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Research Article

Robust Nonfragile H_∞ Filtering for Uncertain T-S Fuzzy Systems with Interval Delay: A New Delay Partitioning Approach

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This paper investigates the problem of robust nonfragile fuzzy H_∞ filtering for uncertain Takagi-Sugeno (T-S) fuzzy systems with interval time-varying delays. Attention is focused on the design of a filter such that the filtering error system preserves a prescribed H_∞ performance, where the filter to be designed is assumed to have gain perturbations. By developing a delay decomposition approach, both lower and upper bound information of the delayed plant states can be taken into full consideration; the proposed delay-fractional-dependent stability condition for the filter error systems is obtained based on the direct Lyapunov method allied with an appropriate and variable Lyapunov-Krasovskii functional choice and with tighter upper bound of some integral terms in the derivation process. Then, a new robust nonfragile fuzzy H_∞ filter scheme is proposed, and a sufficient condition for the existence of such a filter is established in terms of linear matrix inequalities (LMIs). Finally, some numerical examples are utilized to demonstrate the effectiveness and reduced conservatism of the proposed approach.

1. Introduction

During the past several years, fuzzy systems of the T-S model [1, 2] have attracted great interests from the stability and control community [3]. It is well known that the problem of H_∞ filtering is both theoretically and practically important in control and signal processing [4, 5]. The main advantage of H_∞ filtering is that no statistical assumption on the noise signals is needed and, thus, it is more general than classical Kalman filtering [6]. Moreover, the H_∞ filter is designed by minimizing signal estimation error for the bounded disturbances and noises of the worst cases, which is more robust than classical Kalman filtering [7, 8]. For the fuzzy H_∞ filtering problem based on T-S fuzzy models, some important results have been obtained; see for example, [9–14], and the references therein.

Among the literatures, An et al. [11] designed some H_∞ filters for uncertain systems with time-varying distributed delays. In [12, 13], some new delay-dependent H_∞ filter design schemes have been proposed for continuous-time T-S fuzzy systems. Huang et al. [14] improved some existing results on H_∞ filter design for T-S fuzzy systems with time delay. And the H_∞ filter has been shown to be much more robust against unmodeled dynamics [10]. Moreover, Li and Gao [15] proposed a new comparison model by employing a new approximation for delayed state, and then lifting method and simple LK functional method are used to analyze the scaled small gain of this comparison model and developed reduced-order H_∞ filtering [16] and finite frequency H_∞ filtering [17] for discrete-time systems and for 2-D systems [18], and then these new method can also be extended to T-S fuzzy systems case.

On the other hand, the nonfragile control and filtering problems have been attractive topics in theory analysis and practical implement. The nonfragile concept is proposed to this new problem: how to design a controller or filter that will be insensitive to some error in gains [19–21]. For the nonfragile filtering problem, some numerically effective design methods have been obtained [21–31]. Yang et al. [21–24] focused on the nonfragile filtering problem for linear systems and fuzzy system, respectively. However, the time delays are not considered [21–24]. Most recently, Chang and Yang [31] proposed the design of nonfragile H_∞ filter for discrete-time T-S fuzzy systems with multiplicative gain variations and investigated fuzzy modeling and control for a class of inverted pendulum system in [32]; however, they are also not considered as the time delay case. However, time delay, as a source of instability and poor performance, often appears in many dynamic systems, for example, chemical process, biological systems, nuclear reactor, rolling mill systems and communication networks [2, 3], and networked control systems. In particular, a special type of time delay, interval time-varying delays, that is, $h_a \le \tau(t) \le h_b$ and h_a, is not restricted to be zero in practical engineering systems as NCS. Xia and Li [30] concerned with the nonfragile H_∞ filter design problem for uncertain discrete-time T-S fuzzy systems with time delay, whereas the delay is constant case. Li et al. [26] investigated the problem of nonfragile robust H_∞ filtering for a class of T-S fuzzy time-delay systems, whereas the delay is limited to $0 \le \tau(t) \le h$ and $\dot{\tau}(t) \le \mu < 1$. Moreover, when $\dot{\tau}(t) \le \mu < 1$, which does not allow the fast time-varying delay, the restriction will limit the application scope. Therefore, the robust nonfragile fuzzy H_∞ filtering for uncertain nonlinear systems via T-S fuzzy models with interval time-varying delays has not only important theoretical interest but also practical value. And, to best of our knowledge, few results on robust nonfragile fuzzy H_∞ filtering for the above fuzzy systems have been reported in the literatures. This motivates the present research.

In this paper, we will investigate the problem of robust nonfragile fuzzy H_∞ filter designs for uncertain T-S fuzzy systems with interval time-varying delays. Our objective is to design a fuzzy H_∞ filter with the gain perturbations such that the filtering error system is asymptotically stable with a prescribed H_∞ performance. Firstly, based on the Lyapunov stability theory and Finsler lemma, a delay-fractional-dependent sufficient condition is derived since a new LK functional is constructed by developing a variable delay-decomposition method and estimating tightly the upper bound of its derivative through some improved inequalities techniques. Then, based on the above conditions, a sufficient condition for the solvability of the aforementioned system is developed in terms of LMIs. Finally, some numerical examples are provided to illustrate the feasibility and advantage of the proposed design method.

2. Problem Formulation

Consider a nonlinear system with interval time-varying delays which could be approximated by a time-delay T-S fuzzy model with r plant rules.

Plant Rule i. IF $\theta_1(t)$ is N_{i1} and ... and $\theta_p(t)$ is N_{ip}, THEN

$$\begin{aligned}\dot{x}(t) &= A_i(t)x(t) + A_{\tau i}(t)x(t-\tau(t)) + B_i(t)w(t),\\ y(t) &= C_i(t)x(t) + C_{\tau i}(t)x(t-\tau(t)) + D_i(t)w(t),\\ z(t) &= L_i(t)x(t) + L_{\tau i}(t)x(t-\tau(t)) + G_i(t)w(t),\\ x(t) &= \phi(t), \quad \forall t \in [-h_b, 0],\end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $y(t) \in \mathbb{R}^m$ is the measurement vector; $w(t) \in \mathbb{R}^q$ is the disturbance signal vector which belongs to $L_2[0,\infty)$; $z(t) \in \mathbb{R}^p$ is the signal vector to be estimated; $\phi(t)$ is the continuous initial vector function defined on $[-h_b, 0]$; $\theta_1(t), \theta_2(t), \dots, \theta_p(t)$ denote the premise variables; and $N_{i1}, N_{i2}, \dots, N_{ip}$ represent the fuzzy sets, $i = 1, 2, \dots, r$, and r is the number of IF-THEN rules. In what follows, we define $\overline{\tau} := h_b - h_a$ for brevity, and we denote the coefficient matrices of system (1) as

$$\begin{aligned}\chi_i(t) &= \begin{bmatrix} A_i(t), A_{\tau i}(t), B_i(t) \\ C_i(t), C_{\tau i}(t), D_i(t) \\ L_i(t), L_{\tau i}(t), G_i(t) \end{bmatrix}\\ &:= \begin{bmatrix} A_i, A_{\tau i}, B_i \\ C_i, C_{\tau i}, D_i \\ L_i, L_{\tau i}, G_i \end{bmatrix} + \begin{bmatrix} \Delta A_i, \Delta A_{\tau i}, \Delta B_i \\ \Delta C_i, \Delta C_{\tau i}, \Delta D_i \\ \Delta L_i, \Delta L_{\tau i}, \Delta G_i \end{bmatrix}\\ &:= \begin{bmatrix} A_i, A_{\tau i}, B_i \\ C_i, C_{\tau i}, D_i \\ L_i, L_{\tau i}, G_i \end{bmatrix} + \begin{bmatrix} D_{1i} \\ D_{2i} \\ D_{3i} \end{bmatrix} F_{1i}(t) \begin{bmatrix} E_{1i} & E_{2i} & E_{3i} \end{bmatrix},\end{aligned} \tag{2}$$

where A_i, $A_{\tau i}$, B_i, C_i, $C_{\tau i}$, D_i, L_i, $L_{\tau i}$, G_i denotes the nominal part of $\chi_i(t)$, and the uncertainty is considered as time varying but norm bounded; that is, ΔA_i, $\Delta A_{\tau i}, \Delta B_i, \Delta C_i, \Delta C_{\tau i}, \Delta D_i, \Delta L_i, \Delta L_{\tau i}, \Delta G_i$ stands for the uncertain part, D_{ki}, E_{ki} ($k = 1, 2, 3$; $i = 1, 2, \dots, r$) are constant real matrices, and $F_{1i}(t)$ are unknown time-varying matrices satisfying $F_{1i}^T(t)F_{1i}(t) \le I$.

The time-varying delay $\tau(t)$ is assumed to be either differentiable case satisfied with $0 < h_a \le \tau(t) \le h_b$, $\dot{\tau}(t) \le h_d$, where h_d are given bounds, or fast-varying case (i.e., $0 < h_a \le \tau(t) \le h_b$, but no constraints on the delay derivatives, h_d is unknown).

Let $h_i(\theta(t)) = \mu_i(\theta(t))/\sum_{i=1}^r \mu_i(\theta(t))$, $\mu_i(\theta(t)) = \prod_{j=1}^p N_{ij}(\theta_j(t))$, in which $N_{ij}(\theta_j(t))$ is the membership function of $\theta_j(t)$ in N_{ij}. It is assumed that $\mu_i(\theta(t)) \ge 0$, and then $\sum_{i=1}^r h_i(\theta(t)) = 1$. By fuzzy blending, the final output of the fuzzy system (1) is inferred as follows:

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^r h_i(\theta(t))\left[A_i(t)x(t) + A_{\tau i}(t)x(t-\tau(t)) + B_i(t)w(t)\right],\\ y(t) &= \sum_{i=1}^r h_i(\theta(t))\left[C_i(t)x(t) + C_{\tau i}(t)x(t-\tau(t)) + D_i(t)w(t)\right],\\ z(t) &= \sum_{i=1}^r h_i(\theta(t))\left[L_i(t)x(t) + L_{\tau i}(t)x(t-\tau(t)) + G_i(t)w(t)\right],\\ x(t) &= \phi(t), \quad \forall t \in [-h_b, 0].\end{aligned} \tag{3}$$

Motivated by the parallel distributed compensation (PDC), in this paper, we consider the following full order nonfragile fuzzy H_∞ filter.

Rule j. IF $\theta_1(t)$ is N_{j1} and ... and $\theta_p(t)$ is N_{jp}, THEN

$$\dot{x}_f(t) = \left(A_{fj} + \Delta A_{fj}\right) x_f(t) + \left(B_{fj} + \Delta B_{fj}\right) y(t), \quad x_f(0) = 0,$$
$$z_f(t) = \left(C_{fj} + \Delta C_{fj}\right) x_f(t) + \left(D_{fj} + \Delta D_{fj}\right) y(t), \quad (j = 1, 2, \ldots, r), \tag{4}$$

where $x_f(t) \in \mathbb{R}^n$ and $z_f(t) \in \mathbb{R}^p$ are the state and output of the filter, respectively. The filter matrices $A_{fj} \in \mathbb{R}^{n\times n}, B_{fj} \in \mathbb{R}^{n\times m}, C_{fj} \in \mathbb{R}^{p\times n}, D_{fj} \in \mathbb{R}^{p\times m}$ are to be determined, and $[\Delta A_{fj} \;\; \Delta B_{fj}] := D_{4j}F_{2j}(t)[E_{4j} \;\; E_{5j}]$, $[\Delta C_{fj} \;\; \Delta D_{fj}] := D_{5j}F_{3j}(t)[E_{6j} \;\; E_{7j}]$ denotes the time-varying parameters of fuzzy H_∞ filter, and $F_{2j}(t), F_{3j}(t)$ are unknown time-varying matrices satisfying $F_{kj}^T(t)F_{kj}(t) \le I, (k = 2, 3)$. For simplicity, we define $A_{fj}(t) = A_{fj} + \Delta A_{fj}$; $B_{fj}(t) = B_{fj} + \Delta B_{fj}$; $C_{fj}(t) = C_{fj} + \Delta C_{fj}$; and $D_{fj}(t) = D_{fj} + \Delta D_{fj}$. The defuzzified output of fuzzy filter system (4) is inferred as follows:

$$\dot{x}_f(t) = \sum_{j=1}^{r} h_j(\theta(t)) \left[A_{fj}(t) x_f(t) + B_{fj}(t) y(t)\right], \quad x_f(0) = 0,$$
$$z_f(t) = \sum_{j=1}^{r} h_j(\theta(t)) \left[C_{fj}(t) x_f(t) + D_{fj}(t) y(t)\right]. \tag{5}$$

