{-\delta T} L_{\mathit{floating}}^{P}(T)\, dT = \frac{S_0}{r+\delta} \sum_{i=1}^{N} \frac{\prod_{l=1}^{m} (1 - \beta_{i,r+\delta}/\eta_l)}{\prod_{k=1, k\neq i}^{N} (1 - \beta_{i,r+\delta}/\beta_{k,r+\delta})} \frac{1}{\beta_{i,r+\delta} - 1} + \frac{S_0}{r+\delta} - \frac{S_0}{\delta}, \tag{54}$$

where $\beta_{1,r+\delta},\ldots,\beta_{N,r+\delta}$ *are the* N *positive roots of the equation* $\psi_1(z)=r+\delta$ *and*

$$N=\begin{cases} m+1, & \sigma>0,\ \text{or}\ \sigma=0,\ \mu>0,\\ m, & \sigma=0,\ \mu\le 0.\end{cases} \tag{55}$$

Proof. (i) We prove it along the same line as in Cai and Kou [15]. Set $k=\ln(K/S_0)\ge 0$; then

$$L^P_{\text{fix}}(K,T)=S_0e^{-rT}\int_k^\infty e^y\mathbb{P}\left(\sup_{0\le s\le T}X(s)\ge y\right)dy. \tag{56}$$

It follows that

$$\begin{aligned}\int_0^\infty & e^{-\delta T}L^P_{\text{fix}}(K,T)\,dT\\ &=S_0\int_k^\infty e^y\left[\int_0^\infty e^{-(r+\delta)T}\mathbb{P}\left(\sup_{0\le s\le T}X(s)\ge y\right)dT\right]dy\\ &=\frac{S_0}{r+\delta}\int_k^\infty e^y\mathbb{E}\left(e^{-(r+\delta)\tau_y^+}\right)dy.\end{aligned} \tag{57}$$

The result follows from Theorem 2 and (57).

(ii) Since

$$L^P_{\text{floating}}(T)=S_0e^{-rT}\mathbb{E}\left[\exp\left(\sup_{0\le t\le T}X(t)\right)\right]-S_0, \tag{58}$$

it follows that

$$\begin{aligned}\int_0^\infty & e^{-\delta T}L^P_{\text{floating}}(T)\,dT\\ &=S_0\int_0^\infty e^{-(r+\delta)T}\mathbb{E}\left[\exp\left(\sup_{0\le t\le T}X(t)\right)\right]dT-\frac{S_0}{\delta}\\ &=\frac{S_0}{r+\delta}\mathbb{E}\left[\exp\left(\sup_{0\le t\le e(r+\delta)}X(t)\right)\right]-\frac{S_0}{\delta}\\ &=\frac{S_0}{r+\delta}\left[1+\int_0^\infty e^y\mathbb{P}\left(\sup_{0\le s\le e(r+\delta)}X(s)\ge y\right)dy\right]-\frac{S_0}{\delta}\\ &=\frac{S_0}{r+\delta}\left[1+\int_0^\infty e^y\mathbb{E}\left(e^{-(r+\delta)\tau_y^+}\right)dy\right]-\frac{S_0}{\delta}.\end{aligned} \tag{59}$$

The result follows from Theorem 2 and (59). □

5.2. Barrier Options. The generic term barrier options refers to the class of options whose payoff depends on whether or not the underlying prices hit a prespecified barrier during the options' lifetimes. There are eight types of (one dimensional, single) barrier options: up- (down) and-in (out) call (put) options. For more details, we refer the reader to Schoutens [30]. Kou and Wang [10] obtain closed-form price of up-and-in call barrier option under a double exponential jump diffusion model; Cai and Kou [15] obtain closed-form expressions of the up-and-in call barrier option under a double mixed-exponential jump diffusion model. Here, we only illustrate how to deal with the down-and-out call barrier option because the other seven barrier options can be priced similarly. For jump diffusion process (1) with jump size density (33), given a strike price K and a barrier level U, under the risk-neutral probability measure $\mathbb{P}$, the price of down-and-out call option is defined as

$$\text{DOC}=\exp(-rT)\,\mathbb{E}\left[(S(T)-K)^+\mathbf{1}_{(\inf_{0\le t\le T}S(t)>U)}\mid S_0\right],\quad U<S_0. \tag{60}$$

Let $h=\ln(U/S_0)$ and $k=-\ln K$. Then

$$\begin{aligned}\text{DOC}(k,T) &:= \text{DOC}\\ &=\exp(-rT)\,\mathbb{E}_x\left[\left(S_0e^{X(T)}-e^{-k}\right)^+\mathbf{1}_{(\tau_h^->T)}\right].\end{aligned} \tag{61}$$