Defining the augmented state vector $\tilde{x}(t) := \mathrm{col}\{x(t) \;\; x_f(t)\}$, $e(t) := z(t) - z_f(t)$, and $E = [I \;\; 0]$, we can obtain the following filtering error system:

$$\dot{\tilde{x}}(t) = \widetilde{A}(t)\tilde{x}(t) + \widetilde{A}_\tau(t) E\tilde{x}(t - \tau(t)) + \widetilde{B}(t) w(t),$$
$$e(t) = \widetilde{C}(t)\tilde{x}(t) + \widetilde{C}_\tau(t) E\tilde{x}(t - \tau(t)) + \widetilde{D}(t) w(t), \tag{6}$$
$$\tilde{x}(t) = \mathrm{col}\{\phi(t) \;\; 0\}, \quad \forall t \in [-h_b, 0],$$

where

$$\begin{bmatrix} \widetilde{A}(t) & \widetilde{A}_\tau(t) & \widetilde{B}(t) \\ \widetilde{C}(t) & \widetilde{C}_\tau(t) & \widetilde{D}(t) \end{bmatrix} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(t) h_j(t) \begin{bmatrix} \widehat{A}(t) & \widehat{A}_\tau(t) & \widehat{B}(t) \\ \widehat{C}(t) & \widehat{C}_\tau(t) & \widehat{D}(t) \end{bmatrix},$$

$$\widehat{A}(t) = \begin{bmatrix} A_i + \Delta A_i(t) & 0 \\ B_{fj}(C_i + \Delta C_i(t)) & A_{fj} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \Delta B_{fj}(t)(C_i + \Delta C_i(t)) & \Delta A_{fj}(t) \end{bmatrix},$$

$$\widehat{A}_\tau(t) = \begin{bmatrix} A_{\tau i} + \Delta A_{\tau i}(t) \\ B_{fj}(C_{\tau i} + \Delta C_{\tau i}(t)) \end{bmatrix} + \begin{bmatrix} 0 \\ \Delta B_{fj}(t)(C_{\tau i} + \Delta C_{\tau i}(t)) \end{bmatrix},$$

$$\widehat{B}(t) = \begin{bmatrix} B_i + \Delta B_i(t) \\ B_{fj}(D_i + \Delta D_i(t)) \end{bmatrix} + \begin{bmatrix} 0 \\ \Delta B_{fj}(t)(D_i + \Delta D_i(t)) \end{bmatrix},$$

$$\widehat{C}(t) = \left[(L_i + \Delta L_i(t)) - D_{fj}(C_i + \Delta C_i(t)) \;\; -C_{fj}\right] + \left[-\Delta D_{fj}(t)(C_i + \Delta C_i(t)) \;\; -\Delta C_{fj}(t)\right],$$

$$\widehat{C}_\tau(t) = \left[(L_{\tau i} + \Delta L_{\tau i}(t)) - D_{fj}(C_{\tau i} + \Delta C_{\tau i}(t))\right] + \left[-\Delta D_{fj}(t)(C_{\tau i} + \Delta C_{\tau i}(t))\right],$$

$$\widehat{D}(t) = \left[(G_i + \Delta G_i(t)) - D_{fj}(D_i + \Delta D_i(t))\right] + \left[-\Delta D_{fj}(t)(D_i + \Delta D_i(t))\right]. \tag{7}$$

Then the robust fuzzy H_∞ filter design problem to be addressed in this paper can be expressed as follows.

Robust Nonfragile Fuzzy H_∞ Filtering Problem. Given uncertain fuzzy system (3), design a suitable robust nonfragile fuzzy filter in the form of (5) such that the following two requirements are satisfied simultaneously:

(R1) the filtering error system (6) with $w(t) \equiv 0$ is asymptotically stable;

(R2) the H_∞ performance $\|e\|_2 < \gamma\|w\|_2$ is guaranteed for all nonzero $w(t) \in L_2[0, \infty)$ and a prescribed $\gamma > 0$ under zero initial condition.

The following lemmas will be useful in establishing our main results.

Lemma 1 (integral inequalities, Gu et al. [3] and Zhang et al. [33]). *Let $x(t) \in \mathfrak{R}^n$ be a vector-valued function with first-order continuous-derivative entries. Then, for any matrices $M, N \in \mathfrak{R}^{n\times n}$, $Z \in \mathfrak{R}^{2n\times 2n}$, $X = X^T \in \mathfrak{R}^{n\times n}$, and some given scalars $0 \le \tau_1 < \tau_2$, the following integral inequality holds.*

(1) *When $X > 0$ and τ_1, τ_2 are constant values,*

$$(\tau_2 - \tau_1)\int_{t-\tau_2}^{t-\tau_1} x^T(s) X x(s)\, ds \ge \int_{t-\tau_2}^{t-\tau_1} x^T(s)\, ds X \int_{t-\tau_2}^{t-\tau_1} x(s)\, ds,$$
$$-(\tau_2 - \tau_1)\int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s) X \dot{x}(s)\, ds \le \begin{bmatrix} x(t-\tau_1) \\ x(t-\tau_2) \end{bmatrix}^T \begin{bmatrix} -X & X \\ * & -X \end{bmatrix} \begin{bmatrix} x(t-\tau_1) \\ x(t-\tau_2) \end{bmatrix}. \tag{8}$$

(2) *When* $X > 0$ *and* τ_1, τ_2 *are time-varying,* $h = \tau_2 - \tau_1 := h(t) \geq 0$,

$$-\int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s) X \dot{x}(s)\, ds$$
$$\leq \begin{bmatrix} x(t-\tau_1) \\ x(t-\tau_2) \end{bmatrix}^T \left\{ \begin{bmatrix} M + M^T & -M + N^T \\ * & -N - N^T \end{bmatrix} + h \begin{bmatrix} M \\ N \end{bmatrix} X^{-1} \begin{bmatrix} M^T & N^T \end{bmatrix} \right\} \times \begin{bmatrix} x(t-\tau_1) \\ x(t-\tau_2) \end{bmatrix}. \tag{9}$$

(3) *When* τ_1, τ_2 *are time-varying,* $h = \tau_2 - \tau_1 := h(t) \geq 0$, *and* X *is any symmetric matrix,*

$$-\int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s) X \dot{x}(s)\, ds$$
$$\leq \begin{bmatrix} x(t-\tau_1) \\ x(t-\tau_2) \end{bmatrix}^T \left\{ \begin{bmatrix} M + M^T & -M + N^T \\ * & -N - N^T \end{bmatrix} + hZ \right\} \begin{bmatrix} x(t-\tau_1) \\ x(t-\tau_2) \end{bmatrix} \tag{10}$$

with $\begin{bmatrix} X & Y \\ * & Z \end{bmatrix} \geq 0$ *and* $Y = [M \;\; N]$.

Lemma 2 (Wang et al. [34]). *Let* G, L, E, *and* $F(t)$ *be real matrices of appropriate dimensions with* $F(t)$ *being a matrix function satisfying* $F^T(t)F(t) \leq I$. *Then, for any scalar* $\varepsilon > 0$, *we have* $LF(t)E + E^T F^T(t) L^T \leq \varepsilon^{-1} L L^T + \varepsilon E^T E$. *Furthermore, for any scalar* $\varepsilon > 0$ *such that* $I - \varepsilon E^T E > 0$, *we have* $[G + LF(t)E][G^T + E^T F^T(t) L^T] \leq \varepsilon^{-1} L L^T + G^T (I - \varepsilon E^T E) G$.

3. Main Results

In this section, we provide a delay-fractional-dependent sufficient condition for the solvability of robust nonfragile fuzzy H_∞ filtering problem for the system (1), which is formulated in the previous section.

The following proposition will be useful in establishing our main results.

Proposition 3. *For real matrices* P_1, P_2, A, A_τ, B *and* D_i, X_j, $(j = 1, 2, \ldots, 8;\; i = 1, 2, \ldots, 14)$ *with compatible dimensions, the following inequalities are equivalent, where* U *is an extra slack nonsingular matrix:*

$$\begin{bmatrix} \Xi_1 & \Xi_2 & D_2 & 0 & 0 & 0 & A^T P_1 B + P_2^T B & X_1 \\ * & \Xi_3 & 0 & D_4 & D_5 & D_6 & A_\tau^T P_1 B & X_2 \\ * & * & D_7 & D_8 & 0 & 0 & 0 & X_3 \\ * & * & * & D_9 & D_{10} & 0 & 0 & X_4 \\ * & * & * & * & D_{11} & D_{12} & 0 & X_5 \\ * & * & * & * & * & D_{13} & 0 & X_6 \\ * & * & * & * & * & * & B^T P_1 B + D_{14} & X_7 \\ * & * & * & * & * & * & * & X_8 \end{bmatrix} < 0,$$

$$\begin{bmatrix} P_1 - He\{U\} & P_2 + UA & UA_\tau & 0 & 0 & 0 & 0 & UB & 0 \\ * & D_1 & 0 & D_2 & 0 & 0 & 0 & 0 & X_1 \\ * & * & D_3 & 0 & D_4 & D_5 & D_6 & 0 & X_2 \\ * & * & * & D_7 & D_8 & 0 & 0 & 0 & X_3 \\ * & * & * & * & D_8 & D_9 & 0 & 0 & X_4 \\ * & * & * & * & * & D_{10} & D_{12} & 0 & X_5 \\ * & * & * & * & * & * & D_{13} & 0 & X_6 \\ * & * & * & * & * & * & * & D_{14} & X_7 \\ * & * & * & * & * & * & * & * & X_8 \end{bmatrix} < 0, \tag{11}$$

where $\Xi_1 = A^T P_1 A + He\{P_2^T A\} + D_1, \Xi_2 = A^T P_1 A_\tau + P_2^T A_\tau, \Xi_3 = A_\tau^T P_1 A_\tau + D_3$.

Proof. See the Appendix. □

Then, we divide the delay interval $[0, h_a]$ and $[h_a, h_b]$ into four segments: $[h_{i-1}, h_i], i = 1, 2, 3, 4$, where $h_0 = 0, h_1 = h_a/2, h_2 = h_a, h_3 = h_a + \alpha\overline{\tau}, h_4 = h_b, 0 < \alpha < 1$. For simplicity, we denote $\tau_i = h_i - h_{i-1}, r_i = h_i^2 - h_{i-1}^2$, $(i = 1, 2, 3, 4)$, and $\overline{\tau}_0 = h_a - 0$. For the T-S fuzzy filter error system (6), based on the Lyapunov stability theorem, we will give a sufficient condition for the solvability of the fuzzy filter design problem for the system (1) by using the novel delay decomposition approach.

Theorem 4. *Given scalars* $0 < h_a \leq h_b, 0 < \alpha < 1, h_d$ *and* $\gamma > 0$, *the* H_∞ *filter error system* (6), *for all differentiable delay* $\tau(t) \in [h_a, h_b]$ *with* $\dot{\tau}(t) \leq h_d$, *is asymptotically stable and has a prescribed* H_∞ *performance level* γ *if there exist real symmetric matrices,* $Q_k \geq 0, R_k \geq 0, (k = 1, 2, 3, 4), Q_\tau \geq 0, R_\tau \geq 0$, $P = \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix} > 0$, $S = \begin{bmatrix} S_1 & S_2 \\ * & S_3 \end{bmatrix} > 0$, *the nonsingular matrix* $U = \begin{bmatrix} U_1 & 0 \\ U_2 & U_3 \end{bmatrix}$ *and matrices* $Z_m = \begin{bmatrix} Z_{m1} & Z_{m2} \\ * & Z_{m3} \end{bmatrix}, (m = 1, 2)$, $\mathcal{A}_{fj}, \mathcal{B}_{fj}, \mathcal{C}_{fj}, \mathcal{D}_{fj}, M_l, N_l$ *with appropriate dimensions, and* *positive scalars* $\varepsilon_{lij}, \varepsilon_{5ij}$, $(l = 1,2,3,4; i, j = 1,2,\ldots,r)$, *such that the inequalities in* (12) *hold:*

$$\begin{bmatrix} \Pi_{kk}^{m} + \Theta_{kk} & \Gamma_{kk}^{1} & \Gamma_{kk}^{2} \\ * & -\varepsilon_{1kk} I & 0 \\ * & * & -\varepsilon_{2kk} I \end{bmatrix} < 0, \quad (m = 1,2,3,4;\ k = 1,2,\ldots,r),$$

$$\begin{bmatrix} \Pi_{ij}^{m} + \Theta_{ij} + \Pi_{ji}^{m} + \Theta_{ji} & \Gamma_{ij}^{1} & \Gamma_{ij}^{2} & \Gamma_{ji}^{1} & \Gamma_{ji}^{2} \\ * & -\varepsilon_{1ij} I & 0 & 0 & 0 \\ * & * & -\varepsilon_{2ij} I & 0 & 0 \\ * & * & * & -\varepsilon_{1ji} I & 0 \\ * & * & * & * & -\varepsilon_{2ji} I \end{bmatrix} < 0, \quad (m = 1,2,3,4; 0 < i < j \le r),$$

$$I - \varepsilon_{3ij} \Gamma_{ij}^{3} \left(\Gamma_{ij}^{3}\right)^{T} > 0, \qquad I - \varepsilon_{5ij} \Gamma_{ij}^{5} \left(\Gamma_{ij}^{5}\right)^{T} > 0, \quad (0 < i, j \le r),$$

$$\begin{bmatrix} \tau_3 R_3 + (1 - h_d) R_\tau & [M_1 \ \ N_1] \\ * & Z_1 \end{bmatrix} \ge 0,$$

$$\begin{bmatrix} \tau_4 R_4 + (1 - h_d) R_\tau & [M_4 \ \ N_4] \\ * & Z_2 \end{bmatrix} \ge 0, \tag{12}$$

where

$$\Pi_{ij}^{1} = \begin{bmatrix} P_1^1 - He\{U\} & P_2 + \Pi_1 & \Pi_2 & 0 & 0 & 0 & 0 & \Pi_3 & 0 \\ * & \Lambda_1 & 0 & \Lambda_2 & 0 & 0 & 0 & 0 & \Pi_4 \\ * & * & \Lambda_3 & 0 & \Lambda_4 & \Lambda_5 & 0 & 0 & \Pi_5 \\ * & * & * & \Lambda_6 & \Lambda_7 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Lambda_8 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Lambda_9 & \Lambda_{10} & 0 & 0 \\ * & * & * & * & * & * & \Lambda_{11} & 0 & 0 \\ * & * & * & * & * & * & * & \Lambda_{12} & \Pi_6 \\ * & * & * & * & * & * & * & * & -I \end{bmatrix},$$

$$\Pi_{ij}^{2} = \begin{bmatrix} P_1^2 - He\{U\} & P_2 + \Pi_1 & \Pi_2 & 0 & 0 & 0 & 0 & \Pi_3 & 0 & 0 \\ * & \Lambda_1 & 0 & \Lambda_2 & 0 & 0 & 0 & 0 & \Pi_4 & 0 \\ * & * & \Lambda_3^2 & 0 & \Lambda_4^2 & \Lambda_5 & 0 & 0 & \Pi_5 & \tau_3 M_2 \\ * & * & * & \Lambda_6 & \Lambda_7 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Lambda_8^2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Lambda_9 & \Lambda_{10} & 0 & 0 & \tau_3 N_2 \\ * & * & * & * & * & * & \Lambda_{11} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Lambda_{12} & \Pi_6 & 0 \\ * & * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & * & -R_3 \end{bmatrix},$$

$$\Pi_{ij}^{3} = \begin{bmatrix} P_1^3 - He\{U\} & P_2 + \Pi_1 & \Pi_2 & 0 & 0 & 0 & 0 & \Pi_3 & 0 & 0 \\ * & \Lambda_1 & 0 & \Lambda_2 & 0 & 0 & 0 & 0 & \Pi_4 & 0 \\ * & * & \Lambda_3^3 & 0 & 0 & \Lambda_4^3 & \Lambda_5^3 & 0 & \Pi_5 & \tau_4 N_3 \\ * & * & * & \Lambda_6 & \Lambda_7 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Lambda_8^3 & \Lambda_9^3 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Lambda_{10}^3 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Lambda_{11}^3 & 0 & 0 & \tau_4 M_3 \\ * & * & * & * & * & * & * & \Lambda_{12} & \Pi_6 & 0 \\ * & * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & * & -R_4 \end{bmatrix},$$

$$\Pi_{ij}^{4} = \begin{bmatrix} P_1^4 - He\{U\} & P_2 + \Pi_1 & \Pi_2 & 0 & 0 & 0 & 0 & \Pi_3 & 0 \\ * & \Lambda_1 & 0 & \Lambda_2 & 0 & 0 & 0 & 0 & \Pi_4 \\ * & * & \Lambda_3^4 & 0 & 0 & \Lambda_4^3 & \Lambda_5^4 & 0 & \Pi_5 \\ * & * & * & \Lambda_6 & \Lambda_7 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Lambda_8^3 & \Lambda_9^3 & 0 & 0 & 0 \\ * & * & * & * & * & \Lambda_{10}^3 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Lambda_{11}^4 & 0 & 0 \\ * & * & * & * & * & * & * & \Lambda_{12} & \Pi_6 \\ * & * & * & * & * & * & * & * & -I \end{bmatrix},$$

$$\Theta_{ij} = \varepsilon_{1ij}\left(\Upsilon_{ij}^{1}\right)^{T}\Upsilon_{ij}^{1} + \varepsilon_{4ij}^{-1}\Gamma_{ij}^{2}\left(\Gamma_{ij}^{2}\right)^{T} + \varepsilon_{2ij}\varepsilon_{3ij}^{-1}\left(\Upsilon_{ij}^{3}\right)^{T}\Upsilon_{ij}^{3} + \varepsilon_{2ij}\left(\Upsilon_{ij}^{2}\right)^{T}\left[I - \varepsilon_{3ij}\Gamma_{ij}^{3}\left(\Gamma_{ij}^{3}\right)^{T}\right]^{-1}\Upsilon_{ij}^{2}$$
$$+ \varepsilon_{4ij}\varepsilon_{5ij}^{-1}\left(\Upsilon_{ij}^{5}\right)^{T}\Upsilon_{ij}^{5} + \varepsilon_{4ij}\left(\Upsilon_{ij}^{4}\right)^{T}\left[I - \varepsilon_{5ij}\Gamma_{ij}^{5}\left(\Gamma_{ij}^{5}\right)^{T}\right]^{-1}\Upsilon_{ij}^{4}, \tag{13}$$

with

$$P_1^1 = P_1^2 = \begin{pmatrix} \sum_{i=1}^{4} R_i + \tau_3 R_\tau & 0 \\ 0 & 0 \end{pmatrix},$$
$$P_1^3 = P_1^4 = \begin{pmatrix} \sum_{i=1}^{4} R_i + \tau_4 R_\tau & 0 \\ 0 & 0 \end{pmatrix}, \qquad P_2 = P,$$
$$\Pi_1 = \begin{pmatrix} U_1 A_i & 0 \\ U_2 A_i + \mathscr{B}_{fj} C_i & \mathscr{A}_{fj} \end{pmatrix},$$
$$\Pi_2 = \begin{pmatrix} U_1 A_{\tau i} \\ U_2 A_{\tau i} + \mathscr{B}_{fj} C_{\tau i} \end{pmatrix},$$
$$\Pi_3 = \begin{pmatrix} U_1 B_i \\ U_2 B_i + \mathscr{B}_{fj} D_i \end{pmatrix}, \qquad \Pi_4 = \begin{pmatrix} L_i^T - C_i^T \mathscr{D}_{fj}^T \\ -\mathscr{C}_{fj}^T \end{pmatrix},$$
$$\Pi_5 = \left(L_{\tau i}^T - C_{\tau i}^T \mathscr{D}_{fj}^T\right), \qquad \Pi_6 = \left(G_i^T - D_i^T \mathscr{D}_{fj}^T\right),$$
$$\Lambda_1 = \begin{pmatrix} Q_1 + S_1 - R_1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad \Lambda_2 = \begin{pmatrix} S_2 + R_1 \\ 0 \end{pmatrix},$$
$$\Lambda_3 = -(1 - h_d) Q_\tau - N_1 - N_1^T + \tau_3 \left(M_2 + M_2^T\right) + \tau_3 Z_{13},$$
$$\Lambda_4 = -M_1^T + N_1 + \tau_3 Z_{12}^T, \qquad \Lambda_5 = \tau_3 \left(-M_2 + N_2^T\right),$$
$$\Lambda_6 = Q_2 - Q_1 + S_2 - S_1 - R_1 - R_2,$$
$$\Lambda_7 = -S_2 + R_2,$$
$$\Lambda_8 = Q_3 - Q_2 + Q_\tau - S_3 - R_2 + \tau_3 Z_{11} + M_1 + M_1^T,$$
$$\Lambda_9 = Q_4 - Q_3 - R_4 - \tau_3 \left(N_2 + N_2^T\right),$$
$$\Lambda_{10} = R_4, \qquad \Lambda_{11} = -Q_4 - R_4,$$
$$\Lambda_{12} = -\gamma^2 I,$$
$$\Lambda_3^2 = -(1 - h_d) Q_\tau - N_1 - N_1^T + \tau_3 \left(M_2 + M_2^T\right),$$
$$\Lambda_4^2 = -M_1^T + N_1,$$
$$\Lambda_8^2 = Q_3 - Q_2 + Q_\tau - S_3 - R_2 + M_1 + M_1^T,$$
$$\Lambda_3^3 = -(1 - h_d) Q_\tau + M_4 + M_4^T - \tau_4 \left(N_3 + N_3^T\right),$$
$$\Lambda_4^3 = \tau_4 \left(-M_3 + N_3^T\right), \qquad \Lambda_5^3 = -M_4 + N_4^T,$$
$$\Lambda_8^3 = Q_3 - Q_2 + Q_\tau - S_3 - R_2 + R_3,$$
$$\Lambda_9^3 = R_3,$$
$$\Lambda_{10}^3 = Q_4 - Q_3 - R_3 + \tau_4 \left(M_3 + M_3^T\right),$$
$$\Lambda_{11}^3 = -Q_4 - N_4 - N_4^T,$$
$$\Lambda_3^4 = -(1 - h_d) Q_\tau + M_4 + M_4^T - \tau_4 \left(N_3 + N_3^T\right) + \tau_4 Z_{21},$$
$$\Lambda_5^4 = -M_4 + M_4^T + \tau_4 Z_{22},$$
$$\Lambda_{11}^4 = -Q_4 - N_4 - N_4^T + \tau_4 Z_{23},$$
$$\Gamma_{ij}^1 = \left[\begin{pmatrix} U_1 D_{1i} & 0 \\ U_2 D_{1i} & \mathscr{B}_{fj} D_{2i} \end{pmatrix} \ \underbrace{0 \ \cdots \ 0}_{7} \ \left(D_{3i} \ \ -\mathscr{D}_{fj} D_{2i}\right)\right]^T,$$
$$\Gamma_{ij}^2 = \left[\begin{pmatrix} 0 \\ U_3 D_{4j} \end{pmatrix} \ \underbrace{0 \ \cdots \ 0}_{7} \ 0\right]^T,$$
$$\Gamma_{ij}^4 = \left[0 \ \underbrace{0 \ \cdots \ 0}_{7} \ \left(-D_{5j}\right)\right]^T,$$
$$\Gamma_{ij}^3 = E_{5j} D_{2i}, \qquad \Gamma_{ij}^5 = E_{7j} D_{2i},$$
$$\Upsilon_{ij}^1 = \left[0 \ \begin{pmatrix} E_{1i} & 0 \\ E_{1i} & 0 \end{pmatrix} \ \begin{pmatrix} E_{2i} \\ E_{2i} \end{pmatrix} \ 0 \ 0 \ 0 \ 0 \ \begin{pmatrix} E_{3i} \\ E_{3i} \end{pmatrix} \ 0\right],$$
$$\Upsilon_{ij}^2 = \left[0 \ \left(E_{5j} C_i \ \ E_{4j}\right) \ \left(E_{5j} C_{\tau i}\right) \ 0 \ 0 \ 0 \ 0 \ \left(E_{5j} D_i\right) \ 0\right],$$
$$\Upsilon_{ij}^3 = \left[0 \ \left(E_{1i} \ \ 0\right) \ \left(E_{2i}\right) \ 0 \ 0 \ 0 \ 0 \ \left(E_{3i}\right) \ 0\right],$$
$$\Upsilon_{ij}^4 = \left[0 \ \left(E_{7j} C_i \ \ E_{6j}\right) \ \left(E_{7j} C_{\tau i}\right) \ 0 \ 0 \ 0 \ 0 \ \left(E_{7j} D_i\right) \ 0\right],$$
$$\Upsilon_{ij}^5 = \Upsilon_{ij}^3. \tag{14}$$

A suitable filter in the form of (4) *can be given by*

$$A_{fj} = U_3^{-1} \mathscr{A}_{fj}, \qquad B_{fj} = U_3^{-1} \mathscr{B}_{fj},$$
$$C_{fj} = \mathscr{C}_{fj}, \qquad D_{fj} = \mathscr{D}_{fj}, \tag{15}$$
$$(j = 1, 2, \ldots, r).$$