Theorem 10. *For any* $0<\phi<\eta_1-1$ *and* $r+\varphi>\psi_1(\phi+1)$, *then*

$$\begin{aligned}\int_0^\infty\int_{-\infty}^\infty & e^{-\phi k-\varphi T}DOC(k,T)\,dk\,dT\\ &=\frac{S_0^{\phi+1}\left(1-e^{-(\phi+1)(x-h)}\sum_{k=1}^J B_{r+\varphi,k}e^{-R_k(x-h)}\right)}{\phi(\phi+1)(\varphi+r-\psi_1(\phi+1))},\end{aligned} \tag{62}$$

where $-R_1,\ldots,-R_J$ *are the negative roots of the equation* $\psi_2(r)=r+\varphi$ *and*

$$\begin{aligned} J&=\begin{cases} m+1, & \sigma>0,\ \text{or}\ \sigma=0,\ \mu<0,\\ m, & \sigma=0,\ \mu\ge 0,\end{cases}\\ B_{r+\varphi,k}&=\frac{\prod_{k=1}^m\left(1-R_j/\eta_k\right)}{\prod_{k=1,k\ne j}^J\left(1-R_j/R_k\right)}\\ &\quad\cdot\frac{\prod_{i=1,i\ne k}^J\left(1+(\phi+1)/R_i\right)}{\prod_{i=1}^m\left(1+(\phi+1)/\eta_i\right)}.\end{aligned} \tag{63}$$

Proof. Using the same argument as that of the proof of Theorem 5.2 in Cai and Kou [15], we get

$$\begin{aligned}\int_0^\infty\int_{-\infty}^\infty & e^{-\phi k-\varphi T}\text{DOC}(k,T)\,dk\,dT\\ &=\int_0^\infty\int_{-\infty}^\infty e^{-\phi k-(r+\varphi)T}\mathbb{E}_x\left[\left(S_0e^{X(T)}-e^{-k}\right)^+\mathbf{1}_{(\tau_h^->T)}\right]dk\,dT\\ &=\frac{S_0^{\phi+1}}{\phi(\phi+1)}\frac{1}{\varphi+r-\psi_1(\phi+1)}\\ &\quad\times\left(1-\mathbb{E}_x\left[e^{-(r+\varphi)\tau_h^-+(\phi+1)X(\tau_h^-)}\right]\right),\end{aligned} \tag{64}$$

and the result follows from Theorem 4(i). □

6. The Price of the Zero-Coupon Bond

In this section, we give a simple application on the price of the zero-coupon bond under a structural credit risk model with jumps. As in Dong et al. [18], we assume that the total market value of a firm under the pricing probability measure P is given by

$$V(t) = V_0 e^{X(t)-x}, \quad t \geq 0, \tag{65}$$

where V_0 is positive constant and $X(t)$ is defined as (1). For $K > 0$, define the default time as

$$\tau = \inf\{t : V(t) \leq K\}. \tag{66}$$

If we set $x = -\ln(K/V_0)$, then

$$\tau = \inf\{t : X(t) \leq 0\}. \tag{67}$$

Given $T > 0$ and a short constant rate of interest $r > 0$, Dong et al. [18] have shown that the Laplace transform of the fair price $B(0,T)$ of a defaultable zero-coupon bound at time 0 with maturity T is given by

$$\widehat{B}(\gamma) = \frac{1 - E\left[e^{-(\gamma+r)\tau}\right]}{\gamma + r} + \frac{\mathrm{R}E\left[e^{-(\gamma+r)\tau} V(\tau)\mathbf{1}(\tau < \infty)\right]}{K\gamma}, \tag{68}$$

where $R \in [0,1]$ is a constant. When the jump size distribution is a double hyperexponential distribution, a closed-form expression is obtained, but the coefficients cannot be determined explicitly (except for $n = 2$). Now applying the result in Section 3.2, we get the following result.

Corollary 11. *If the process $X(t)$ is defined as (1) has jump size density (33), one has*

$$\begin{aligned} \widehat{B}(\gamma) = {} & \frac{1 - \sum_{j=1}^{J} C_j e^{-\rho_j x}}{\gamma + r} \\ & + \frac{R}{\gamma} \sum_{j=1}^{J} C_j \frac{\prod_{i=1, i\neq j}^{J} (1 + 1/\rho_i)}{\prod_{i=1}^{m} (1 + 1/\eta_i)} e^{-\rho_j x}, \end{aligned} \tag{69}$$

where $-\rho_1, \ldots, -\rho_J$ are the negative roots of the equation $\psi_2(\rho) = \gamma + r$ and

$$\begin{aligned} J &= \begin{cases} m+1, & \sigma > 0, \text{ or } \sigma = 0, \ \mu < 0, \\ m, & \sigma = 0, \ \mu \geq 0, \end{cases} \\ C_j &= \frac{\prod_{k=1}^{m} \left(1 - \rho_j/\eta_k\right)}{\prod_{k=1, k\neq j}^{J} \left(1 - \rho_j/r_k\right)}, \quad j = 1, \ldots, J. \end{aligned} \tag{70}$$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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