Proof. The delay-dependent LK functional can be constructed as follows:

$$V(t, \tilde{x}_t) = V_1(t, \tilde{x}_t) + V_2(t, \tilde{x}_t) + V_3(t, \tilde{x}_t) + V_4(t, \tilde{x}_t), \tag{16}$$

where $\widetilde{x}_t$ denotes the function $\widetilde{x}(s)$ defined on the interval $[t-h_b, t]$ and

$$
\begin{aligned}
V_1(t,\widetilde{x}_t) &= \widetilde{x}^T(t)P\widetilde{x}(t),\\
V_2(t,\widetilde{x}_t) &= \sum_{i=1}^{4}\int_{t-h_i}^{t-h_{i-1}}\widetilde{x}^T(s)E^TQ_iE\widetilde{x}(s)\,ds\\
&\quad+\int_{t-\tau(t)}^{t-h_2}\widetilde{x}^T(s)E^TQ_\tau E\widetilde{x}(s)\,ds,\\
V_3(t,\widetilde{x}_t)&\\
&=\int_{t-h_1}^{t}\begin{bmatrix}\widetilde{x}(s)\\ \widetilde{x}\left(s-\dfrac{\overline{\tau}_0}{2}\right)\end{bmatrix}^T\begin{bmatrix}E&0\\0&E\end{bmatrix}^T\begin{bmatrix}S_1&S_2\\ *&S_3\end{bmatrix}\begin{bmatrix}E&0\\0&E\end{bmatrix}\\
&\quad\times\begin{bmatrix}\widetilde{x}(s)\\ \widetilde{x}\left(s-\dfrac{\overline{\tau}_0}{2}\right)\end{bmatrix}ds,\\
V_4(t,\widetilde{x}_t)&=\sum_{i=1}^{4}\tau_i\int_{-h_i}^{-h_{i-1}}\int_{t+\theta}^{t}\dot{\widetilde{x}}^T(s)E^TR_iE\dot{\widetilde{x}}(s)\,ds\,d\theta\\
&\quad+\int_{-\tau(t)}^{-h_2}\int_{t+\theta}^{t}\dot{\widetilde{x}}^T(s)E^TR_\tau E\dot{\widetilde{x}}(s)\,ds\,d\theta,
\end{aligned}
\tag{17}
$$

with $P=\begin{bmatrix}P_1&P_2\\ *&P_3\end{bmatrix}>0, Q_k>0, R_k>0, (k=1,2,3,4), Q_\tau\geq 0, R_\tau\geq 0, S=\begin{bmatrix}S_1&S_2\\ *&S_3\end{bmatrix}>0$ being real symmetry matrices to be determined.

Taking the derivative of (16) with respect to t along the trajectory of the filtering error system (6), we have

$$
\begin{aligned}
&\dot{V}_1(t,\widetilde{x}_t)\\
&=\dot{\widetilde{x}}^T(t)P\widetilde{x}(t)+\widetilde{x}^T(t)P\dot{\widetilde{x}}(t)\\
&=\left[\widetilde{A}(t)\widetilde{x}(t)+\widetilde{A}_\tau(t)E\widetilde{x}(t-\tau(t))+\widetilde{B}(t)w(t)\right]^TP\widetilde{x}(t)\\
&\quad+\widetilde{x}^T(t)P\left[\widetilde{A}(t)\widetilde{x}(t)+\widetilde{A}_\tau(t)E\widetilde{x}(t-\tau(t))+\widetilde{B}(t)w(t)\right],\\
&\dot{V}_2(t,\widetilde{x}_t)\\
&=\sum_{i=1}^{4}\left[\widetilde{x}^T(t-h_{i-1})E^TQ_iE\widetilde{x}(t-h_{i-1})\right.\\
&\qquad\left.-\widetilde{x}^T(t-h_i)E^TQ_iE\widetilde{x}(t-h_i)\right]\\
&\quad+\widetilde{x}^T(t-h_2)E^TQ_\tau E\widetilde{x}(t-h_2)\\
&\quad-(1-\dot{\tau}(t))\widetilde{x}^T(t-\tau(t))E^TQ_\tau E\widetilde{x}(t-\tau(t))\\
&\leq\sum_{i=1}^{4}\left[x^T(t-h_{i-1})Q_ix(t-h_{i-1})-x^T(t-h_i)Q_ix(t-h_i)\right]\\
&\quad+x^T(t-h_2)Q_\tau x(t-h_2)\\
&\quad-(1-h_d)x^T(t-\tau(t))Q_\tau x(t-\tau(t)),\\
&\dot{V}_3(t,\widetilde{x}_t)=\begin{bmatrix}x(t)\\x(t-h_1)\end{bmatrix}^T\begin{bmatrix}S_1&S_2\\ *&S_3\end{bmatrix}\begin{bmatrix}x(t)\\x(t-h_1)\end{bmatrix}\\
&\qquad-\begin{bmatrix}x(t-h_1)\\x(t-h_2)\end{bmatrix}^T\begin{bmatrix}S_1&S_2\\ *&S_3\end{bmatrix}\begin{bmatrix}x(t-h_1)\\x(t-h_2)\end{bmatrix},\\
&\dot{V}_4(t,x_t)=\sum_{i=1}^{4}\tau_i^2\dot{x}^T(t)R_i\dot{x}(t)+(\tau(t)-h_2)\dot{x}^T(t)R_\tau\dot{x}(t)\\
&\qquad-\sum_{i=1}^{4}\tau_i\int_{t-h_i}^{t-h_{i-1}}\dot{x}^T(s)R_i\dot{x}(s)\,ds-(1-\dot{\tau}(t))\\
&\qquad\times\int_{t-\tau(t)}^{t-h_2}\dot{x}^T(s)R_\tau\dot{x}(s)\,ds.
\end{aligned}
\tag{18}
$$

For any $t\geq 0$, it is a fact that $h_a\leq\tau(t)\leq h_a+\alpha\tau$ or $h_a+\alpha\tau\leq\tau(t)\leq h_b$, $(0<\alpha<1)$. In the case of $h_a\leq\tau(t)\leq h_a+\alpha\tau$; that is, $\tau(t)\in[h_2,h_3], k=3$, suitably using the integral inequalities in Lemma 1, the following inequalities are true:

$$
\begin{aligned}
&(\tau(t)-h_2)\dot{x}^T(t)R_\tau\dot{x}(t)\\
&\quad\leq\alpha\overline{\tau}\dot{x}^T(t)R_\tau\dot{x}(t)=\tau_3\dot{x}^T(t)R_\tau\dot{x}(t),\\
&-\tau_i\int_{t-h_i}^{t-h_{i-1}}\dot{x}^T(s)R_i\dot{x}(s)\,ds\\
&\quad\leq\begin{bmatrix}x(t-h_{i-1})\\x(t-h_i)\end{bmatrix}^T\begin{bmatrix}-R_i&R_i\\ *&-R_i\end{bmatrix}\begin{bmatrix}x(t-h_{i-1})\\x(t-h_i)\end{bmatrix},\\
&\qquad\qquad(i=1,2,4),\\
&-\tau_3\int_{t-h_3}^{t-h_2}\dot{x}^T(s)R_3\dot{x}(s)\,ds-(1-\dot{\tau}(t))\\
&\quad\times\int_{t-\tau(t)}^{t-h_2}\dot{x}^T(s)R_\tau\dot{x}(s)\,ds\\
&\quad\leq-\tau_3\int_{t-h_3}^{t-h_2}\dot{x}^T(s)R_3\dot{x}(s)\,ds-(1-h_d)\\
&\qquad\times\int_{t-\tau(t)}^{t-h_2}\dot{x}^T(s)R_\tau\dot{x}(s)\,ds\\
&\quad=-\int_{t-\tau(t)}^{t-h_2}\dot{x}^T(s)(\tau_3R_3+(1-h_d)R_\tau)\dot{x}(s)\,ds\\
&\qquad-\tau_3\int_{t-h_3}^{t-\tau(t)}\dot{x}^T(s)R_3\dot{x}(s)\,ds\\
&\quad\leq\begin{bmatrix}x(t-h_2)\\x(t-\tau(t))\end{bmatrix}^T\\
&\qquad\times\left\{\begin{bmatrix}M_1+M_1^T&-M_1+N_1^T\\ *&-N_1-N_1^T\end{bmatrix}+\rho\cdot\alpha\overline{\tau}\cdot Z_1\right\}\\
&\qquad\times\begin{bmatrix}x(t-h_2)\\x(t-\tau(t))\end{bmatrix}
\end{aligned}
$$

$$+\begin{bmatrix} x(t-\tau(t)) \\ x(t-h_3) \end{bmatrix}^T \times \left\{ \tau_3 \begin{bmatrix} M_2+M_2^T & -M_2+N_2^T \\ * & -N_2-N_2^T \end{bmatrix} + (1-\rho)\cdot\alpha\overline{\tau}\cdot \begin{bmatrix} M_2 \\ N_2 \end{bmatrix} \tau_3 R_3^{-1} \begin{bmatrix} M_2 \\ N_2 \end{bmatrix}^T \right\} \times \begin{bmatrix} x(t-\tau(t)) \\ x(t-h_3) \end{bmatrix} \quad (19)$$

with $\begin{bmatrix} \tau_3 R_3+(1-h_d)R_\tau & [M_1 \ \ N_1] \\ * & Z_1 \end{bmatrix} \geq 0$ and $\rho = (\tau(t)-h_2)/\alpha\overline{\tau}, 0 \leq \rho \leq 1$.

It follows from (18)-(19) that

$$\begin{aligned} &\dot{V}(t,\tilde{x}_t) + e^T(t)e(t) - \gamma^2 w^T(t)w(t) \\ &\leq \xi^T(t)[\Omega_1]\xi(t) \\ &\quad + \rho\cdot \begin{bmatrix} x(t-h_2) \\ x(t-\tau(t)) \end{bmatrix}^T \{\alpha\overline{\tau}\cdot Z_1\} \begin{bmatrix} x(t-h_2) \\ x(t-\tau(t)) \end{bmatrix} \\ &\quad + (1-\rho)\cdot \begin{bmatrix} x(t-\tau(t)) \\ x(t-h_3) \end{bmatrix}^T \left\{\alpha\overline{\tau}\cdot \begin{bmatrix} M_2 \\ N_2 \end{bmatrix} \tau_3 R_3^{-1} \begin{bmatrix} M_2 \\ N_2 \end{bmatrix}^T \right\} \\ &\quad \times \begin{bmatrix} x(t-\tau(t)) \\ x(t-h_3) \end{bmatrix} \\ &= \xi^T(t)\left[\rho\cdot\Omega_{1\rho} + (1-\rho)\cdot\Omega_{1(1-\rho)}\right]\xi(t) \end{aligned} \quad (20)$$

with $\begin{bmatrix} \tau_3 R_3+(1-h_d)R_\tau & [M_1 \ \ N_1] \\ * & Z_1 \end{bmatrix} \geq 0$, where

$$\xi(t) := \mathrm{col}\left\{[x(t) \ \ x_f(t)] \ \ x(t-\tau(t)) \ \ x(t-h_1) \ \ x(t-h_2) \ \ x(t-h_3) \ \ x(t-h_4) \ \ w(t)\right\} \quad (21)$$

and $\Omega_{1\rho}$ is defined as follows:

$$\begin{aligned} \Omega_{1\rho} &= \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & 0 & 0 & 0 & \Omega_{17} \\ * & \Omega_{22} & 0 & \Omega_{24} & \Omega_{25} & 0 & \Omega_{27} \\ * & * & \Omega_{33} & \Omega_{34} & 0 & 0 & 0 \\ * & * & * & \Omega_{44} & 0 & 0 & 0 \\ * & * & * & * & \Omega_{55} & \Omega_{56} & 0 \\ * & * & * & * & * & \Omega_{66} & 0 \\ * & * & * & * & * & * & \Omega_{77} \end{bmatrix} + \begin{bmatrix} \widetilde{C}^T(t) \\ \widetilde{C}_\tau^T(t) \\ 0 \\ 0 \\ 0 \\ 0 \\ \widetilde{D}^T(t) \end{bmatrix} \begin{bmatrix} \widetilde{C}^T(t) \\ \widetilde{C}_\tau^T(t) \\ 0 \\ 0 \\ 0 \\ 0 \\ \widetilde{D}^T(t) \end{bmatrix}^T, \\ \Omega_{11} &= \widetilde{A}^T(t)E^T RE\widetilde{A}(t) + He\left\{P\widetilde{A}(t)\right\} + \Lambda_1, \\ R &= \sum_{i=1}^{4} R_i + \tau_3 R_\tau, \\ \Omega_{12} &= P\widetilde{A}_\tau(t) + \widetilde{A}^T(t)E^T RE\widetilde{A}_\tau(t), \\ \Omega_{13} &= \Lambda_2, \\ \Omega_{17} &= P\widetilde{B}(t) + \widetilde{A}^T(t)E^T RE\widetilde{B}(t), \\ \Omega_{22} &= \widetilde{A}_\tau^T(t)E^T RE\widetilde{A}_\tau(t) + \Lambda_3, \\ \Omega_{24} &= \Lambda_4, \\ \Omega_{25} &= \Lambda_5, \\ \Omega_{27} &= \widetilde{A}_\tau^T(t)E^T RE\widetilde{B}(t), \\ \Omega_{33} &= \Lambda_6, \\ \Omega_{34} &= \Lambda_7, \\ \Omega_{44} &= \Lambda_8, \\ \Omega_{55} &= \Lambda_9, \\ \Omega_{56} &= \Lambda_{10}, \\ \Omega_{66} &= \Lambda_{11}, \\ \Omega_{77} &= \widetilde{B}^T(t)E^T RE\widetilde{B}(t) + \Lambda_{12}. \end{aligned} \quad (22)$$

Since $0 \leq \rho \leq 1$, applying the convex combination method, we conclude that if

$$\Omega_{1\rho} < 0, \qquad \Omega_{1(1-\rho)} < 0, \quad (23)$$

then

$$\dot{V}(t,\tilde{x}_t) + e^T(t)e(t) - \gamma^2 w^T(t)w(t) < 0. \quad (24)$$

For $\Omega_{1\rho} < 0$, by Schur complement, we have

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & 0 & 0 & 0 & \Omega_{17} & \widetilde{C}^T(t) \\ * & \Omega_{22} & 0 & \Omega_{24} & \Omega_{25} & 0 & \Omega_{27} & \widetilde{C}_\tau^T(t) \\ * & * & \Omega_{33} & \Omega_{34} & 0 & 0 & 0 & 0 \\ * & * & * & \Omega_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & \Omega_{55} & \Omega_{56} & 0 & 0 \\ * & * & * & * & * & \Omega_{66} & 0 & 0 \\ * & * & * & * & * & * & \Omega_{77} & \widetilde{D}^T(t) \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0. \quad (25)$$

By using Proposition 3, we have the following inequality, which is equivalent to (27):

$$\begin{bmatrix} P_1 - He\{U\} & P_2 - U\widetilde{A}(t) & U\widetilde{A}_\tau(t) & 0 & 0 & 0 & \Omega_{17} & U\widetilde{B}(t) & 0 \\ * & \Lambda_1 & 0 & \Lambda_2 & 0 & 0 & 0 & 0 & \widetilde{C}^T(t) \\ * & * & \Lambda_3 & 0 & \Lambda_4 & \Lambda_5 & 0 & 0 & \widetilde{C}_\tau^T(t) \\ * & * & * & \Lambda_6 & \Lambda_7 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Lambda_8 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Lambda_9 & \Lambda_{10} & 0 & 0 \\ * & * & * & * & * & * & \Lambda_{11} & 0 & 0 \\ * & * & * & * & * & * & * & \Lambda_{12} & \widetilde{D}^T(t) \\ * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{26}$$

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(t)h_j(t) \begin{bmatrix} P_1 - He\{U\} & P_2 - U\widehat{A}_{ij}(t) & U\widehat{A}_{\tau ij}(t) & 0 & 0 & 0 & 0 & U\widehat{B}_{ij}(t) & 0 \\ * & \Lambda_1 & 0 & \Lambda_2 & 0 & 0 & 0 & 0 & \widetilde{C}_{ij}^T(t) \\ * & * & \Lambda_3 & 0 & \Lambda_4 & \Lambda_5 & 0 & 0 & \widetilde{C}_{\tau ij}^T(t) \\ * & * & * & \Lambda_6 & \Lambda_7 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Lambda_8 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Lambda_9 & \Lambda_{10} & 0 & 0 \\ * & * & * & * & * & * & \Lambda_{11} & 0 & 0 \\ * & * & * & * & * & * & * & \Lambda_{12} & \widetilde{D}_{ij}^T(t) \\ * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0. \tag{27}$$

Let $\mathscr{A}_{fj} = U_3A_{fj}, \mathscr{B}_{fj} = U_3B_{fj}, \mathscr{C}_{fj} = C_{fj}, \mathscr{D}_{fj} = D_{fj}$, and, using Lemma 2, for the case of $\Omega_{1\rho}$, we have the following equation:

$$\begin{aligned} \Pi^1 &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(t)h_j(t)\left[\Pi_{ij}^1 + He\left\{\Gamma_{ij}^1 \operatorname{diag}\{F_{1i}(t), F_{1i}(t)\}\Upsilon_{ij}^1\right\}\right. \\ &\quad + He\left\{\Gamma_{ij}^2 F_{2j}(t)\left[\Upsilon_{ij}^2 + \Gamma_{ij}^3 F_{1i}(t)\Upsilon_{ij}^3\right]\right\} \\ &\quad \left. + He\left\{\Gamma_{ij}^4 F_{3j}(t)\left[\Upsilon_{ij}^4 + \Gamma_{ij}^5 F_{1i}(t)\Upsilon_{ij}^5\right]\right\}\right] \\ &\le \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(t)h_j(t)\left[\Pi_{ij}^1 + \varepsilon_{1ij}^{-1}\Gamma_{ij}^1\left(\Gamma_{ij}^1\right)^T + \varepsilon_{1ij}\left(\Upsilon_{ij}^1\right)^T\Upsilon_{ij}^1\right. \\ &\quad + \varepsilon_{2ij}^{-1}\Gamma_{ij}^2\left(\Gamma_{ij}^2\right)^T + \varepsilon_{2ij}\left[\Upsilon_{ij}^2 + \Gamma_{ij}^3 F_{1i}(t)\Upsilon_{ij}^3\right]^T \\ &\quad \times\left[\Upsilon_{ij}^2 + \Gamma_{ij}^3 F_{1i}(t)\Upsilon_{ij}^3\right] + \varepsilon_{4ij}^{-1}\Gamma_{ij}^4\left(\Gamma_{ij}^4\right)^T \\ &\quad + \varepsilon_{4ij}\left[\Upsilon_{ij}^4 + \Gamma_{ij}^5 F_{1i}(t)\Upsilon_{ij}^5\right]^T \\ &\quad \left.\times\left[\Upsilon_{ij}^4 + \Gamma_{ij}^5 F_{1i}(t)\Upsilon_{ij}^5\right]\right] \\ &\le \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(t)h_j(t)\left[\Pi_{ij}^1 + \varepsilon_{1ij}^{-1}\Gamma_{ij}^1\left(\Gamma_{ij}^1\right)^T + \varepsilon_{1ij}\left(\Upsilon_{ij}^1\right)^T\Upsilon_{ij}^1\right. \\ &\quad + \varepsilon_{2ij}^{-1}\Gamma_{ij}^2\left(\Gamma_{ij}^2\right)^T + \varepsilon_{4ij}^{-1}\Gamma_{ij}^4\left(\Gamma_{ij}^4\right)^T \\ &\quad + \varepsilon_{2ij}\varepsilon_{3ij}^{-1}\left(\Upsilon_{ij}^3\right)^T\Upsilon_{ij}^3 \\ &\quad + \varepsilon_{2ij}\left(\Upsilon_{ij}^2\right)^T\left[I - \varepsilon_{3ij}\Gamma_{ij}^3\left(\Gamma_{ij}^3\right)^T\right]^{-1}\Upsilon_{ij}^2 \\ &\quad + \varepsilon_{4ij}\varepsilon_{5ij}^{-1}\left(\Upsilon_{ij}^5\right)^T\Upsilon_{ij}^5 \\ &\quad \left.+ \varepsilon_{4ij}\left(\Upsilon_{ij}^4\right)^T\left[I - \varepsilon_{5ij}\Gamma_{ij}^5\left(\Gamma_{ij}^5\right)^T\right]^{-1}\Upsilon_{ij}^4\right] \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(t)h_j(t)\left[\Pi_{ij}^1 + \varepsilon_{1ij}^{-1}\Gamma_{ij}^1\left(\Gamma_{ij}^1\right)^T\right. \\ &\quad \left. + \varepsilon_{2ij}^{-1}\Gamma_{ij}^2\left(\Gamma_{ij}^2\right)^T + \Theta_{ij}\right] \\ &= \sum_{i=1}^{r} h_i^2(t)\left[\Pi_{ii}^1 + \Theta_{ii} + \varepsilon_{1ii}^{-1}\Gamma_{ii}^1\left(\Gamma_{ii}^1\right)^T\right. \\ &\quad \left. + \varepsilon_{2ii}^{-1}\Gamma_{ii}^2\left(\Gamma_{ii}^2\right)^T\right] \\ &\quad + \sum_{i<j}^{r} h_i(t)h_j(t)\left[\Pi_{ij}^1 + \Theta_{ij} + \Pi_{ji}^1 + \Theta_{ji} + \varepsilon_{1ij}^{-1}\Gamma_{ij}^1\left(\Gamma_{ij}^1\right)^T\right. \\ &\quad + \varepsilon_{2ij}^{-1}\Gamma_{ij}^2\left(\Gamma_{ij}^2\right)^T \\ &\quad \left. + \varepsilon_{1ji}^{-1}\Gamma_{ji}^1\left(\Gamma_{ji}^1\right)^T + \varepsilon_{2ji}^{-1}\Gamma_{ji}^2\left(\Gamma_{ji}^2\right)^T\right], \end{aligned} \tag{28}$$

where

$$\begin{aligned} \Theta_{ij} &= \varepsilon_{1ij}\left(\Upsilon_{ij}^1\right)^T\Upsilon_{ij}^1 + \varepsilon_{4ij}^{-1}\Gamma_{ij}^4\left(\Gamma_{ij}^4\right)^T \\ &\quad + \varepsilon_{2ij}\varepsilon_{3ij}^{-1}\left(\Upsilon_{ij}^3\right)^T\Upsilon_{ij}^3 \end{aligned}$$

$$+ \varepsilon_{2ij}\left(\Upsilon_{ij}^{2}\right)^{T}\left[I-\varepsilon_{3ij}\Gamma_{ij}^{3}\left(\Gamma_{ij}^{3}\right)^{T}\right]^{-1}\Upsilon_{ij}^{2}$$

$$+ \varepsilon_{4ij}\varepsilon_{5ij}^{-1}\left(\Upsilon_{ij}^{5}\right)^{T}\Upsilon_{ij}^{5} + \varepsilon_{4ij}\left(\Upsilon_{ij}^{4}\right)^{T}\left[I-\varepsilon_{5ij}\Gamma_{ij}^{5}\left(\Gamma_{ij}^{5}\right)^{T}\right]^{-1}\Upsilon_{ij}^{4}. \tag{29}$$

If (12) when $m = 1$ hold, then $\Pi^1 < 0$, which implies that the first term of (23) is true. Similar to the above process, if (12) when $m = 2$ hold and we also have $\Omega_{1(1-\rho)} < 0$, then (24) hold.

Meanwhile, if $h_a + \alpha\tau \le \tau(t) \le h_b$; that is, $\tau(t) \in [h_3, h_4]$, $k = 4$, similar to the above deduction process, we also can obtain the conclusion that if (12) hold, then (24) hold.

So far, when assuming the zero disturbance input, from (18)–(19), we can obtain that

$$\dot{V}(t,\tilde{x}_t) \le \tilde{\xi}^T(t)\begin{bmatrix}\Omega_{11} & \Omega_{12} & \Omega_{13} & 0 & 0 & 0\\ * & \Omega_{22} & 0 & \Omega_{24} & \Omega_{25} & 0\\ * & * & \Omega_{33} & \Omega_{34} & 0 & 0\\ * & * & * & \Omega_{44} & 0 & 0\\ * & * & * & * & \Omega_{55} & \Omega_{56}\\ * & * & * & * & * & \Omega_{66}\end{bmatrix}\tilde{\xi}(t), \tag{30}$$

where

$$\tilde{\xi}(t) := \operatorname{col}\left\{\left[x(t)\ \ x_f(t)\right]\ \ x(t-\tau(t))\ \ x(t-h_1)\ \ x(t-h_2)\ \ x(t-h_3)\ \ x(t-h_4)\right\}. \tag{31}$$

By Schur complement, the inequalities in (25) imply $\dot{V}(t,\tilde{x}_t) < 0$. We can conclude that filtering error system (6) with $w(t) = 0$ is asymptotically stable. Now, to establish the H_∞ performance for system (6), assume zero-initial condition and consider the following index:

$$J = \int_{t=0}^{\infty}\left[e^T(t)e(t) - \gamma^2 w^T(t)w(t)\right]dt. \tag{32}$$

Under zero initial condition and (24), we have $J \le -\int_{t=0}^{\infty}\dot{V}(t,\tilde{x}_t)dt = -V(\infty) + V(0) = -V(\infty) < 0$, which means that $\|e\|_2 < \gamma\|w\|_2$. Thus, this completes the proof. ☐

Remark 5. It may be noted that, in the above, no approximation of the delay term is involved excepting exploiting a convex combination of the uncertain terms involved. In fact, Lemma 1 plays a key effect on the present results, which is different from the common Jensen's inequality. Although their similarity can be established following the equivalency results in [41], if $h = \tau_2 - \tau_1 := h(t)$ is uncertain and required to be approximated with its lower or upper bound then use of (9) or (10) would be beneficial since the free variables $Z_j = \left[\begin{smallmatrix} Z_{j1} & Z_{j2}\\ * & Z_{j3}\end{smallmatrix}\right]$ and M_i, N_i are introduced. Such a feature leads to less conservative results compared to the existing ones as is shown in the next section using numerical examples.

Remark 6. In Proposition 3, by introduction of the auxiliary slack matrix variable U, matrices P_1, P_2, $D_i, X_j (j = 1, 2, \dots, 8;\ i = 1, 2, \dots, 14)$ and $A_i, A_{\tau i}, B_i$ are decoupled. This novel technique is proposed in this paper to transform the nonlinear matrix inequalities (25) into a set of LMIs, which is different from the existing literatures.

Remark 7. In the proof of Theorem 4, the interval $[h_a, h_b]$ is divided into two variable subintervals $[h_a, h_a + \alpha\overline{\tau}]$ and $[h_a + \alpha\overline{\tau}, h_b]$; meanwhile, the lower bound of the delay $[0, h_a]$ is also divided into two equal subintervals $[0, h_a/2]$ and $[h_a/2, h_a]$ for the sake of simplification. Therefore, the information of delayed state $x(t - h_a/2)$ and $x(t - h_a - \alpha\overline{\tau})$ can be fully taken into account. And it is clear that the LK functional (16) are more general than the existing ones [26, 35, 37]. Moreover, since the variable delay decomposition approach in this paper is introduced in constructing the LK functional and the upper bound of its derivative is also estimated by suitably utilizing integral inequalities in Lemma 1, the proposed result is much less conservative and is more general than some existing ones. Meanwhile, the stability criteria of proposed approach are also different when the tuning delay-fractional parameter α is varying. Examples below show that the proposed method yields less conservatism than the existing ones and also show that the delay decomposition is different; the maximum upper bound of the delay may be different.

Without considering the filter gain uncertainties, that is, $F_{2j}(t) = 0$ and $F_{3j}(t) = 0$, the following corollary gives a delay-fractional-dependent condition of designing a standard fuzzy H_∞ filter for the uncertain system (1) as [12–14], which system uncertainties have not been considered.

Corollary 8. *For uncertain system* (1)*, given scalars* $0 < h_a \le h_b, 0 < \alpha < 1, h_d$ *and* $\gamma > 0$*, the* H_∞ *filter error system* (6)*, for all differentiable delay* $\tau(t) \in [h_a, h_b]$ *with* $\dot{\tau}(t) \le h_d$*, is asymptotically stable and has a prescribed* H_∞ *performance level* γ *if there exist real symmetric matrices* $P = \left[\begin{smallmatrix} P_1 & P_2\\ * & P_3\end{smallmatrix}\right] > 0$, $Q_l > 0, R_l > 0$, $Q_\tau \ge 0, R_\tau \ge 0$, $S = \left[\begin{smallmatrix} S_1 & S_2\\ * & S_3\end{smallmatrix}\right] > 0$*, the nonsingular matrix* $U = \left[\begin{smallmatrix} U_1 & 0\\ U_2 & U_3\end{smallmatrix}\right]$ *and matrices* $Z_m = \left[\begin{smallmatrix} Z_{m1} & Z_{m2}\\ * & Z_{m3}\end{smallmatrix}\right]$, $(m = 1, 2)$, $\mathscr{A}_{fj}, \mathscr{B}_{fj}, \mathscr{C}_{fj}, \mathscr{D}_{fj}$, M_l, N_l, $(l = 1, 2, 3, 4)$ *with appropriate dimensions, and positive scalars* $\tilde{\varepsilon}_{1ij}$, $(i, j = 1, 2, \dots, r)$*, such that the inequalities in* (33) *hold:*

$$\begin{bmatrix}\Pi_{kk}^{m} + \tilde{\Theta}_{kk} & \Gamma_{kk}^{1}\\ * & -\varepsilon_{1kk}I\end{bmatrix} < 0,$$

$$(m = 1, 2, 3, 4;\ k = 1, 2, \dots, r),$$

$$\begin{bmatrix} \Pi_{ij}^{m}+\widetilde{\Theta}_{ij}+\Pi_{ji}^{m}+\widetilde{\Theta}_{ji} & \Gamma_{ij}^{1} & \Gamma_{ji}^{1} \\ * & -\varepsilon_{1ij}I & 0 \\ * & * & -\varepsilon_{1ji}I \end{bmatrix}<0,$$
$$(m=1,2,3,4; 0<i<j\leq r),$$
$$\begin{bmatrix} \tau_3R_3+(1-h_d)R_\tau & [M_1\ \ N_1] \\ * & Z_1 \end{bmatrix}\geq 0,$$
$$\begin{bmatrix} \tau_4R_4+(1-h_d)R_\tau & [M_4\ \ N_4] \\ * & Z_2 \end{bmatrix}\geq 0$$
$$(i=1,2,\ldots,r), \tag{33}$$

where Π_{ij}^{m}, Γ_{ij}^{1} *are defined in* (14), $\widetilde{\Theta}_{ij}=\widetilde{\varepsilon}_{1ij}(\Upsilon_{ij}^{1})^{T}\Upsilon_{ij}^{1}$. *The filter parameters are given by* (15).

Moreover, if the above LMIs are feasible with $Q_\tau=0$ *and* $R_\tau=0$, *then the fuzzy* H_∞ *filtering problem is solvable for all fast-varying delays in* $[h_a,h_b]$. *Meanwhile a suitable fuzzy* H_∞ *filter is designed as* (15).

Similarly, without considering the system uncertainties, that is, $F_{1j}(t)=0$, the following corollary gives a delay-fractional-dependent condition of designing a nonfragile fuzzy H_∞ filter for the nominal case of system (1) as [24], in which time delay has not been considered.

Corollary 9. *For the nominal case of system* (1), *given scalars* $0<h_a\leq h_b$, $0<\alpha<1$, h_d *and* $\gamma>0$, *the* H_∞ *filter error system* (6), *for all differentiable delay* $\tau(t)\in[h_a,h_b]$ *with* $\dot{\tau}(t)\leq h_d$, *is asymptotically stable and has a prescribed* H_∞ *performance level* γ *if there exist real symmetric matrices* $P=\begin{bmatrix} P_1 & P_2 \\ * & P_3\end{bmatrix}>0,\ldots,Q_\tau\geq 0,R_\tau\geq 0,S=\begin{bmatrix} S_1 & S_2 \\ * & S_3\end{bmatrix}>0$, *the nonsingular matrix* $U=\begin{bmatrix} U_1 & 0 \\ U_2 & U_3\end{bmatrix}$ *and matrices* $Z_m=\begin{bmatrix} Z_{m1} & Z_{m2} \\ * & Z_{m3}\end{bmatrix}$, $(m=1,2)$, $\mathscr{A}_{fj},\mathscr{B}_{fj},\mathscr{C}_{fj},\mathscr{D}_{fj},M_l,N_l,(l=1,2,3,4)$ *with appropriate dimensions, and positive scalars* $\widehat{\varepsilon}_{2ij},\widehat{\varepsilon}_{4ij}$, $(i,j=1,2,\ldots,r)$, *such that the inequalities in* (34) *hold:*

$$\Pi_{kk}^{m}+\widehat{\Theta}_{kk}<0,\quad (m=1,2,3,4;\ k=1,2,\ldots,r),$$
$$\Pi_{ij}^{m}+\widehat{\Theta}_{ij}+\Pi_{ji}^{m}+\widehat{\Theta}_{ji}<0,\quad (m=1,2,3,4; 0<i<j\leq r),$$
$$\begin{bmatrix} \tau_3R_3+(1-h_d)R_\tau & [M_1\ \ N_1] \\ * & Z_1 \end{bmatrix}\geq 0,$$
$$\begin{bmatrix} \tau_4R_4+(1-h_d)R_\tau & [M_4\ \ N_4] \\ * & Z_2 \end{bmatrix}\geq 0, \tag{34}$$

where Π_{ij}^{m} *is defined in* (14), $\widehat{\Theta}_{ij}=\widehat{\varepsilon}_{2ij}^{-1}\Gamma_{ij}^{2}(\Gamma_{ij}^{2})^{T}+\widehat{\varepsilon}_{2ij}(\Upsilon_{ij}^{2})^{T}\Upsilon_{ij}^{2}+\widehat{\varepsilon}_{4ij}^{-1}(\Gamma_{ij}^{4})^{T}\Gamma_{ij}^{4}+\widehat{\varepsilon}_{4ij}(\Upsilon_{ij}^{4})^{T}\Upsilon_{ij}^{4}$. *The filter parameters are given by* (15).

Moreover, if the above LMIs are feasible with $Q_\tau=0$ *and* $R_\tau=0$, *then the fuzzy* H_∞ *filtering problem is solvable for all fast-varying delays in* $[h_a,h_b]$. *Meanwhile a suitable fuzzy* H_∞ *filter is designed as* (15).

Remark 10. When considering both no system uncertainties and no filter gain perturbations, Corollary 8 further reduces a delay-fractional-dependent sufficient condition for designing a standard fuzzy H_∞ filter for the nominal case of system (1).

Remark 11. Given $0\leq h_a\leq h_b$, Theorem 4 and Corollaries 8 and 9 provide delay-fractional-dependent stabilization conditions for uncertain systems (1) in the form of LMIs. They can be verified using recently developed standard algorithms in MATLAB Toolbox. Meanwhile, it is worthy of mentioning that the variable delay decomposition approach proposed in this paper can be applied to the further stability analysis along with a new model transformation [15], and the corresponding stability criteria with less conservatism and small computing burden may be derived. Moreover, the proposed results can be extended to reduced-order H_∞ filtering for T-S fuzzy system based on the proposed method in [16], finite frequency H_∞ filtering [17], and even the above analysis and filtering for 2-D systems [18], and neutral system [42], and the corresponding results will appear in the near future.

4. Numerical Examples

In this section, three numerical examples are given to show the effectiveness and reduced conservatism of the proposed method in this paper.

Example 12 (example of [26]). Consider the uncertain system (1), in which the parameters are given as

$$A_1=\begin{bmatrix} -2.63 & 0.13 \\ 1.25 & -2.50\end{bmatrix},\qquad A_2=\begin{bmatrix} -2.38 & 0 \\ -0.25 & -1.38\end{bmatrix},$$
$$A_{\tau 1}=\begin{bmatrix} -1.1 & 0.1 \\ -0.8 & -0.9\end{bmatrix},\qquad A_{\tau 2}=\begin{bmatrix} -0.9 & 0 \\ -1.1 & -1.2\end{bmatrix},$$
$$B_1=B_2=\begin{bmatrix} -0.5 \\ 1.0\end{bmatrix},$$
$$C_1=\begin{bmatrix} -0.2 & 0.1 \\ 0 & 0.05\end{bmatrix},\qquad C_2=\begin{bmatrix} 0.3 & 1.0 \\ 0.1 & -0.5\end{bmatrix},$$
$$C_{\tau 1}=\begin{bmatrix} 0.5 & 1.0 \\ 0.2 & -0.3\end{bmatrix},\qquad C_{\tau 2}=\begin{bmatrix} 1.0 & -0.2 \\ 0.2 & -0.5\end{bmatrix},$$
$$D_1=\begin{bmatrix} 0.1 \\ -0.1\end{bmatrix},\qquad D_2=\begin{bmatrix} 0.2 \\ 0.3\end{bmatrix},$$
$$L_1=\begin{bmatrix} 1.0 & -0.5 \\ 0.2 & -0.3\end{bmatrix},\qquad L_2=\begin{bmatrix} -0.2 & 0.3 \\ 0.1 & 0\end{bmatrix},$$
$$L_{\tau 1}=L_{\tau 2}=\begin{bmatrix} 0 & 0 \\ 0 & 0\end{bmatrix},\qquad G_1=G_2=0 \tag{35}$$

and the membership function are defined as $h_1(t)=\sin^2(x_1(t))$, $h_2(t)=\cos^2(x_1(t))$.

Meanwhile, the system uncertainty and filter gain variant are assumed as

$$\begin{aligned}
&D_{11}=\begin{bmatrix}0\\-0.5\end{bmatrix},\quad D_{12}=\begin{bmatrix}0.8\\0.1\end{bmatrix},\quad D_{21}=\begin{bmatrix}0\\0.3\end{bmatrix},\\
&D_{22}=\begin{bmatrix}-0.3\\0.6\end{bmatrix},\quad D_{31}=0,\quad D_{32}=0,\\
&E_{11}=\begin{bmatrix}0 & 0.3\end{bmatrix},\quad E_{12}=\begin{bmatrix}0.2 & 0\end{bmatrix},\quad E_{13}=0.1,\\
&E_{21}=\begin{bmatrix}0.5 & 0\end{bmatrix},\quad E_{22}=\begin{bmatrix}0 & -0.2\end{bmatrix},\quad E_{23}=0,\\
&D_{41}=\begin{bmatrix}-0.5\\0.1\end{bmatrix},\quad D_{51}=\begin{bmatrix}1.0\\0.1\end{bmatrix},\quad D_{42}=\begin{bmatrix}0.5\\1.0\end{bmatrix},\\
&D_{52}=\begin{bmatrix}-0.2\\0.1\end{bmatrix},\quad E_{41}=\begin{bmatrix}0 & -0.4\end{bmatrix},\quad E_{51}=0,\\
&E_{42}=\begin{bmatrix}0 & -0.4\end{bmatrix},\quad E_{61}=\begin{bmatrix}0.1 & 0\end{bmatrix},\quad E_{71}=0,\\
&E_{62}=\begin{bmatrix}0 & -0.4\end{bmatrix},\quad E_{72}=0,\\
&F_{ki}(t)=\sin(t),\quad (i=1,2;\ k=1,2,3).
\end{aligned} \tag{36}$$

We assume that the time delay is $\tau(t)=0.3+0.2\sin(t)$; that is, $h_a=0.1, h_b=0.5, h_d=0.2$. In this case, we can calculate the optimal performance level $\gamma_{\min}=0.473$, while there is $\gamma=1.600$ in [26]. When $\gamma=0.5$, applying Theorem 4 and using the MATLAB LMI Toolbox, a desired nonfragile fuzzy H_∞ filter can be constructed to solve the LMIs in (12). The parameters can be chosen as follows (other matrices are omitted for space saving):

$$\begin{aligned}
A_{f1}&=\begin{bmatrix}-9.8761 & 2.2818\\ 3.0212 & -13.2159\end{bmatrix},\\
B_{f1}&=\begin{bmatrix}0.1519 & 0.7476\\ -0.5081 & -0.1281\end{bmatrix},\\
C_{f1}&=\begin{bmatrix}-2.1237 & 2.5492\\ -0.6212 & 1.1097\end{bmatrix},\\
A_{f2}&=\begin{bmatrix}-9.8772 & 5.8011\\ 0.4140 & -8.7436\end{bmatrix},\\
B_{f2}&=\begin{bmatrix}0.7381 & 0.2902\\ -1.1894 & 0.1668\end{bmatrix},\\
C_{f2}&=\begin{bmatrix}0.6040 & -0.8878\\ -0.1693 & 0.0722\end{bmatrix},\\
D_{f1}&=\begin{bmatrix}-0.0531 & 0.1305\\ -0.0539 & 0.0444\end{bmatrix},\\
D_{f2}&=\begin{bmatrix}0.1493 & -0.0561\\ -0.0109 & -0.0027\end{bmatrix}.
\end{aligned} \tag{37}$$

Next, we apply the fuzzy filter (5) to the given T-S fuzzy system with interval time-varying delay and obtain the simulation results as Figures 1–3, where the disturbance input $w(t)$ is given as $w(t)=1/(2+5t^2+t), t\geq 0$. Figure 1 shows the state response $x(t)$ under the initial condition $\phi(t)=[0,0]^T, t\in[-0.5,0]$. Figure 2 shows the filter state response $x_f(t)$. Figure 3 shows the error response $e(t):=z(t)-z_f(t)$. From these simulation results, it can be seen that the designed nonfragile robust fuzzy H_∞ filter satisfies the specified performance requirement. Moreover, when there is no external disturbance (i.e., $w(t)=0$), the state response $x(t)$ is shown in Figure 4 under initial condition $\phi(t)=[-0.5,0.5]^T, t\in[-0.5,0]$. It is also clear that the system (6) with $w(t)=0$ is stable.

In order to further show the advantage of our method, the following example is considered.

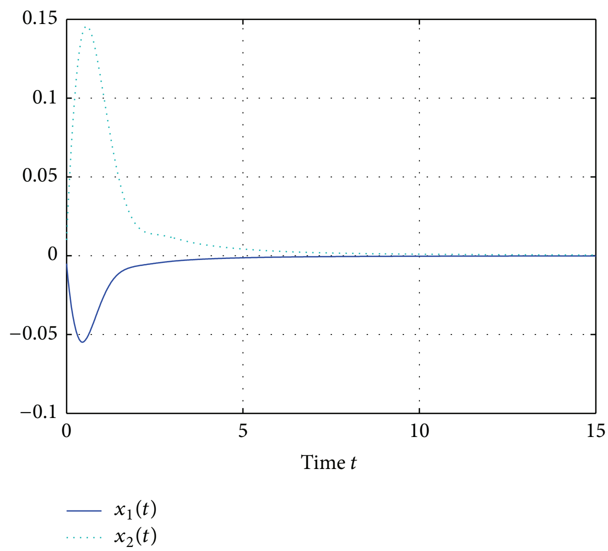

FIGURE 1: Response of the state $x(t)$.

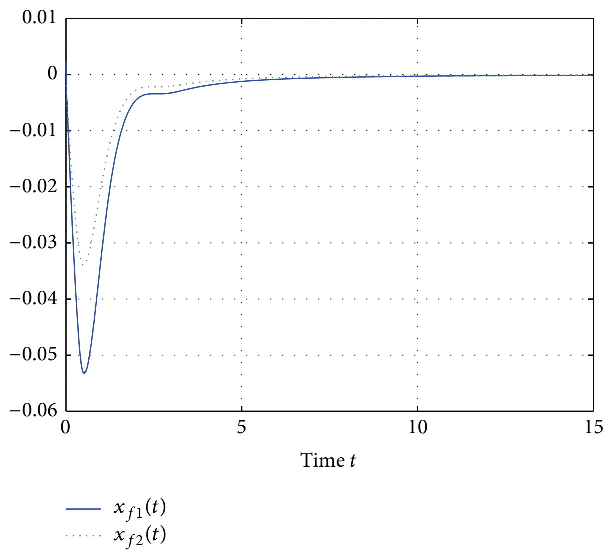

FIGURE 2: Response of the filter state $x_f(t)$.

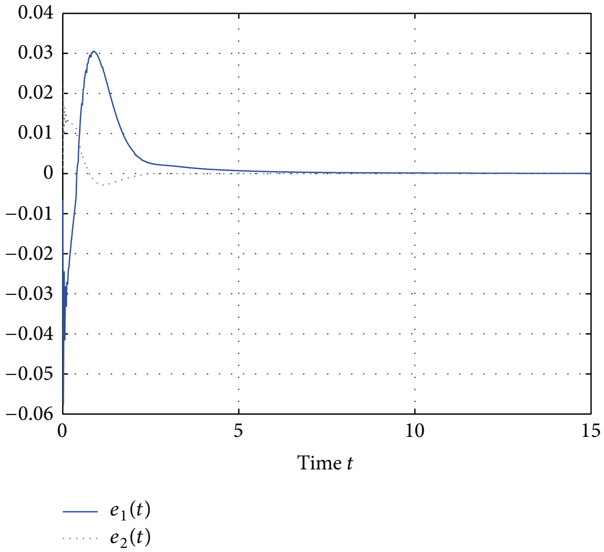

Figure 3: Error response $e(t)$.

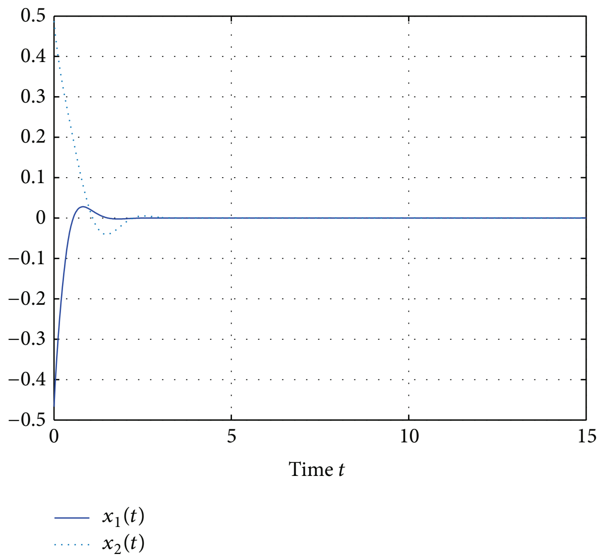

Figure 4: Response of the state $x(t)$ when $w(t) = 0$.

Example 13 (example of [13, 14]). Consider the following fuzzy system without system uncertainties, whose parameters are

$$A_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -0.9 & 0 \\ 0 & -0.5 & -1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -0.9 & 0.2 & 0 \\ -0.2 & -0.5 & 0 \\ 0 & -0.1 & -0.8 \end{bmatrix},$$
$$A_{\tau 1} = \begin{bmatrix} -0.8 & 0.2 & -0.1 \\ 0.1 & -0.8 & 0 \\ -0.4 & 0.25 & -1 \end{bmatrix},$$
$$A_{\tau 2} = \begin{bmatrix} -1 & 0.5 & 0.1 \\ 0.5 & -1 & 0 \\ -0.8 & 0.9 & -0.25 \end{bmatrix},$$
$$C_1 = [0.5 \;\; 0.4 \;\; 0], \qquad C_2 = [0.5 \;\; -1 \;\; 0],$$
$$C_{\tau 1} = [1 \;\; -0.5 \;\; 0.5], \qquad C_{\tau 2} = [1 \;\; 0.1 \;\; -0.5],$$
$$L_1 = [0.5 \;\; 0 \;\; 0], \qquad L_2 = [1 \;\; -0.5 \;\; 0],$$
$$L_{\tau 1} = [0.1 \;\; 0.5 \;\; 0.5], \qquad L_{\tau 2} = [0.1 \;\; 0 \;\; 0.5],$$
$$B = [0 \;\; 0 \;\; 0.5]^T, \qquad D = 0.25, \qquad G = 0,$$
$$h_1 = \left(1 - \frac{1}{1 + e^{(-6x_2 + 1.5\pi)}}\right)\left(\frac{1}{1 + e^{(-6x_2 - 1.5\pi)}}\right),$$
$$h_2 = 1 - h_1. \tag{38}$$

In the implementation of the nonfragile fuzzy filter, we consider that the filter gain perturbations have

$$\begin{aligned} D_{41} &= [-0.5 \;\; 0.1 \;\; 0]^T, & D_{51} &= 0.1, \\ D_{42} &= [0.5 \;\; 1.0 \;\; 0]^T, & D_{52} &= -0.1, \\ E_{41} &= [0 \;\; -0.4 \;\; 0], & E_{51} &= 0, \\ E_{42} &= [0 \;\; -0.4 \;\; 0], & E_{52} &= 0, \\ E_{61} &= [0 \;\; -0.4 \;\; 0], & E_{71} &= 0, \\ E_{62} &= [0 \;\; -0.4 \;\; 0], & E_{72} &= 0. \end{aligned} \tag{39}$$

We assume that the time delay is $\tau(t) = 0.3 + 0.25\cos(t)$; that is, $h_a = 0.05, h_b = 0.55, h_d = 0.25$. From this fuzzy system, by using the Matlab LMI control Toolbox to solve LMIs in (34) of Corollary 9, we can calculate the optimal performance level $\gamma_{\min} = 0.31$. And the filter matrices can be obtained as follows:

$$A_{f1} = \begin{bmatrix} -2.0606 & 0.8132 & -4.7777 \\ -7.0837 & -15.6430 & 13.9274 \\ 0.3648 & 1.4454 & -11.4656 \end{bmatrix},$$
$$B_{f1} = \begin{bmatrix} 0.0766 \\ -0.3242 \\ -0.3563 \end{bmatrix}, \qquad C_{f1} = \begin{bmatrix} -1.3364 \\ -1.9659 \\ -1.8168 \end{bmatrix}^T,$$
$$D_{f1} = -0.0093;$$

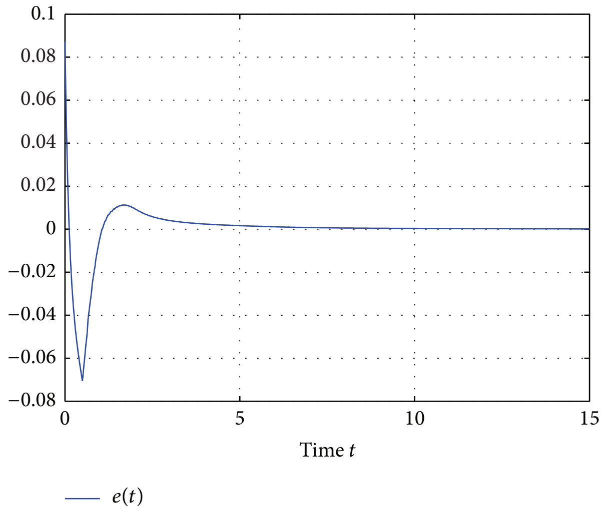

FIGURE 5: Error response $e(t)$.

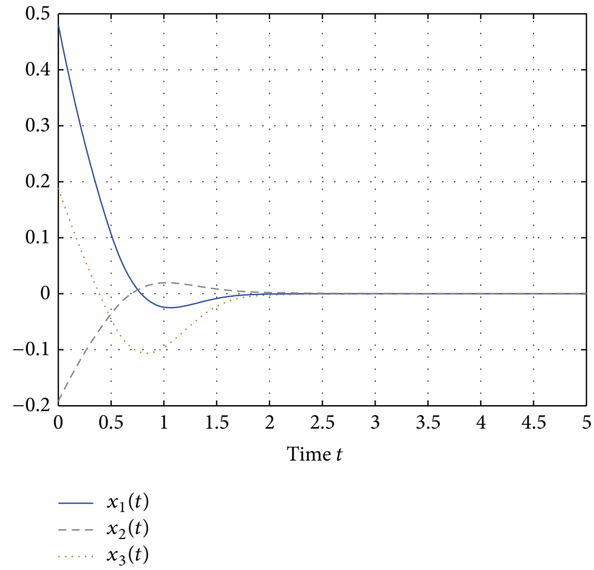

FIGURE 6: Response of the state $x(t)$ when $w(t) = 0$.

$$A_{f2} = \begin{bmatrix} -4.8970 & -0.5771 & 4.6164 \\ 0.9201 & -2.8290 & 0.8770 \\ 0.9644 & 1.8330 & -5.8828 \end{bmatrix},$$
$$B_{f2} = \begin{bmatrix} 0.3196 \\ -0.2579 \\ -0.2458 \end{bmatrix}, \qquad C_{f2} = \begin{bmatrix} 0.0127 \\ 0.9834 \\ -4.9001 \end{bmatrix}^T,$$
$$D_{f2} = -0.1073. \tag{40}$$

In order to illustrate the importance of the proposed nonfragile fuzzy filter design method, we give a contrastive analysis based on the example. Take Corollary 9 for example, the fuzzy filter consisting of (40) is nonfragile; that is, when the filter has gain perturbations, the optimal performance level $\gamma_{\min} = 0.31$ is always guaranteed for any filter gain variant as (39). Based on this filter, we can obtain the simulation results of signal error $e(t) := z(t) - z_f(t)$ as Figure 5 where the disturbance input $w(t)$ is given as $w(t) = 1/(1 + 2t^2 + 3t), t \geq 0$ and filter gain variant is assumed as $F_{2j}(t) = \sin(t), F_{3j}(t) = \cos(t), (j = 1, 2)$. From Figure 5 under initial condition $\phi(t) = [0.2, -0.2, 0.1]^T, t \in [-0.55, 0]$, it can be seen that the designed nonfragile fuzzy H_∞ filter with filter perturbations in (39) can stabilize the system (38). Moreover, when there is no external disturbance (i.e., $w(t) = 0$), the state response $x(t)$ is shown in Figure 6 under initial condition $\phi(t) = [0.5, -0.2, 0.2]^T, t \in [-0.55, 0]$. It is also clear that the system (38) with $w(t) = 0$ is asymptotically stable.

Correspondingly, for the system (38), by Remark 10 with the H_∞ performance level $\gamma = 0.30$, we can obtain the following standard fuzzy filter matrices as follows:

$$A_{f1} = \begin{bmatrix} -2.9808 & 0.5070 & -1.7769 \\ 0.3429 & -3.6529 & 0.4105 \\ -0.8253 & 0.1616 & -6.1823 \end{bmatrix},$$

TABLE 1: Comparison with minimum performance level γ for various h_b ($h_d = 0.2$).

Method	$h_b = 0.5$	$h_b = 0.6$	$h_b = 0.8$
Lin et al. [9]	0.35–0.38	0.38–0.44	0.44–0.63
Su et al. [13]	0.24–0.26	0.27–0.31	0.31–0.45
Huang et al. [14]	0.24–0.26	0.27-0.28	0.31–0.33
An et al. [12]	0.218	0.241	0.301
Remark 10 ($\alpha = 0.7$ and $h_a = 0$)	0.203	0.229	0.287
Corollary 9 ($\alpha = 0.7$ and $h_a = 0$)	0.257	0.291	0.330

TABLE 2: Comparison with maximum values on h_b for various γ ($h_d = 0.2$).

Method	$\gamma = 0.3$	$\gamma = 0.4$	$\gamma = 0.5$
Lin et al. [9]	0.12–0.33	0.52–0.70	0.67–0.97
Su et al. [13]	0.59–0.78	0.70–1.08	0.70–1.13
Huang et al. [14]	0.72–0.78	1.00–1.09	1.06–1.17
An et al. [12]	0.80	1.16	1.19
Remark 10 ($\alpha = 0.65$ and $h_a = 0$)	0.82	1.19	1.23
Corollary 9 ($\alpha = 0.65$ and $h_a = 0$)	0.67	0.93	1.07

$$B_{f1} = \begin{bmatrix} 2.8762 \\ -0.3514 \\ -1.2524 \end{bmatrix}, \qquad C_{f1} = \begin{bmatrix} -0.0429 \\ 0.0231 \\ -1.4800 \end{bmatrix}^T,$$
$$D_{f1} = -0.2639;$$
$$A_{f2} = \begin{bmatrix} -3.0616 & 0.1748 & -5.0208 \\ 0.4286 & -2.6990 & 1.9728 \\ -0.1665 & 0.1401 & -1.3033 \end{bmatrix},$$

Table 3: Comparison with maximum values on h_b for various h_a (h_d unknown).

Method\h_a	0.4	0.8	1.0	1.2
Peng et al. [35]	1.0183	1.1817	1.2776	1.3816
Peng and Han [36]	1.1800	1.3100	1.3700	1.4300
Tian et al. [37]	1.2647	1.3032	1.3528	1.4214
An and Wen [38]	1.2770	1.3110	1.3580	1.4190
Peng and Fei [39]	1.3200	1.3200	1.3800	1.4200
Souza et al. [40]	1.2836	1.3394	1.4009	1.4815
Theorem 4 ($\alpha = 0.7$)	1.5274	1.5361	1.5762	1.6340
Theorem 4 ($\alpha = 0.8$, $h_d = 0.3$)	1.8791	1.8583	1.8290	1.7864

$$B_{f2} = \begin{bmatrix} 2.4661 \\ -0.2198 \\ -0.8735 \end{bmatrix}, \qquad C_{f2} = \begin{bmatrix} -0.4849 \\ 0.1575 \\ -2.1071 \end{bmatrix}^T, \qquad D_{f2} = -0.1058. \tag{41}$$

Now, we assume that the filter (41) has gain perturbations using (39) and $F_{2j}(t) = F_{3j}(t) = 0.8$. In this case, by Theorem 4, the minimum H_∞ performance $\gamma_{\min} = 0.56$ is obtained, which implies that the H_∞ performance $\gamma = 0.30$ cannot guarantee that the corresponding matrix inequalities in (12) are feasible. Meanwhile, in the case of no filter gain perturbations, by Remark 10, we can calculate the optimal performance level $\gamma_{\min} = 0.251$.

What is more, based on this example, to compare with the existing ones in [9, 12–14], we assumed that $h_a = 0$ and $h_d = 0.2$. According to Remark 10 and Corollary 9, we compare the minimum H_∞ performance level γ for the given different h_b and k_1, k_2, δ in [9, 13, 14] with any k_1, k_2, δ in this paper, which is shown by Tables 1 and 2. From these simulation results, our approach yields less conservative than the existing results.

Furthermore, we will give another example to show the effectiveness and merit of nonlinear system via T-S fuzzy models.

Example 14 (Example 1 of [39]). Consider the following time-delayed nonlinear system:

$$\begin{aligned} \dot{x}_1(t) &= 0.5\left(1 - \sin^2(\theta(t))\right)x_2(t) - x_1(t - \tau(t)) \\ &\quad - \left(1 + \sin^2(\theta(t))\right)x_1(t), \\ \dot{x}_2(t) &= \operatorname{sgn}\left(|\theta(t)| - \frac{\pi}{2}\right)\left(0.9\cos^2(\theta(t)) - 1\right)x_1(t - \tau(t)) \\ &\quad - x_2(t - \tau(t)) - \left(0.9 + 0.1\cos^2(\theta(t))\right)x_2(t) \end{aligned} \tag{42}$$

which can be exactly expressed as a nominal T-S delayed system (5) with the following rules [35, 36, 39, 40]:

$$\begin{aligned} &R^1\text{: if } \theta(t) \text{ is } \pm\frac{\pi}{2}, \text{ then } \dot{x}(t) = A_1 x(t) + A_{\tau 1} x(t - \tau(t)), \\ &R^2\text{: if } \theta(t) \text{ is } 0, \text{ then } \dot{x}(t) = A_2 x(t) + A_{\tau 2} x(t - \tau(t)). \end{aligned} \tag{43}$$

The membership functions for above rules 1 and 2 are

$$h_1(x_1(t)) = \sin^2(x_1(t)), \qquad h_2(x_1(t)) = \cos^2(x_1(t)) \tag{44}$$

with the following system parameters:

$$\begin{aligned} A_1 &= \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \qquad A_{\tau 1} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix}, \qquad A_{\tau 2} = \begin{bmatrix} -1 & 0 \\ 0.1 & -1 \end{bmatrix}. \end{aligned} \tag{45}$$

To compare with the existing results, we assume that h_d is unknown. The improvement of this paper is shown in Table 3. If the delay is fast time-varying case, the LMIs in Theorem 4 are feasible with $1.2 \le \tau(t) \le 1.6340$. If the additional information $h_d = 0.3$ is given, larger upper bounds of the delay can be computed by Theorem 4, which is shown at the last row of Table 3. From Table 3, it also can be seen that the proposed method yields less conservative than the existing ones.

5. Conclusion

This paper deals with the robust nonfragile fuzzy H_∞ filter design problem for uncertain T-S fuzzy systems with interval time-varying delays. An LMI approach has been developed by introducing a new delay decomposition method and then the sufficient condition for the existence of the nonfragile fuzzy H_∞ filter has been given in terms of LMIs. It has been shown that the designed filter guarantees not only the robust stability but also a prescribed H_∞ performance level of the fuzzy H_∞ filtering error system for all admissible uncertainties. Three numerical examples are utilized to illustrate the effectiveness and reduced conservatism of the proposed method.

Appendix

Proof of Proposition 3. Motivated by [30, 43], we can rewrite the second inequality of Proposition 3 as

$$\begin{bmatrix} \Sigma_3 \\ \Sigma_1 \end{bmatrix}^T \begin{bmatrix} \Sigma_0 & \Sigma_2 \\ \Sigma_2^T & 0 \end{bmatrix} \begin{bmatrix} \Sigma_3 \\ \Sigma_1 \end{bmatrix} < 0, \tag{A.1}$$

where

$$\begin{aligned} \Sigma_1 &= \begin{bmatrix} -I & A & A_\tau & 0 & 0 & 0 & 0 & B & 0 \end{bmatrix}, \\ \Sigma_2^T &= \begin{bmatrix} U^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ \Sigma_3 &= \operatorname{diag}\{I \ \ I \ \ I \ \ I \ \ I \ \ I \ \ I \ \ I \ \ I\}, \\ \Sigma_0 &= \begin{bmatrix} P_1 & P_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & D_1 & 0 & D_2 & 0 & 0 & 0 & 0 & X_1 \\ * & * & D_3 & 0 & D_4 & D_5 & D_6 & 0 & X_2 \\ * & * & * & D_7 & D_8 & 0 & 0 & 0 & X_3 \\ * & * & * & * & D_8 & D_9 & 0 & 0 & X_4 \\ * & * & * & * & * & D_{10} & D_{12} & 0 & X_5 \\ * & * & * & * & * & * & D_{13} & 0 & X_6 \\ * & * & * & * & * & * & * & D_{14} & X_7 \\ * & * & * & * & * & * & * & * & X_8 \end{bmatrix}. \end{aligned} \tag{A.2}$$

Then, we choose the orthogonal complement of Σ_1 as

$$\Sigma_{1\perp} = \begin{bmatrix} A & A_\tau & 0 & 0 & 0 & 0 & B & 0 \\ I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}, \quad \text{(A.3)}$$

which satisfies $\Sigma_1 \Sigma_{1\perp} = 0$. Moreover, $\begin{bmatrix} \Sigma_1^T & \Sigma_{1\perp} \end{bmatrix}$ is of column full rank. Then, it follows that (A.1) is equivalent to the following matrix inequality:

$$\Sigma_{1\perp}^T \begin{bmatrix} \Sigma_3 \\ \Sigma_1 \end{bmatrix}^T \begin{bmatrix} \Sigma_0 & \Sigma_2 \\ \Sigma_2^T & 0 \end{bmatrix} \begin{bmatrix} \Sigma_3 \\ \Sigma_1 \end{bmatrix} \Sigma_{1\perp} < 0 \quad \text{(A.4)}$$

which can be further reduced to

$$\Sigma_{1\perp}^T \Sigma_0 \Sigma_{1\perp} < 0. \quad \text{(A.5)}$$

Thus, we have shown that the second inequality of Proposition 3 is equivalent to (A.5).

It is also easily seen that the first matrix inequality of Proposition 3 can be rewritten as (A.5).

This completes the proof. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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